

Solution of the problem of Plateau

# BULLETIN

(NEW SERIES)

of the

AMERICAN MATHEMATICAL SOCIETY



Given a contour  $P$  in euclidean space of  $n$  dimensions; to <sup>prove</sup> demonstrate the existence of a minimal surface  $M$  bounded by  $P$ , and at the same time an informal representation of  $M$  on the interior of a circle  $C$ .

This formulation differs from the usual one in two respects.

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Susan Friedlander  
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Book Reviews

Robert L. Devaney

1° The number of dimensions of the containing euclidean space is an arbitrary integer  $n$ . Indeed, as <sup>particular</sup> the value of  $n$  is of no essential importance, and to assume a special value for  $n$  would produce no simplification either in method or result.

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2° We demand not merely a <sup>minimal</sup> surface  $M$  bounded by  $P$  but also a <sup>in add.</sup> conformal

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BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY

CONTENTS

Vol. 45, No. 2

April 2008

ARTICLES

Barry Mazur, <a href="#">Finding meaning in error terms</a> .....	185
George A. Elliott and Andrew S. Toms, <a href="#">Regularity properties in the classification program for separable amenable C*-algebras</a> .....	229
J. M. Landsberg, <a href="#">Geometry and the complexity of matrix multiplication</a> .....	247
A. Stoimenow, <a href="#">Tait's conjectures and odd crossing number amphicheiral knots</a> .....	285

MATHEMATICAL PERSPECTIVES

Jeremy Gray and Mario Micalef, <a href="#">About the cover: The work of Jesse Douglas on minimal surfaces</a> .....	293
<a href="#">Selected Mathematical Reviews</a> .....	303

BOOK REVIEWS

John R. Harper (Reviewer), <a href="#">Generalized cohomology</a> , by Akira Kono and Dai Tamaki .....	309
Mark A. Ronan (Reviewer), <a href="#">Quadrangular algebras</a> , by Richard M. Weiss .....	315
John B. Garnett (Reviewer), <a href="#">Harmonic measure: Geometric and analytic points of view</a> , by Luca Capogna, Carlos E. Kenig, and Loredana Lanzani .....	321
Yulij Ilyashenko (Reviewer), <a href="#">The monodromy group</a> , by H. Zoladek .....	329
Leonid Friedlander (Reviewer), <a href="#">Extremum problems for eigenvalues of elliptic operators</a> , by Antoine Henrot .....	335

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## BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY

## CONTENTS

Vol. 45, No. 2

April 2008

## ARTICLES

- Barry Mazur**, Finding meaning in error terms ..... 185
- George A. Elliott and Andrew S. Toms**, Regularity properties in the classification program for separable amenable  $C^*$ -algebras ..... 229
- J. M. Landsberg**, Geometry and the complexity of matrix multiplication ..... 247
- A. Stoimenow**, Tait's conjectures and odd crossing number amphicheiral knots ..... 285

## MATHEMATICAL PERSPECTIVES

- Jeremy Gray and Mario Micalef**, About the cover: The work of Jesse Douglas on minimal surfaces ..... 293
- Selected Mathematical Reviews ..... 303

## BOOK REVIEWS

- John R. Harper** (Reviewer), Generalized cohomology, by Akira Kono and Dai Tamaki ..... 309
- Mark A. Ronan** (Reviewer), Quadrangular algebras, by Richard M. Weiss ..... 315
- John B. Garnett** (Reviewer), Harmonic measure: Geometric and analytic points of view, by Luca Capogna, Carlos E. Kenig, and Loredana Lanzani ..... 321
- Yulij Ilyashenko** (Reviewer), The monodromy group, by H. Zoladek ..... 329
- Leonid Friedlander** (Reviewer), Extremum problems for eigenvalues of elliptic operators, by Antoine Henrot ..... 335



0273-0979(200804)45:2;1-6