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SELECTED MATHEMATICAL REVIEWS

A selection of Mathematical Reviews concerning editions of Euler's works

MR0590427 (82c:01069) 01A75 (76-03)

Eulerus, Leonhardus [Euler, Leonhard]

Commentationes mechanicae ad theoriam machinarum pertinentes. (Latin) [Commentaries on mechanics pertaining to the theory of machines]

Edited and with a preface by Charles Blanc and Pierre de Haller.
Leonhardi Euleri Opera Omnia, Series Secunda: Opera Mechanica et Astronomica, XVI.
Orell Füssli, *Zurich*, 1979. xviii+327 *pp*. $70.65.

This volume, a sequel to *Opera Omnia, Series Secunda, Vol. XV* [1957, MR0090534 (19,826i)], contains nine papers in French, Latin, or German by Leonhard Euler and his son Johann Albrecht, from the field of hydrodynamics. More particularly, there are four papers on windmills and kites, one on balloons, one on the construction of a manometer, one on the motion of the blood in the arteries, one on the Archimedean screw and one on the improvement of the water wheel.

The starting point for Euler's theory of the action of the wind or a fluid upon a plane is Newton's sine-square law: $N = S\rho v^2 \sin^2\alpha$ (S = area of the plane, ρ = density of the fluid, v = velocity, α = angle of inclination). Although well aware that especially for fluids this law does not correctly describe the situation (Newton himself had doubts that it is applicable to fluids other than air), Euler did not hesitate to make use of it in connection with stream fillets, to which he applied Bernoulli's equation. He had in fact, in his translation of Robin's *New principles of gunnery*, in 1745 given a proof that a steady flow of a fluid past an obstacle exerts no force on that obstacle (usually called d'Alembert's paradox). Due to the fact that some of his basic hypotheses did not represent the actual reality sufficiently exactly, Euler's studies of windmills did not lead to practical consequences. It was only in the 19th and 20th century that the mathematical tools for successfully handling such problems have been worked out. At the end of his very systematic paper on the motion of the fluid in the Archimedean screw, Euler clearly pinpoints the difficulty: the dynamical investigations lead to equations whose solution still lies beyond the limits of mathematical analysis.

The paper on balloons is not only the last memoir written by the great mathematician (published in 1784) but was of pressing importance, too, at the time of the Montgolfières. Euler's solution of the differential equation derived by him gave a good approximation for the first phase of ascension, though for higher altitudes it was not satisfactory.

Published for the first time in complete form in this volume is Euler's investigation of the motion of blood—considered by him as the motion of a liquid through an elastic tube. Again he pushed his dynamical studies to a point where the mathematical difficulties became insurmountable.

Among the three papers by Johann Albrecht Euler is a long memoir summarizing his father's results about water mills and hydraulic machines, and one about kites which at that time were used for experiments with atmospheric electricity.

In a preface of about ten pages the editors have outlined the main contents of the papers published in this volume, and have given numerous references to related papers and to introductions of other volumes in this series.

From MathSciNet, June 2007

C. J. Scriba

MR0595168 (82h:01075) 01A75 (01A5 10-03 26-03 35-03 41-01)

Eulerus, Leonhardus [Euler, Leonhard]

Commercium epistolicum (French) [Correspondence].

Correspondence with A. C. Clairaut, J. d'Alembert, and J. L. Lagrange.
Edited and with an introduction by Adolf P. Juškevič [A. P. Yushkevich] and René Taton. With the assistance of Charles Blanc, Ašot T. Grigorijan [A. T. Grigor′jan], Walter Habicht and Guy Picolet.
Leonhardi Euleri Opera Omnia, Series Quarta A: Commercium Epistolicum, V.
Birkhäuser Verlag, Basel-Boston, *Mass.*, 1980. viii+611 *pp*. $100.00.
ISBN 3-7643-0868-0

This is the second volume of the new, fourth series of Euler's *Opera omnia* to be published (for the initial volume, IV A 1, containing summaries of the whole correspondence, see MR0497632 (58 #15920)). For mathematicians it surely must be one of the most interesting volumes in this sequence of Euler's correspondences, so that we should be grateful that this volume IV A 5 precedes all others. Here we have four of the great masters of the 18th century, exchanging their thoughts on problems from pure and applied mathematics and related fields. (With Laplace, Euler seems to have had no correspondence at all, but for one letter from Laplace.)

The Euler-Clairaut correspondence, beginning in 1740 when Euler was 33 and Clairaut 27 years of age, consists of 60 letters (only 15 by Euler—more have not been discovered). Until 1744 the main topics were problems from mathematics (integration of differential forms, properties of homogeneous functions, some number theory) and from fluid mechanics. Clairaut in 1736/37 had accompanied Maupertuis on his expedition to Lapland for determining the shape of the earth, and was now working at the theory of figures of equilibrium of a spherical fluid and writing his book *Théorie de la figure de la terre*, published in 1743. From 1744 to 1752 much space is devoted to the theory of the Moon and other questions from celestial mechanics. For a while the opinion was predominant that Newton's law of gravitation would have to be modified for small distances, before Clairaut succeeded in deducing the lunar theory solely from Newton's law; this filled Euler with "une satisfaction infinie". Another success of Clairaut was the prediction of the return of Halley's comet in 1759, definitely establishing that the comets of 1531, 1607 and 1759 all were identical with Halley's of 1682.

The Euler-d'Alembert correspondence, consisting of 40 letters (of which 14 were written by Euler), covers the years from 1746–1751 and 1763–1773. During the first period Euler was somewhat reserved towards d'Alembert, who was favoured by

King Frederick II, but after the Frenchman's visit to Berlin in 1763 their relations became rather friendly. In their scientific studies, though, differences of opinion were not infrequent. In the matter of logarithms of negative numbers, the Leibniz-Bernoulli controversy was revived; in lunar theory d'Alembert in contrast to Euler never felt a need to modify Newton's law; in the theory of the precession of the equinoxes and the nutation of the axis of the Earth, Euler and d'Alembert quarrelled about priorities. And last but not least there were the disputes about admissible solutions to the differential equation of the vibrating string, actually rooted in the question whether nondifferentiable or discontinuous functions are to be admitted in mathematical physics.

The correspondence between Euler and Lagrange stretches from 1754 to 1775. There are 18 letters by Euler and 19 by Lagrange. While both shared a common widespread interest that ranged from all kinds of applications of mathematics to its purest parts, including number theory, their mathematical styles were totally opposed: Euler's prolixity contrasted sharply to Lagrange's concise way of writing; where Euler illustrated his theories by concrete examples, Lagrange's utmost aim was generality and systematic exposition. Nevertheless, both valued highly each other's works and often felt inspired to extend or complement the newest research of the partner. As for topics, variational calculus, minimal surfaces and related problems are discussed at the beginning, Diophantine equations and other problems from number theory towards the end (about 1770), the vibrating string, the three-body problem and the propagation of plane and spherical waves in between.

In an introduction (in French) of 63 pages the editors give a very useful survey of the three correspondences, and in numerous detailed notes to each letter they analyse the mathematical contents and give cross-references within the letters as well as to all books and papers mentioned or referred to by the correspondents. They also point out errors in the traditional historical accounts and provide new insights into the closely knit network of international scientific relations in the 18th century. Supplied with three appendices and half a dozen indexes, this edition is an example of scholarship in the history of mathematics. Adorned by four portraits and seven facsimiles and beautifully printed, it is also an example of craftsmanship that is becoming ever more rare.

From MathSciNet, June 2007

C. J. Scriba

MR0715928 (85d:01030) 01A75 (01A50)

Euler, Leonhard

Einleitung in die Analysis des Unendlichen. Teil 1. (German)

[Introduction to the analysis of the infinite. Part I]

Reprint of the 1885 edition.

With an introduction by Wolfgang Walter.

Springer-Verlag, *Berlin*, 1983. 21+xi+319 *pp*. \$20.70. *ISBN* 3-540-12218-4

On 4 July 1744 Euler wrote to Christian Goldbach: "I have meanwhile finished a new book entitled 'Introductio in analysin infinitorum', wherein I treat the higher parts of algebra and geometry. There I solve a large number of difficult problems without applying infinitesimal calculus—almost nothing of this can be found anywhere else. After I had made a plan for a complete treatise on infinitesimal analysis I noticed that many things ought to be presented first which do not actually belong there and which are not dealt with anywhere; from these topics the present book has arisen as a preparation for the study of infinitesimal analysis."

Euler divided this Introductio into two parts. The first one (whose German translation is the reprint under review) contains what Euler considered to be prerequisites for the study of infinitesimal calculus. (The second part deals with the theory of curves and surfaces, as preparation to the applications of the calculus to geometry. Maser's German translation comprises only the first part.) Divided into 18 chapters, Part I of the Introductio presents the following topics: functions, their classification, transformations and development into infinite series; exponential functions, logarithms, and circular (trigonometric) functions and their series; infinite products, summation, recurrent series; angular sections; applications to the partition of numbers; continued fractions.

Thus it is the concept of functions (here still defined as analytical expression) that with Euler attains the central position in higher mathematics. The rules of common algebra are extended to infinite series and infinite products, problems of convergence are hardly mentioned, and where we would apply a limiting process, Euler is operating with infinitely small or infinitely large numbers. But although these foundational problems were only settled satisfactorily during the 19th century, the Introductio is a fascinating book to read. One of its highlights is the derivation of $\sum_1^\infty 1/n^2 = \pi^2/6$ and, more generally, the determination of the values of the function $\zeta(n)$ for even values of n; another is the introduction of generating functions into additive number theory in connection with problems of the partition of numbers.

The reprint of this German translation (first published by Julius Springer, Berlin, 1885) is equipped with a new introduction by W. Walter , briefly summarizing Euler's life and the contents of the first part of the Introductio. (The date of Halley's publication cited on p. 16 of this introduction should read 1695.)

The Latin original, first published in 1748 in Lausanne by M.-M. Bousquet and edited again in 1783 and 1797, was republished within Leonhardi Euleri Opera Omnia, Series prima, volumen octavum, in 1922 (B. G. Teubner, Leipzig and Berlin). There are two French translations (1786 and 1796), another German version (by J. A. C. Michelsen, 1788), and a Russian translation (1961), but—as far as I know—no translation into the English language.

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C. J. Scriba

MR0759160 (86k:01049) 01A75 (30-03)

Euler, Leonhard

Zur Theorie komplexer Funktionen. (German) [On the theory of complex functions]

Edited by H. Wussing.
With an introduction and notes by A. P. Juschkewitsch.
Translated from the Latin and French by W. Purkert, H. Maser, H. Müller, R. Thiele, O. Neumann, A. Wangerin and E. Schuhmann.

Ostwalds Klassiker der Exakten Wissenschaften [Ostwald's Classics of the Exact Sciences], 261.
Akademische Verlagsgesellschaft Geest & Portig K.-G., Leipzig, 1983. 263 *pp.* MDN 34.00.

The core of this collection consists of seven publications (or excerpts of them), translated into German, on the theory of complex functions, which were written by Leonhard Euler between 1744 and 1783 (the year of his death). Part I deals with the derivation of the formulas of de Moivre and Euler as given in Sections 132, 133, 138–140 of the *Introductio in analysin infinitorum* (Lausanne 1748, reprint in *Leonhardi Euleri Opera Omnia, Series Prima*, Vol. 9 (=EO I, **9**); Eneström-No. 101 (=E 101). In part II one finds the essay, originally written in French, "On the controversy between Leibniz and Bernoulli regarding the logarithms of negative and imaginary numbers" (EO I, **17**, 195–232; E 168), in which Euler finally clarifies the problems connected with the multivaluedness of the logarithm. The "Reflections on orthogonal trajectories", reproduced in part III, were presented by Euler to the St. Petersburg Academy in 1768; they mark the beginnings of the theory of conformal mappings (EO I, **28**, 99–119; E 390). The paper "On the mapping of a spherical surface onto a plane" (EO I, **28**, 248–275; E 490) evolved in 1775 in connection with the making of good maps of the Russian empire; in it Euler establishes, among other things, the conditions for angle- and area-preserving mappings. The last three papers deal with integral theory in the complex domain. In parts V (EO I, **19**, 1–44; E 656) and VI (EO I, **19**, 268–286; E 694)—both from the year 1777—Euler starts from the assumption that the complex functions under consideration, together with their argument, are transformed into the complex conjugate value; he shows the validity of the Cauchy-Riemann differential equations for the real and imaginary part of an analytic function (with real values on the real axis), and he discusses the integrals

$$\int \frac{z^{m-1}dz}{1+z^n} \qquad \text{and} \qquad \int \frac{z^{m-1}dz}{(a+bz^n)^\lambda}.$$

Finally, in part VII containing "On the values of the integrals extending from $x = 0$ to $x = \infty$" (EO I, **19**, 217–227; E 675; presented to the St. Petersburg Academy in 1781 and 1783), the two so-called improper Fresnel integrals

$$\int_0^\infty \frac{\cos x\,dx}{\sqrt{x}} = \int_0^\infty \frac{\sin x\,dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2}}$$

and the integral $\int_0^\infty x^{-1}\sin x\,dx = \pi/2$ are computed with the help of complex substitutions; on the way to this (as earlier, 1765, and E 368) Euler represents the gamma function by the integral $\int_0^\infty x^{n-1}e^{-x}\,dx$.

These seven papers of Euler are commented on in detail in an appendix of about 20 pages. The footnotes, aside from factual explanations, contain many bibliographical hints. But what really makes this collection so valuable is the introduction of 40 pages by A. P. Yushkevich. In it the author gives a survey on Euler's life and work, on the evolution of the theory of complex numbers up to Euler and on Euler's contributions to the theory of analytic functions (including those which are not emphasized in this collection). For this the reader will be especially grateful (as he will for the use of modern notation in the translations, the latter being mostly of recent date), since it will enable him to appreciate Euler's achievements in a historical context.

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C. J. Scriba