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ABOUT THE COVER: LUCA DA PACIOLI AND LEONARDO'S DRAWINGS OF POLYHEDRA

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Luca da Pacioli (1446/7–1517) was a Franciscan friar and Renaissance polymath, traditionally thought to be a protégé of Piero della Francesca (1420–1492), one of the foremost painters of the Florentine-Umbrian school. There is no doubt that Pacioli was a mentor to Leonardo da Vinci (1452–1520), though there are conflicting views on the relationships and the influence of any of the three on the others [1]. Luca da Pacioli and Piero della Francesca were born in the same small town, Borgo di Sansepolcro, southeast of Florence and on the border between Tuscany and Umbria. Piero della Francesca, because of his eminence as a Renaissance painter, is usually referred to only as Piero, just as the name Leonardo usually refers to Leonardo da Vinci, not one of the other Leonardos of history. Piero, like Leonardo, combined his ability as a painter with an interest in science and mathematics. Unlike Leonardo, his contributions in mathematics were primarily relevant to art—perspective, proportion, and, by extension, architecture. Leonardo's work is well known, touching on mechanics, hydrodynamics, anatomy, architecture, music, military engineering, and even ideas anticipating human flight. Luca da Pacioli's contributions were more modest. His first published book, *Summa de arithmetica, geometria, proportioni, et proportionalità*, was published in Venice in 1497 and was widely used as a textbook. Highly peripatetic, early in his career Pacioli moved to Urbino, where he tutored the son of Federigo da Montefeltro, Duke of Urbino, immortalized by one of the most famous portraits of Renaissance Italy. This profile portrait by Piero hangs in the Uffizi. Federigo's image is so striking that Piero used it on at least one other occasion, in his *Virgin and Child Enthroned with Saints and Angels* (and incidentally, with Federigo da Montefeltro himself kneeling in the foreground). It is sometimes called the Montefeltro Altarpiece since it was commissioned by the Duke, and it is now in the Pinacoteca di Brera in Milan. Some have

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claimed to see the likeness of Pacioli in this picture, but that claim is debunked by Field in [2].

Urbino under Federigo's reign was a center of culture and hence probably was influential in Pacioli's growth as a mathematician and scientist. Federigo's library was particularly famous, and there are reports that Pacioli used it extensively. The library is still a destination for many visitors and is known for its extraordinary *trompe l'œil* marquetry. Pacioli later traveled to Milan, where he worked for the powerful Sforza family, and to Mantua, to work in the court of Isabella d'Este. He also taught at various times at the Universities of Perugia, Pisa, Naples, Rome, and Bologna, all significant centers of learning of that time.

A 1495 portrait of Pacioli by Jacopo de Barbari (though its attribution is in question) shows him standing at a table, drawing a geometric figure on a slate, pointing to a line in an open book of Euclid (or Pacioli's *Summa* as claimed in [2]) with a model of an octahedron on the table and a glass rhombicuboctahedron half filled with water hanging from the ceiling. It is the ideal Renaissance portrait joining together mathematics and art. A second figure in this painting (easily found with a Google search) is not firmly identified, but two plausible conjectures are Guidobaldi Montefeltro, whom Pacioli had tutored when he was a child, and Albrecht Dürer. Guidobaldi would by that time have been the Duke since his father died in 1482.

As influential as the *Summa* was (it did spell out double-entry bookkeeping for which Pacioli is remembered by accountants), mathematically far more interesting was Pacioli's *De divina proportione,* that dealt with principles of proportion and, whence the title, the ever-admired "golden" section. It also touches on architecture and the design of Roman letters. Like the *Summa* this book does not have much material in it that could be considered original with Pacioli. Some think that the first sections were largely stolen from unpublished work of Piero. The last section is, however, the reason this book is remembered. It contains sixty-one drawings of solids, attributed to Leonardo [4]. And spectacular they are. Originally drawings, these renderings appeared in the 1509 volume as woodcuts. Names and descriptions are shown above the figure in Latin and below in Greek. What is original about them is the portrayal of the polyhedra as solids but also as skeletal figures where the edges have been thickened but the faces are open, like windows, so that the viewer can see inside the figures to check on the structure on the back that one would otherwise not see. This eliminates the ambiguity that shows up in other portrayals of polyhedra, like the figure in Dürer's *Melancholia I* where one can conjecture various possibilities for the unseen faces. This is almost certainly the first time anyone had drawn polyhedra this way, and it is unclear whether Leonardo drew these using only his own imagination or whether he drew them from wooden models of the figures that may have been available to him at that time. Colored reproductions of these drawings exist in a lavish book privately printed in 1956 by Giovanni Mardersteig in Verona at his Officina Bodoni [4]. In *De divina proportione* we find the first printed appearance of the rhombicuboctahedron, the same polyhedron as the hanging glass model in the portrait of Pacioli described above. The painting, now in Naples, preceded *De divina proportione* by fourteen years. Those with a good eye can see, we are told, a reflection of the Palazzo Ducale in Urbino in one of the glass faces of the rhombicuboctahedron.

Long interested in architecture, Pacioli was a friend of the architect Leone Battista Alberti, but it was another architect, Giorgio Vasari, who attacked Pacioli for plagiarism, saying that he stole ideas in his two major works from Piero. In 1942 Taylor [5] wrote a full length detailed biography of Pacioli that made it clear that he was aware of the accusations of plagiarism but largely discounted them. Further, he went so far as to suggest obliquely that Pacioli's contributions to mathematics surpassed those of his mentor, Piero, and perhaps Piero stole ideas of Pacioli's. The biography comes across at times as something of a hagiography. The plagiarism issue is complex and later scholarship has been less kind to Pacioli. Still, he cannot be dismissed. The fact that he codified and promulgated double-entry bookkeeping, even though before his time it was commonly used in Venice, was a significant contribution. One can observe that Euclid may not have contributed to geometry many results of his own, still, his organization of the known geometry of his day was so important we still call it *Euclidean* geometry. Pacioli was to a great extent writing textbooks and thereby succeeded in spreading knowledge to a wide audience. Nevertheless, his use of Leonardo's representation of polyhedra may remain the contribution for which he is best known.

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