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ABOUT HERMANN WEYL'S "RAMIFICATIONS, OLD AND NEW, OF THE EIGENVALUE PROBLEM"

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Immediately following the commentary below, this previously published article is reprinted in its entirety: Hermann Weyl, *Ramifications, old and new, of the eigenvalue problem*, Bulletin of the American Mathematical Society **56** (1950), 115–139.

From time to time, over the past five decades, I've read parts of this wonderful article [1], and—each time—I admire it again and am again intrigued by the way Hermann Weyl gingerly threads through so much mathematics and physics, allowing himself to be led by the unifying theme of *eigenvalue*, and by his more personal mathematical reflections. I don't know any other expository mathematical essay like this. Weyl manages to sketch for us revealing shreds of intellectual biography, of his own and of some of his contemporaries. At the same time, almost *sotte voce*, he offers illuminating explanations of the underlying mathematics while recounting how mathematics was practiced, how it commingled with physics, and how it changes in temperament from the classical to the more structural function-space approach to the same problem, in the eras when the ideas he discusses were brewing.

Weyl's expository essay begins with the (Wilbraham–)Gibbs phenomenon, and manages—in less than three pages—to explain it from scratch; to mention its effect for different conductivities on the circle which, in turn, allows him to comment, *en passant*, on his own encounter with Diophantine approximation; and to give tantalizing hints of higher-dimensional Gibbs phenomena along nonsmooth discontinuity loci. To begin a general expository article about *eigenvalues* with this fundamental and curious quirk inherent in eigenfunction expansions (Gibb's phenomena) is, I think, a stroke of brilliance. There is also a seamlessness to the writing so that the passage from eigenfunction expansions for ordinary differential equations to the beautiful ideas behind asymptotic distributions of eigenvalues for oscillating membranes, and thence to Zeta-functions of Laplacians is deft. Along the way we get the hint of an unfinished project about which Weyl scholars might know the inside story (I don't):

> I feel that these informations about the proper oscillations of a membrane, valuable as they are, are still very incomplete. I have certain conjectures on what a complete analysis of their asymptotic behavior should aim at; but since for more than 35 years I have

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> made no serious attempt to prove them, I think I had better keep them to myself.

Weyl ends his story with a discussion of the Peter–Weyl formula, comments on Bohr's theory of almost-periodic functions, and hints regarding then-current harmonic analysis. He signs off saying:

> I hope you have taken this lecture for what it was meant to be: a *Plauderei*, the chat of a man who has reached the age where it is more pleasant to remember the past than to look forward into the future. Even so, it gives him a little satisfaction to see that the issues to which the efforts of his youth were dedicated have kept alive over the years and are still in the process of unfolding their implications.

They still are!

References

[1] Hermann Weyl, *Ramifications, old and new, of the eigenvalue problem*, Bulletin of the American Mathematical Society **56** (1950), 115–139. MR0034940 (11,666i)

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