BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 50, Number 3, July 2013, Pages 513–521
S 0273-0979(2012)01403-9
Article electronically published on December 18, 2012

Plato's ghost: the modernist transformation of mathematics, by Jeremy Gray, Princeton University Press, Princeton, 2008, x+515 pp., ISBN 978-0-691-13610-3

Modern mathematics contains many elements taken for granted today, several of comparatively recent vintage. Its language, structures, and contents all reflect a consensus view that emerged after 1900 when mathematics achieved the status of an autonomous scientific discipline. With that status came a new-found freedom from earlier epistemological concerns, a matter to which Bertrand Russell once alluded when he wrote that "mathematics is the subject in which we never know what we are talking about or whether what we are saying is true" [1, p. 286]. Today many mathematicians might respond to Russell's quip by saying that he missed the point, and yet one must remember that he made this remark when the modern paradigm for mathematical knowledge was still asserting itself against a far older, long established view. The shift from classical to modern came quite suddenly during a calamitous era of human history, and for several eminent research mathematicians this transition was a painful process.

In today's world, philosophers pay rather scant attention to mathematics and mathematicians even less to philosophy. Both groups surely would agree, though, that mathematicians are engaged in producing (or perhaps discovering) a special type of knowledge that requires no outside referent in the physical world. Mathematicians legitimize their work by proving theorems (or claiming they can do so) and throughout most of the twentieth century that was all they were expected to do. A theorem was a theorem; one did not need to philosophize about its meaning or justify its larger importance outside the realm of specialists for whom alone it had any relevance. Publish or perish was the watchword of the day and specialized research flourished, especially when funding agencies were flush with money. That era is now history, too, and we have become accustomed to life in the new age of the media-savvy mathematician. The vulnerability and ultimate absurdity of 1960s-style purism was already brought out nicely by Davis and Hersh [6] in their delightful parody of the real-world opinions of an expert on "Riemannian hyper-squares", who found himself in the awkward position of having to explain the importance of his research to a layman.

Mathematical knowledge has always had an esoteric character. Yet something very vital and important took place in mathematics during the period from roughly 1890 to 1930. In physics, this was the era that saw the waning explanatory power of classical mechanics and electromagnetism, theories that no longer provided firm foundations for new phenomena and laboratory findings. Black-body radiation, particle physics, and the failure to detect the earth's motion relative to the electromagnetic ether all raised seemingly insurmountable problems that left physicists groping for a way forward. What eventually emerged through the work of Einstein, Bohr, and many others came to be known as modern physics, a familiar and oft-told success story. But the other side of this tale can be just as illuminating. In his *Night Thoughts of a Classical Physicist* [12], Russell McCormmach provided a

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2010 *Mathematics Subject Classification.* Primary 01A60, 01-02.

window on the mental world of an old German physicist who had lost his bearings in the new age of relativity and the quantum.

Something similar took place in mathematics, though to describe just what happened and why poses a major challenge for historians, in part because of the far more subtle ways in which mathematical theories evolve and stabilize. For while mathematicians generally presume the universal validity of their work, some even ascribing to it an eternal, transcendent quality, historians are bound to the opposite prejudice: for us, mathematics is a highly stable, yet contingent body of knowledge that remains tied to its producers and practitioners. Studying the history of modern mathematics therefore requires scrutiny of the social and cultural conditions under which mathematics was made. So how can the historian begin to grasp the monumental transformation that brought the world of modern mathematics into being?

In *Plato's Ghost*—an image taken from Yeats' poem "What Then?"—Jeremy Gray takes up this challenging theme of modernism by drawing on several recent historical studies dealing with foundational issues. Much of his text describes episodes in the history of mathematics from roughly 1820 to 1930 chosen to illustrate how modernism crept into the picture incrementally as modern analysis, algebra, and geometry gradually evolved. One senses from his introduction how daunting this task proved to be; there he takes pains to explain what he tried to accomplish and what he chose to leave out of his account. My own sense is that he left too much in; certainly this is not an easy read.

Before ticking off what is new in his book, Gray cites the views of Francis Bacon regarding the activity of the historian. That caught my eye, as it signals perhaps why I find this book so difficult to read and review. Bacon is, of course, mainly remembered for his influential, though largely mundane views about induction in scientific methodology rather than what he had to say about historical method. At any rate, he once likened the historians' task with that of raising a gigantic shipwreck from the ocean floor. One must proceed carefully in both cases, trying to leave all the pieces intact and their relative positions undisturbed. This metaphor would seem apt when describing the work ethic of the modern archaeologist, but the historian? Having spent a good deal of time in archives, I can see how those who work with archival sources on a daily basis could identify with Bacon's image, but historians go there in order to find artifacts they can work with, not to preserve them but to use them as supporting evidence for their arguments. Moreover, for every hundred pages of archival documents the historian looks at, the researcher will be lucky if even one page is worth citing. Perhaps in Bacon's day historians understood their craft as something like sifting through the wreckage, but surely no longer. So when Jeremy Gray writes that he has "tried to bring more to the surface than has been attempted before" (p. 3) the reader can rest assured that this is so. But what about sifting, organizing, and assessing this new information?

Gray sets about doing that by following important developments in the major branches of mathematics before, during, and after the onset of modernism. The threshold stage was reached during the two decades from 1880 to 1900, a period that saw new breakthroughs in geometry, analysis, and algebra, as well as the advent of modern logic and Cantorian set theory. These achievements were then absorbed and systematized after the turn of the century when axiomatization came into vogue. After describing how modernist tendencies affected each of the main branches of pure mathematics, Gray steps back to assess concurrent trends in mathematical physics, theories of measurement, and the popularization of these new mathematical sciences. He then goes on to discuss modern ideas about natural and artificial languages, related theories of knowledge, and in particular the psychological underpinnings of mathematical knowledge. All of these were matters of intense interest during the period leading up to the outbreak of the Great War.

Passing over that disruptive break, Gray takes up some of the famous conflicts that ensued immediately afterward during the era when communism and fascism began to encroach upon the European political scene. L. E. J. Brouwer, supported by Hermann Weyl, openly challenged Hilbert's formalist credo as a false ideology, one they intended to overturn by adopting the principles of a revamped intuitionism. Hilbert, assisted by Paul Bernays, rose to meet this challenge as an answer to Weyl's "foundations crisis" manifesto [19], in which he declared that "Brouwer is the revolution." During the ensuing battle much turned on one central question. Ever since he announced his second Paris problem [10], Hilbert had staked his reputation on demonstrating that the continuum could be axiomatized by showing that his axioms for the real numbers were consistent. By so doing—as he stressed in his Paris lecture—Hilbert sought to refute the views of Kronecker, who had denied the possibility of arithmetizing the continuum in a rigorous manner. Indeed, Hilbert emphasized that proving the consistency of his axiom system was tantamount to demonstrating the existence of the real numbers, just as he assumed that the same could be shown for Cantor's number classes and cardinal numbers. By the 1920s, however, he came to realize (something he never dreamed of in 1900) that proving consistency even of the Peano axioms for elementary arithmetic was no easy matter. His collaboration with Bernays, beginning in 1918, thus marks an essentially new period in foundations research: the emergence of proof theory [11].

As Weyl's enthusiasm for intuitionism waned, Brouwer found himself backed in a corner, while Hilbert became increasingly scornful of all opposition. In 1928, in the wake of the politically charged ICM in Bologna, the clash between Göttingen's elderly tyrant and his Dutch rival ended with Brouwer's ouster from the editorial board of *Mathematische Annalen* [3]. Needless to say, Hilbert and Brouwer never reconciled after this dramatic rupture. The 1920s represented the hotly contested phase in the foundations debates; after Brouwer retired from the scene, the polemical language died down quickly. Perhaps not coincidentally, in 1930 leading representatives of the three leading "philosophical schools" convened at a conference in Königsberg that was devoid of the customary fireworks. Rudolf Carnap spoke on behalf of logicism, John von Neumann represented formalism, whereas Arend Heyting served as spokesman for intuitionism. In the meantime, Heyting had given a formalization of intuitionist logic, a development Brouwer had approved, albeit with the understanding that this formal language contained nothing new or even important for intuitionist mathematics. At any rate, this conference successfully canonized the three approaches that would for the next several decades come to dominate discussions in philosophy of mathematics

In the meantime, Hilbert—aided by a small entourage of younger experts, most notably Paul Bernays—began to close in on the resolution of his second Paris problem, which called for an internal proof that the axioms for arithmetic are consistent. But then along came Kurt Gödel, who made a quiet announcement of his incompleteness theorems at the Königsberg conference, and suddenly Hilbert's game was up. Gödel's results had devastating implications for Hilbert's proof-theoretic program, for he proved that the proposition which asserts "this system of axioms is consistent" was formally undecidable for axiom systems that contain elementary arithmetic. Thus, the only way forward would require changing the rules of Hilbert's proof theory, and that, indeed, was how Gerhard Gentzen was able to prove consistency five years later.

Gray sticks with the master narrative, however, leaving proof theory with Gödel and moving on to the wider terrain of mathematical activity, which he treats as a kind of coda to the main themes of his book. For modern mathematical Platonism had yet to emerge as an integral part of the new mathematics. It would eventually become the unofficial ideology of the "working mathematician", explicitly acknowledged by such luminaries as David Mumford and Alain Connes, but later ridiculed by Davis and Hersh in [6]. We learn from Jeremy Gray (p. 444) that the term "mathematical Platonism" in its modern sense was first coined by none other than Paul Bernays in 1935. Still, the pre-eminent Platonist of the twentieth century was surely Kurt Gödel, and so this book ends, appropriately enough, with two extended quotations in which the great logician affirms the centrality of ideas that go back to Plato and Kant.

Much is new in this massive survey, but as noted already, Gray has mainly drawn on and synthesized results from earlier historical studies. He also makes a point of contrasting his picture of this gradual transformational process with the one offered some twenty years ago by Herbert Mehrtens in his *Moderne—Sprache—Mathematik (M-S-M)* [14], an ambitious study of the crises and conflicts that led to the birth of modern mathematics during the early decades of the twentieth century. Mehrtens's provocative book focused largely on developments in Germany from 1900 up until the onset of the Nazi era. Not that anyone doubted whether fundamental changes had taken place during these years, but Mehrtens was the first to draw strong parallels between modernist movements in the arts and the less transparent transformation that altered the character of mathematical research after 1900. Most importantly, he took a far broader view of what was at stake, arguing that the shift toward an abstract, axiomatic style in mathematics had something to do with modernization in general and the complex changes that produced a culture of modernity. More precisely put, he offered a sweeping interpretation of how modern mathematics emerged after breaking with traditional research practices as part of a trend to create disciplinary autonomy for professional mathematicians. Not coincidentally, these modernists were often staunch advocates of Cantor's *Mengenlehre* and adopted the Cantorian stance, according to which "the essence of mathematics lies in its freedom."

Gray pays tribute to Mehrtens's book by way of an honest appraisal in which he spells out what he sees as its limitations and weaknesses (pp. 9–12). Those pages make for must reading, especially in view of the circumstance that few in the English-speaking world are aware of this earlier study, except perhaps by way of second- or third-hand opinion. Written in a rather prolix German, *M-S-M* never found a translator and has now been out of print for some time. Mehrtens's Foucaultian approach and postmodern jargon also serve to set his book apart from conventional studies in the history of mathematics. In fact, the thrust of its argument concerns cultural criticism rather than historical matters per se, as for example when Mehrtens argues that formal languages and logics served as tools for the assembly line production of mathematical commodities. Far from celebrating the ultimate victory of modern mathematics, he paints a harsh picture that underscores moral bankruptcy and dehumanizing influences, as when mathematicians willingly lent their technical expertise to the military. Clearly the shadow of the Nazi era hangs heavily over this picture of events, which was written during the midst of the *Historikerstreit* of the late 1980s when debates regarding the historical significance of the Holocaust were in full flame. Jeremy Gray takes a far broader and essentially internationalist perspective while narrating events that mainly point us forward. This approach results, not surprisingly, in a far smoother and less acrimonious picture, but at the price of diminishing the human drama and downplaying the passionate debates that fill Mehrtens's earlier account.

Rather than hewing to a strictly chronological narrative, Gray opted to present his findings in three phases conforming to the three main chapters of the book. In these he traces developments both in classical mathematics and the new fields of set theory and symbolic logic. He expends much space describing or explaining that work, far more than Mehrtens allotted for this purpose in his account. Much of the ground he covers can be found in more detailed studies by authors such as Corry [2], Ferreiros [8], Peckhaus [15], and Epple [7]. Indeed, the thrust of Gray's account follows a by now familiar narrative that leads from Dedekind's and Cantor's "arithmetization of analysis" to Hilbert's axiomatization of arithmetic and his defense of "classical mathematics" in the face of intuitionism by means of proof theory, ending with Gödel's refutation of the original formalist program. Gray rightly emphasizes the "inter-disciplinary" character of these developments, in which the larger epistemological stakes were very high, though he pays rather scant attention to the human dimensions of this story. One misses the dialectical tensions that do so much to enliven Mehrtens' version of the key developments.

Mehrtens's analysis of modernism centers on the role of language, operating both within mathematics proper but also as part of informal discourse about the nature of mathematics, particularly when mathematicians reflect on their everyday work. As mathematical knowledge grew more remote and esoteric, its practitioners became increasingly preoccupied with such concerns. As they did so, Mehrtens sees them as falling into two major camps whose leaders engaged in a struggle over the future direction of research: the moderns and their opponents, a mixed bag of individuals who reacted as countermoderns. According to Mehrtens, the hallmark of the moderns was an understanding of their craft as being exclusively concerned with the production of knowledge expressible within a formal language system, itself governed by a system of explicitly given, incontrovertible rules. The moderns saw mathematical language as self-sufficient; it needed no independent referent to justify the existence of the entities about which one spoke. Countermoderns, on the other hand, contended that mathematical knowledge expresses truths pertaining to objects whose ontological status does not derive solely from their function within a given formalized mathematical theory.

This shifting divide only gradually emerged during the latter half of the nineteenth century when Cantor's set theory created much controversy, particularly among French analysts. It widened quickly after 1900, however, when Hilbert began to promote Cantor's ideas along with his own ambitious program for axiomatizing mathematics. In the meantime, younger and more radical thinkers, Hausdorff and Brouwer, adopted positions at the respective far wings of the modernist and countermodernist camps. Hausdorff, who considered Zermelo's axiom of choice wholly unproblematic, quietly advanced the modernist cause on three fronts: set theory, topology, and measure theory [16]. Brouwer, on the other hand, sought to restrict set theory and began to explore a new concept of the continuum based on free choice sequences. The Dutch topologist also viewed Hilbert's position on foundational matters as untenable, and eventually he came to challenge his views openly, framing the ideological debate that followed as formalism vs. intuitionism [4], [5].

Already in the early 1870s much effort had been expended on freeing the real numbers from all vestiges of "geometrical intuition", For Weierstrass, Cantor, and Dedekind the real number system was a purely arithmetical construct, whereas Kronecker spoke out vociferously against this conception. By 1900 Hilbert famously proclaimed that Kronecker had been dead wrong; Hilbert maintained that the very existence of the "arithmetized continuum" could be proved, once and for all, by demonstrating the consistency of his axioms for a complete ordered Archimedean field. The algebraist Kronecker could only turn over in his grave, but the topologist Brouwer mounted a strong counterattack. Thus behind Brouwer's intuitionism stood his fierce opposition to the general notion that the continuum could be reduced to an arithmetical object; for him the intuitive continuum was far richer than Dedekind's or Cantor's atomistic conceptions of an infinite point set whose individual elements were captured in their entirety by means of arithmetical properties alone. He was also steadfastly opposed to Hilbert's formalist methods. Brouwer was ferociously committed to his vision and knew full well that this placed him on a collision course with Hilbert, the most influential mathematician of the era. His intuitionist conception of the continuum aimed at nothing less than overturning Hilbert's formalized axiomatic methods, an approach Brouwer regarded as contentless.

Gray recounts all these developments, but treats the underlying conflict rather episodically, referring the interested reader to the studies by van Dalen [4], [5] and by Hesseling [9]. The latter investigation focused on the reception of Brouwer's intuitionism in the 1920s, concluding that Mehrtens's modernist/countermodernist framework worked well in accounting for these developments. Gray nevertheless skirts these central issues, evidently convinced that intuitionism was merely the intellectual offspring of a quirky fanatic and that Brouwer's demanding research program was doomed to lose out in the end. Citing a passage in which Weyl passionately defended intuitionism for its intellectual honesty, Gray clearly sides with George Pólya, who thought it mistaken when mathematicians seek some deeper meaning beyond the realm of scientific truth. By dismissing Weyl as a "true believer", Gray seems to overlook the emotional context that framed this debate. For Weyl's opening salvo in his crisis paper was itself directed against a certain "intellectual dishonesty" [19, p. 86] that had crept into the body of mathematics, and he clearly identified this deleterious trend with Hilbert's influence and his overblown rhetorical claims (on Weyl's relationship with Hilbert, see [18]). Weyl's sense of dismay clearly echoes what Gray tells us about why, in the wake of the Great War, the Dutch mathematicians Mannoury and Brouwer became intensely interested in the relationship between language and thought. Unfortunately, *Plato's Ghost* only takes note of that war without really trying to come to grips with its devastating impact.

Yet I find Mehrtens's account of this central chapter in the emergence of modern mathematics flawed as well. For him, the foundations crisis was largely to be seen as an ideological conflict within the German mathematical community, so in *M-S-M* he does little more than set the scene for this part of the story. Far from writing a conventional narrative history, Mehrtens aimed to place the technical issues that divided formalists and intuitionists within the broader context of debates over the nature and meaning of mathematical knowledge in modern industrialized societies. Thus, his 600-page book devotes just ten pages to the "foundations crisis" of the 1920s. He deals with this as the first of four "Continuation Stories", the other episodes being "The Paradise of Machines" (Turing), racism and intuition (*Anschauung* in the sense of Bieberbach), and "Structure and Architecture: Bourbaki". By thereby minimizing the status of the epistemological issues that separated Hilbert from Brouwer, Mehrtens sought to embed their battle within the larger context of a power struggle between moderns and countermoderns. This he interprets as being symptomatic of a deeper divide acutely felt throughout Germany during the post-war years: the sense of disequilibrium and loss of meaning.

Mehrtens addresses the theme of modernization in mathematics by analyzing the main social, political, military, technological, and industrial forces that shaped Gottingen's dynamic community from 1900 to 1933. Its principal leaders, Klein and Hilbert, fall on opposite sides of the modern/countermodern divide, a tension Mehrtens exploits to the full. Together they launched an open-ended research community with a distinctly modernist character—attracting numerous talented foreigners, Jews, women, and other disenfranchised elements—while maintaining the traditionally autocratic and elitist structures that supported Germany's professoriate with its mandarin mentality. Perhaps the single most dynamic and influential mathematician in Göttingen during the Weimar years was Emmy Noether, the "mother of modern algebra" and a far purer representative of the modern style and spirit in mathematics than Hilbert. By fixating on foundations issues and slighting the 1920s, Mehrtens (who is not otherwise prone to overlook the accomplishments of women) manages to write 584 pages without once mentioning Emmy Noether's name! Noether's work paved the way for the culminating phase in the emergence of modern mathematics in which the notion of mathematical structures assumed a dominant position. Mehrtens only makes passing remarks in reference to these developments, which had far more impact on Bourbaki and other leading representatives of modern mathematics than did set theory, logic, and proof theory.

Oddly enough, *Plato's Ghost* also leaves Noether out of the picture, offering instead a potpourri of assorted mathematicians and philosophers, some of whom can only be called obscure. Jeremy Gray's account generally avoids delving into controversial political topics, and he objects to Mehrtens's tendency to treat his principal actors as if they were playing roles from central casting in a Hegelian drama; indeed, he finds *M-S-M* overly fixated on the special conditions that prevailed in Germany. In particular, he rejects the argument that certain countermoderns, most notably Bieberbach, were merely taking their views to the next stage when they became virulent anti-moderns after 1933 (on Bieberbach's career, see [13]). In Gray's version, the older countermoderns (Klein, Frege, et al.) appear more like traditionalists rather than arch-nationalists whose world came crashing down after the Great War. For the most part, however, he writes exclusively about mathematical and philosophical ideas. Politics thus recedes very much into the background, rarely rearing its ugly head. There are no good guys and bad guys anymore, except perhaps for Brouwer, who in this account seems to have gotten his just due when he was banished from Hilbert's sphere of influence within the German mathematical community. But there are winners and losers, and Gray focuses on the forward edge of modernism throughout his book, which lacks the dialectical elements so prominent in Mehrtens's interpretation.

Despite his stated misgivings, Gray also focuses very heavily on events in Germany, supplementing these by reactions in France (especially Poincaré's numerous writings), along with briefer discussions of research on axiomatics in Italy and the United States. In short, he describes the larger international trends toward modern mathematics, an important venue being the early International Congresses, particularly the Second ICM held in Paris in 1900. Beyond enlarging the geographical scope of Mehrtens's original study, Gray also widens the range of knowledge that he sees as relevant for assessing the transformation to the new modern consensus. This task—which takes him well beyond the realm of mathematics and into prolonged discussions of parallel developments in mainstream philosophy—poses a daunting challenge, and in places the reader is bound to find *Plato's Ghost* a rather bumpy ride. Its aim is to show how developments that transformed the foundations of mathematics were embedded in a larger critical discourse that took place simultaneously in a number of fields. In the course of this study Gray uncovers many new and unexpected things, too many to mention here.

One gathers from Jeremy Gray's introduction that he regards *Plato's Ghost* as an updated version of the themes Mehrtens dealt with twenty years ago. I share some of his misgivings about that book, a few of which are discussed in my essay review [17]. Mehrtens goes off the deep end, for example, when he tries to find parallels between the formal languages of modern mathematics and the commando rhetoric of contemporary dictatorships. In many ways, Gray's book offers a richer and more balanced account of how modernist ideas gradually gained inroads within pure mathematics. Both studies, whatever their shortcomings, represent singular efforts to understand the complex manner in which modern mathematics eventually emerged, one of the most challenging problems facing the historian.

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