

INTRODUCTORY COMMENTS

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As the readers will observe, this issue of the *Bulletin of the American Mathematical Society* is somewhat unusual. All the articles concern topology. Last year John Milnor sent me the draft of an article based on his Abel lecture at the 2014 International Congress of Mathematicians in Seoul and inquired if it might fit in the *Bulletin* even though it is written in a very non-standard style. The Editors decided that this “lightning tour” of topology, concentrating on low dimensional manifolds, by a mathematician who made so many seminal contributions to topology would indeed appeal to the readers of the *Bulletin*. We then began to put together a whole issue related to certain aspects of John Milnor’s work.

The items in the Articles section of this issue of the *Bulletin* are the following:

- 1 “Topology through the centuries: low dimensional manifolds” by John Milnor, based on his 2014 ICM Abel Lecture.
- 2 “Topological manifolds and smooth manifolds” by John Milnor, based on his 1962 ICM Fields Medal Lecture, reprinted from *Proceedings of the International Congress of Mathematicians, 1962*.
- 3 “Introduction to eight papers on exotic spheres” by John Milnor, reprinted from volume 3 of his *Collected Works, 2007*.
- 4 Selected letters in the correspondence between Michel Kervaire and John Milnor over the period 1958–1961. Comments on this correspondence, which is now in the Kervaire Nachlass, are given by Andrew Ranicki and Claude Weber.
- 5 “Groups of homotopy spheres: I” by Michel Kervaire and John Milnor, reprinted from the *Annals of Mathematics, 1963*.

Thus the narrative of this issue begins with the most recent International Congress of Mathematicians in Seoul and moves backwards in time to the 1962 ICM in Stockholm where Milnor was awarded a Fields Medal. The citation given by Hasler Whitney on this occasion begins with the observation that the research in the 1950s by Milnor and others opened a vast field of work which now acquired a name: differential topology. Whitney concludes with the words “Differential topology is a strong young field with a bright present and a great outlook, and that its vitality is largely due to the fine achievements of Milnor.”

The list of Milnor’s articles in his collected works on what are called exotic spheres begins with a 1956 paper that Whitney remarks astounded the mathematical world, namely his proof that the 7-sphere was capable of several differential structures. The list concludes with the famous 1963 paper by Kervaire and Milnor, “Groups of homotopy spheres: I”. We reprint Milnor’s own discussion as to how these exotic sphere papers came to be written. We then present some previously unpublished material related to this paper, namely items from the correspondence

between Kervaire and Milnor over the period of their collaboration from 1958 to 1961 that preceded the writing of their 1963 article. We conclude this section by reproducing in full the Kervaire–Milnor article that originally appeared in the *Annals of Mathematics*, vol. 77, no. 3, May 1963.

The Mathematical Perspectives section of this issue contains selected Mathematical Reviews. Our choice begins with the very lucid review by Frank Adams of the 1963 Kervaire–Milnor article and continues with reviews of papers that show the development of the subject in later years.

In this issue the *Bulletin* readers have a rare chance to see behind the scenes of a collaboration that produced one of the most influential articles of 20th century topology. Nowadays letters between collaborators have been replaced by such useful tools as e-mail, skype, dropbox, polymath, etc. We hope 21st century mathematicians will find the “old fashioned” Kervaire–Milnor correspondence illuminating.

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Photo of John Milnor from 1965 that appears with the Kervaire–Milnor *Annals* article.

MME. AIMÉE KERVAIRE

Photo of Michel Kervaire from 1956 that appears with the Kervaire–Milnor *Annals* article.

Kervaire’s correspondence with John Milnor between 1958 and 1961.

JOHN W. MILNOR

Photo of John Milnor in the reprint of the “Introduction to Exotic Spheres”, *Collected Works of John Milnor, III. Differential Topology*, American Mathematical Society, 2007.

Milnor’s correspondence with Michel Kervaire between 1958 and 1961.

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Photo of Friedrich Hirzebruch from 1976 in the reprint of the “Introduction to Exotic Spheres”, *Collected Works of John Milnor, III. Differential Topology*, American Mathematical Society, 2007.

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Photo of René Thom from the early 1970s in the reprint of the “Introduction to Exotic Spheres”, *Collected Works of John Milnor, III. Differential Topology*, American Mathematical Society, Providence, RI, 2007.

INSTITUT MITTAG-LEFFLER

By John Milnor, “Topological manifolds and smooth manifolds”, *Proc. Internat. Congr. Mathematicians*, Stockholm, 1962.

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By Michel Kervaire and John Milnor, “Groups of homotopy spheres: I”, *Annals of Mathematics*, Volume 77, Number 3, May 1963.

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