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Origins of mathematical words: a comprehensive dictionary of Latin, Greek, and Arabic roots, by Anthony Lo Bello, Johns Hopkins University Press, Baltimore, 2013, xviii+350 pp., US $49.95, ISBN 978-1-4214-1098-2

Enlightening symbols: a short history of mathematical notation and its hidden powers, by Joseph Mazur, Princeton University Press, Princeton, NJ, 2014, xxiv+285 pp., US $29.95, ISBN 978-0-691-15463-3

The two books under review are about the language of mathematics and its relation to the history of the subject. But they approach language and history from very different directions. As their titles suggest, the first book is mainly about words and the second is mainly about symbols. This alone makes a big difference, because the replacement of words by symbols is one of the characteristic features of mathematics.

Let me begin with *Origins*, the book on words. As its subtitle suggests, Lo Bello wishes to trace the roots of mathematical terminology, which are mostly in Greek and Latin, and to a lesser extent in Arabic. Lo Bello is an expert in these languages, and he deploys his knowledge not only to explain the words of mathematics, old and new, but also to denounce the ignorance of many coinages—for mixing Greek and Latin, adding superfluous suffixes, forming plurals incorrectly, and so on. Beyond exposing linguistic errors—and this may be the most entertaining feature of the book—Lo Bello uses the platform of etymology to launch attacks on the language and culture of today, both inside and outside mathematics. Reading *Origins of Mathematical Words* is a guilty pleasure because Lo Bello, by his own admission, is a "perpetual complainant" who relentlessly attacks all forms of political correctness and modernity.

An entry from the book, which presents his attitude in a nutshell, is:

> **-logy** The Greeks appended the suffix -λογια from λόγοζ, *word*, *reason*, to certain words to indicate *the study of* .... Modern words formed on the analogy of this construction, such as *geology* and *topology*, are unobjectionable. Incorrect formations, however, are words like *sociology*, where the Greek suffix is appended to the Latin stem. Outright contemptible are things like *cosmetology* and *mixology*, which seek to throw the mantle of learning over low activities.

From this and other entries the reader will learn that Lo Bello is at war with many aspects of the modern world. Among them are:

- American spelling (which gets its own entry),
- computer terminology (see the entry **PowerPoint**, for example),
- the language of educators and administrators,
- levity (which "is unbecoming in mathematics", see p. 24),
- slang and jargon (see the entry **cant**),
- mixing Greek and Latin in the same word,
- modern teaching of English,
- changes in English usage due to ignorance and mistakes.

2010 *Mathematics Subject Classification.* Primary 01-00.

Anyone who dislikes at least a few of the above is sure to enjoy browsing in Lo Bello's book or even reading it from cover to cover (as I did). The book is organized like an encyclopedia, with entries ranging in size from one line to several pages. In particular, the entry **mathematics** is a spirited eight-page essay in which Lo Bello airs both his love of mathematics and hatred for most of the above. The author's combative style invites the reader to argue with him, but his linguistic knowledge is so intimidating that this may be a mistake. Nevertheless I venture a couple of criticisms.

The first is about **American spelling**, which Lo Bello defines as the system established by Noah Webster's *Dictionary* of 1828. In Lo Bello's opinion, Webster committed many crimes, such as removing the letter *u* from certain words, thereby removing the distinction between *author* (which came into English before the Norman invasion in 1066) and *honour* (which came into English from French after 1066). He concludes that "if you learn the English spelling you know when the word came into the language ... If you learn the American spelling, you cannot tell." But, though Lo Bello scolds Webster for this erasure of history, he silently accepts Webster's change of *centre* to *center* and *metre* to *meter*. This change similarly erases the history of these words in Greek, Latin, and French—all of which place the *r* immediately after the *t*. Lo Bello's entry **meter** even claims, disingenuously, that the "meter is the unit of measurement in the scientific system defined by the French revolutionary government."

My second criticism is about the language of computer technology, which Lo Bello denounces in many entries throughout the book. Under the entry **-oid** he mentions respectable examples, such as *spheroid* ("spherelike") which goes back to ancient Greece, but then adds:

> In modern times this practice of making *-oid* words has gotten out of control, the latest invention being a device called the *android*.

As even a casual reader of science fiction will know, the word *android* is not a recent invention. Indeed, it is found as far back as Ephraim Chambers' *Cyclopedia* of 1728 and, as far as I can tell, it is correctly derived from the Greek for "manlike".

Tempting as it is to argue with Lo Bello, I should turn to the main goal of his book, which is to give a critical, historical analysis of the vocabulary of mathematics. Mathematics is a very old discipline, with many concepts that received names thousands of years ago and have remained stable ever since. This accounts for the prevalence of names derived from Greek, such as *sphere*, *ellipse*, *parabola*, and *hyperbola* as well as the abstract terms *theorem* and *hypothesis*. A stream of new names, sometimes displacing Greek names, flowed from Latin, which became the language of scholarly communication in Europe in medieval times. Latin gave us (often through the mediation of French) basic words such as *line*, *point*, and *plane*, and more abstract terms such as *condition* and *implication*. Greek and Latin also have suffixes that turn nouns into adjectives, and this is where trouble begins.

Greek has a suffix -*ικός* that turns *ellipse* into *elliptic*, Latin has a suffix *-alis* that turns *condition* into *conditional*. But this means, alas, that the right adjective to go with *sphere* is *spheric*—not *spherical*. The suffix *-ical* contains not only an absurd mixture of Greek and Latin, but also a redundancy. Even more regrettably, the same applies to *mathematical*. Lo Bello has a field day exposing such errors, but it should be pointed out that most of them were not committed by ignorant moderns, but by our classically trained forebears in the 19th century and earlier. It may be true that our system of names is in chaos, but if so the rot set in centuries ago. Perhaps there is no really scientific system for creating new names, and the best we can do is learn where the existing names came from, and the (sometimes competing) rules for building new names from old. For this purpose, Lo Bello's book is a wonderful guide. I should add, however, that this is a dictionary of word *origins*, and their *meanings* are generally assumed known by the reader. For a dictionary of both origins and meanings, the reader will be better served by the book Schwartzman (1994), also recommended by Lo Bello.

Let me leave the subject of names with one more example, which also serves as a segué to the use of symbols. Lo Bello's entries on **orient** and **orientation** tell us that *orient* comes from the Latin for *to rise*. Through its use in *sol oriens*, meaning *rising sun*, it came to mean the *East*. Then, from the Latin *orientatio* meaning *determination of one's bearings by looking at the morning sun*, we get **orientation**. This train of thought is curiously similar to the construction of the Chinese character for *east*, where the rising sun is implied by showing it behind a tree (see Figure 1, in which the character for *sun* is superimposed on the character for *tree*).

日 + 木 = 東

FIGURE 1. Sun behind a tree equals east

The history of mathematical symbols is the subject of a classic work, Cajori (1929). Cajori's book is extremely thorough and scholarly, though not an easy read (with footnotes taking up more than half of many pages) and with some gaps, such as Chinese mathematics. Nevertheless, by adopting a telegraphic style without much commentary, and by providing facsimile pages of historic mathematics books, Cajori is able to present an enormous amount of information in the 800 pages of his two volumes. His book is also a *tour de force* of typesetting, containing a range of symbols even LaTeXwould find hard to handle. Mazur's *Enlightening Symbols* is a slimmer and less ambitious work, but also much more readable and reflective.

Mazur summarizes his book as follows on p. xix:

> It is chiefly a history of mathematical symbols; however, it is also an exploration of how symbols affect mathematical thought, and of how they invoke a wide range of subconscious inspirations.

As a history, *Enlightening Symbols* covers highlights from Cajori and more recent research. Mazur is a master story teller, using a much greater word-to-symbol ratio than Cajori. The necessary symbols are there, of course, but not to an extent that impedes the narrative. There are also reproductions of historic mathematical texts, though (unusually for Princeton University Press) they are not as clear as one would like. For example, the picture of Euclid's *Elements* on p. 86 is completely unreadable, even though it is taken from a clear online image. The pictures in Cajori's book put these modern ones to shame. Leaving this disappointment aside, how does *Enlightening Symbols* fare in its mission to explore how symbols affect mathematical thought?

Mazur divides his study of symbols into two parts—numerals and algebra—before drawing some general conclusions about the power of symbolism in the third part of the book. The story of numerals, in a nutshell, is about stringing together numerals for small numbers (usually 1, 2, 3, 4, 5, 6, 7, 8, 9) to make numerals for large numbers. Many cultures found similar ways of doing this, apparently independently, before finding the essentially optimal system that we use today—the so-called Hindu or Arabic numerals using not only numerals for 1, 2, 3, 4, 5, 6, 7, 8, 9 but also the long-overlooked symbol 0 for zero, which originated in India. Realizing the need for zero was undoubtedly a major contribution to written mathematics, and hence to mathematical thought. But numerals never *had* to be written. As Mazur shows, with illustrations of many ancient devices, computation was done perfectly well for thousands of years with counting rods or the similar device, the abacus, where zero corresponds to an empty space. Indeed, as he remarks on p. 32, concerning Chinese counting rods:

> The fundamental operations of arithmetic in this rod system are identical with those of the Hindu-Arabic system.

After making this important point quite early in the book, Mazur lingers rather too long, in my opinion, on the story of Arabic numerals and the zero symbol. It is an interesting story, but one that has often been told before.

Arithmetic with Arabic numerals came to Europe in medieval times. The best known, though not the first, source is Fibonacci's *Liber abbaci* (Book of calculation) of 1202. As one can see from the title of the book, calculation was then synonymous with abacus calculation. Fibonacci's goal was to explain how calculation could be done in writing, that is, *without* the abacus. This goal was not easily achieved, because there was really nothing wrong with abacus calculation, other than the lack of a written record. In the marketplace, particularly in Asia, the abacus was used until the 1970s, when it was finally overtaken by the electronic calculator. Among mathematicians, written calculation caught on earlier, paving the way for more general forms of calculation in algebra and calculus.

Mazur takes up the story of algebra in the second part of his book, finally getting past Arabic numerals on p. 80. The word "algebra" comes from the Arabic title of a book by al-Khwārizmi from around 830 CE, a time generally reckoned to mark the recognition of algebra as a separate discipline—one focused on the solving of equations. However, with hindsight we can interpret some of Euclid's propositions as algebra, for example

$$(a+b)^2 = a^2 + b^2 + 2ab.$$

As Euclid shows in the *Elements*, Book II, Proposition 4, a square of side $a+b$ (the left-hand side) equals a square of side a plus a square of side b plus twice the rectangle with adjacent sides a and b (the right-hand side). The algebra of al-Khwārizmi and his successors systematically developed the algebraic part of Euclid, while sticking faithfully to his geometric methods of proof. Despite an early attempt at symbolic algebra by Diophantus (around 100 CE), whose works were known in the Arab world, geometric proofs persisted in algebra until the 16th century. Mazur glosses over this point in his treatment of Arabic algebra; it is made more clearly in the recent history of algebra by Katz and Parshall (2014). But he correctly points out that the great algebraic breakthrough of the early 16th century—the solution of cubic equations—was explained by Cardano (1545) in geometric language.

Algebra as we know it, with symbols for unknowns, addition, subtraction, multiplication, division, and equality did not develop until the late 16th century. Only then was it possible to contemplate algebraic objects, such as polynomials and equations, and hence to use truly algebraic methods of proof. As Mazur reminds us on p. 92, Dantzig (2005) said it best: "the letter liberated algebra from the slavery of the word." This is another important point, but again we have to wait a while before seeing what it means. There are some long, albeit interesting, historical digressions before we see the outcome of algebra's liberation. It happens on p. 150, in a chapter entitled **The Explosion**.

By 1637, when Descartes published his *La Géométrie*, algebraic notation had become almost the same as we use today. With the notation came the ability to manipulate polynomials and solve equations with such ease that *algebra and geometry changed places*. Instead of using geometry to explain algebra, algebra could be used to explain geometry—more simply and often almost mechanically. Readers of the *Bulletin* will be well aware of what came next: algebraic geometry, calculus, and the great explosion of differential geometry and mathematical physics in the 18th and 19th centuries. Mazur only hints at these developments as he is approaching the end of his book, though he takes time to praise Leibniz's notation in calculus and Newton's contributions to physics.

In the final part of the book, Mazur sums up the ways in which symbolism brought enlightenment to mathematics in the period before the 20th century. This is a reasonable place to stop, since the 20th century brought a flood of new ideas in symbolism—in new areas such as logic and computation—that would require an entirely different book. He also discusses some recent psychological experiments on the 'reading' of formulas, which seem to reveal an ability to detect the syntax of symbols similar to our ability to read the syntax of words. More on this, and on the related question of spatial visualization, may be found in the review of Mazur's book by Arianhod (2015). At any rate, the experiments seem to confirm what mathematicians have believed since the time of Descartes: in mathematics, symbols often capture our thoughts better than words.

References

Arianrhod, Robyn. 2015. *Enlightening symbols: a short history of mathematical notation and its hidden powers*, Notices Amer. Math. Soc. **62**, no. 2, 148–151. MR3243610

Cajori, F. 1929. *A History of Mathematical Notations, Volume 1: Notations in elementary mathematics, Volume 2: Notations mainly in higher mathematics*, Open Court, Chicago.

Cardano, G. 1545. *Ars magna.* 1968 translation *The great art or the rules of algebra* by T. Richard Witmer, with a foreword by Oystein Ore., The M.I.T. Press, Cambridge, MA–London.

Dantzig, T. 2005. *Number, the Language of Science*, Pi Press, New York. Reprint of the 1954 edition, edited by Joseph Mazur.

Katz, Victor J. and Karen Hunger Parshall. 2014. *Taming the unknown. A history of algebra from antiquity to the early twentieth century*, Princeton University Press, Princeton, NJ. MR3237138

Schwartzman, Steven. 1994. *The words of mathematics. An etymological dictionary of mathematical terms used in English*, MAA Spectrum, Mathematical Association of America, Washington, DC. MR1270906 (95e:00013)

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