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ABOUT THE COVER: A HIDDEN PRAISE OF MATHEMATICS

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We have all given reasons to show how important pure mathematics is. In particular, I have often heard (and repeated) statements along the lines of "our abstract theories will eventually be useful to build important physical (chemical, biological, ...) theories." But wouldn't it be great if someone like Albert Einstein himself said that?

Well, the news is he did—in fact on the very first page of his main article on the General Theory of Relativity, published in Annalen der Physik, 1916 [1]. But this page got lost in translation....

The year 2005 brought a world celebration for the centennial anniversary of the *Annus Mirabilis* 1905, in which Einstein, at only 26, published four of his most famous papers. Buenos Aires, where I live and teach, was not an exception, and a series of lectures were organized [2]. I was invited to present a public 60-minute lecture, in which my aim was to sketch the evolution of the abstract ideas in geometry that Einstein had at his disposal to describe his brilliant theory of relativity. I am not an expert in the subject, so I spent several months collecting information.

One of my first "moves" was to get the Dover book on physics "The Principle of Relativity" [3]. This inexpensive and great collection of original papers on the special and general theory of relativity by A. Einstein, H.A. Lorentz, H. Minkowski, and H. Weyl, with notes by A. Sommerfeld, is the most common source for Einstein's article [1]. This review paper presents and explains for the first time the whole theory of relativity in a consistent, systematic and comprehensive way.

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I am grateful to David Eisenbud for listening to my story and suggesting to write it for the Bulletin. I also thank my friends Michael Singer, Daniel Arias and Adrián Paenza for their thorough reading of the first version of this note.

Some time later I discovered the fantastic online archives [4] containing the original manuscripts that Einstein donated to the Hebrew University of Jerusalem. There, I found the jewel which is featured as the cover of this issue of the Bulletin of the American Mathematical Society and which follows this article: a facsimile of Einstein's handwritten first page of his breakthrough paper, which, as is said in [5], "is arguably the most valuable Einstein manuscript in existence."

I do not speak German, but I could of course easily recognize on this handwritten page the names of several mathematicians plus five occurrences of words starting with "Mathemat...". I was amazed since I did not remember anything similar in the popular Dover book. Then, I realized that the Albert Einstein archives also offer an online English translation (extracted from [6]), which includes this first page and acknowledges the fact that it was missing in the circulating English version.

These are Albert Einstein's words on the first page of his most important paper on the theory of relativity:

> The theory which is presented in the following pages conceivably constitutes the farthest-reaching generalization of a theory which, today, is generally called the "theory of relativity"; I will call the latter one—in order to distinguish it from the first named—the "special theory of relativity," which I assume to be known. The generalization of the theory of relativity has been facilitated considerably by Minkowski, a mathematician who was the first one to recognize the formal equivalence of space coordinates and the time coordinate, and utililzed this in the construction of the theory. The mathematical tools that are necessary for general relativity were readily available in the "absolute differential calculus," which is based upon the research on non-Euclidean manifolds by Gauss, Riemann, and Christoffel, and which has been systematized by Ricci and Levi-Civita and has already been applied to problems of theoretical physics. In section B of the present paper I developed all the necessary mathematical tools—which cannot be assumed to be known to every physicist—and I tried to do it in as simple and transparent a manner as possible, so that a special study of the mathematical literature is not required for the understanding of the present paper. Finally, I want to acknowledge gratefully my friend, the mathematician Grossmann, whose help not only saved me the effort of studying the pertinent mathematical literature, but who also helped me in my search for the field equations of gravitation.

So, indeed, he was not only paying homage to the work of the differential geometers who had built the geometry theories he used as the basic material for his general physical theory, but he also acknowledged H. Minkowski's idea of a four dimensional "world", with space and time coordinates. In fact, Einstein is even more clear in his recognition of the work of Gauss, Riemann, Levi-Civita and Christoffel in [7], where one could, for instance, read, "Thus it is that mathematicians long ago solved the formal problems to which we are led by the general postulate of relativity."

After Einstein's theory of special relativity was published in 1905, several mathematicians—including D. Hilbert and F. Klein—became very interested in his developments. While spacetime can be viewed as a consequence of Einstein's theory, it

was first explicitly proposed mathematically by H. Minkowski [8]. Minkowski delivered a famous lecture at the University of Köln on 21 September 1908, in which he showed how the theory of special relativity follows naturally from just a simple fundamental hypothesis about the metric in spacetime. It is interesting to note that Einstein's first reaction had been to qualify this contribution as a "superfluous learnedness", and to critically assert that "since the mathematicians have attacked the relativity theory, I myself no longer understand it any more" [9]. This opinion changed soon, in part due to his mathematical friend M. Grossmann.

Einstein's acknowledgment of the importance of abstract mathematics is also shown in his reaction to Emmy Noether's paper [10]. In this paper, Noether was able to find a deep connection between differentiable symmetries of the action of a physical system and conservation laws, under the form known as Noether's theorem (see the article [11] by Nina Byers for a full account). It was her work in the theory of invariants which led to formulations for several concepts in the general theory of relativity. When Einstein received Noether's paper he wrote to Hilbert, "Yesterday I received from Miss Noether a very interesting paper on invariant forms. I am impressed that one can comprehend these matters from so general a viewpoint. It would not have done the old guard at Göttingen any harm had they picked up a thing or two from her..." [11]. (Despite Hilbert's and Klein's many efforts, Noether was for many years prevented from having an official position at Göttingen because she was a woman.)

In 1935, Einstein wrote an obituary [12] for Noether, who had emigrated to the U.S. in 1933 following her dismissal from her academic position because she was Jewish. Einstein once again praises Emmy Noether and shows the great appreciation that he came to develop for mathematics. In this obituary he writes, "Within the past few days a distinguished mathematician, Professor Emmy Noether, formerly connected with the University of Göttingen and for the past two years at Bryn Mawr College, died in her fifty-third year. In the judgment of the most competent living mathematicians, Fräulein Noether was the most significant creative mathematical genius thus far produced since the higher education of women began. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians. Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulas are discovered necessary for the deeper penetration into the laws of nature."

Was it a conspiracy?

But, why was the first page by Einstein, containing his ebullient praise of mathematics, missing in the English version? Was it a conspiracy? I don't think so.

The translation was made in fact by the mathematical physicist G.B. Jeffery together with W. Perrett, a lecturer in German. I contacted the expert Tilman Sauer, who published an historical account of Einstein's 1916 paper, where he lists all the existing versions [13].

Sauer offered me the following clarifications: "The original publication of the paper on general relativity appeared in the journal "Annalen der Physik" and was

also reprinted separately by the publisher J.A. Barth. An early English translation by S.N. Bose, University of Calcutta Press, 1920, does include the first page. The existence of this translation was apparently unbeknownst to Einstein.... The first page is also missing in the very popular German collection of papers on relativity edited by Otto Blumenthal and published by Teubner.

A third edition of the Blumenthal collection was planned in 1919, which would include some of Einstein's later papers about general relativity. However, in a letter from 26 February 1919 [4, Nr. 41-991], Barth vetoed republication of the 1916 Annalen paper, since it still had several hundred copies of the separate printing of the paper in stock. On 12 December 1919, Barth retracted his veto since his stock of copies had almost been sold [4, Nr. 41-993] and on 21 December 1919, Einstein sent his only remaining copy of the Annalen version to Teubner for inclusion in the new edition of the Blumenthal collection: "With the same post I am sending you the only copy of my Annalen-paper that I still have. I ask you to use this copy for the printing, since on page 60 and 63 a few minor errors are corrected".... Now, apparently, Teubner sent the offprint that the typesetters had used (the page breaks are different!) back to Einstein, since the latter wrote on 20 April 1920 to Max Born [14]: "My last copy of the requested article is going out to you in the same post. It had been botched up like that at Teubner's press." According to the letters exchanged by Jeffery and Einstein ([15], [4, Nr. 13 424], [4, Nr. 13 426]), this was the German version used for the English translation.

And Sauer concludes, "My conjecture thus is that it was the typesetters from Teubner publishing house who missed to include the first page: maybe Einstein's copy was frail and the first page was (almost?) loose and fell off? It's sort of curious that we seem to have no evidence that Einstein was even aware that Teubner had reprinted his 1916 review paper without the first page."

It's funny to think that only one deteriorated copy of this seminal paper was left, and that this may have been the reason to keep hidden this praise of mathematics and mathematicians.

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