

Solution of the problem of Plateau

# BULLETIN

(NEW SERIES)

of the

AMERICAN MATHEMATICAL SOCIETY



Given a contour  $P$  in euclidean space of  $n$  dimensions; to <sup>prove</sup> demonstrate the existence of a minimal surface  $M$  bounded by  $P$ , and at the same time an informal representation of  $M$  on the interior of a circle  $C$ .

This formulation differs from the usual one in two respects.

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Susan Friedlander  
Chief Editor

Book Reviews

Robert L. Devaney

1° The number of dimensions of the containing euclidean space is an arbitrary integer  $n$ . Indeed, as <sup>particular</sup> the value of  $n$  is of no essential importance, and to assume a special value for  $n$  would produce no simplification either in method or result.

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2° We demand not merely a <sup>minimal</sup> surface  $M$  bounded by  $P$  but also a <sup>in add.</sup> conformal

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