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ABOUT THE COVER: THE NEWTON–LEIBNIZ CONTROVERSY

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Isaac Newton (1642–1727) and Gottfried Wilhelm von Leibniz (1646–1716) are well known for having invented (or discovered) the calculus in the late 17th century. Right? Well, it may be a bit more complicated than that. Let us run through a few dates to review just what was happening at that time. There is evidence that Newton had some ideas about the calculus as early as his famous *annus mirabilis*, 1666, and there were hints of calculus in the work of his predecessors in England, in particular the writings of John Wallis and Isaac Barrow. The history is complicated by the fact that Leibniz visited London in 1673 and by that time, in a sense, calculus was probably in the air. But the notes he took on his travels, when examined well after the trip, did not shed light on any conversations Leibniz had with colleagues in England. By 1676, both Newton and Leibniz were claiming that they had solved problems of series and quadrature. Newton went so far as to send off some of his ideas on these subjects to Henry (Heinrich) Oldenburg, the secretary of the Royal Society. In a general way John Collins and James Gregory were aware of Newton's work, but they made no mention of fluxions (Newton's word) specifically. Leibniz asked Newton to explain his methods, and Newton answered with an explanation of fluxions and fluents using an anagram:

$$6a\ cc\ d\ \text{æ}\ 13e\ ff\ 7i\ 3l\ 9r\ 4o\ 4qrr\ 4s\ 8t\ 12vx.$$

It did not help. It stands for the sentence, "*Data æquatione quotcunque fluentes quantitates involvente fluxiones invenire, et vice versa.*" Or, "Having any given equation involving never so many flowing quantities to find the fluxions, and vice versa." It still didn't help. Leibniz went ahead and forwarded to John Collins an explanation of the principle and notation involved in the differential calculus. That part of the epic ended with the death of Oldenburg (1677). Only later did Leibniz publish, in 1684, his famous paper in the *Acta Eruditorum*: "*Nova methodus pro maximis et minimis.*" So while Newton and his friends continued to withhold

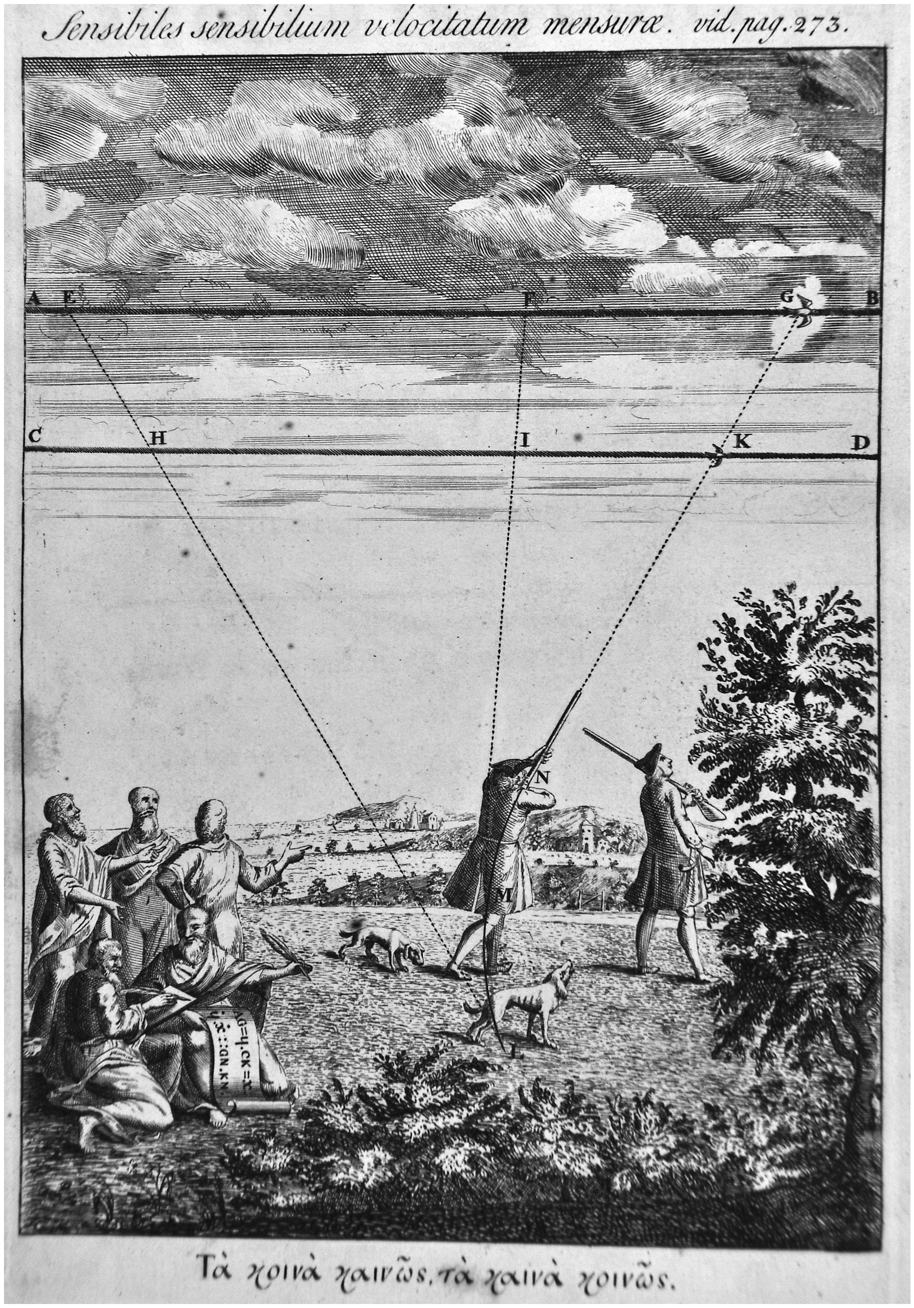

FIGURE 1. CLEAR COPY OF THE COVER. Frontispiece of John Colson's edition of Newton's *The Method of Fluxions and Infinite Series*, 1736, that illustrates a problem in the text of finding and using fluxions, the best position for a hunter to take to kill two birds with one shot, if the paths and velocities of the two birds are well defined in advance.

information about their brand of calculus, Leibniz put it out there in one of the most widely read scientific journals of that period, and it swept the Continent, particularly with the help of Jacques Bernoulli (1655–1705) and his brother Jean (1667–1748), the Marquis de l'Hôpital (1661–1704), as well as Christiaan Huygens (1629–1695) with his famous work on the catenary. As was inevitable, though, while this was happening on the Continent, some of the catenary work was being carried out by James Gregory on the other side of the Channel.

Leibniz enjoyed for 15 years, the widespread view that he was the man who discovered calculus, without much discussion of the matter until the Swiss Fatio de Duillier, a mildly disreputable young man, claimed in 1699 that in fact Newton had discovered calculus first and Leibniz had used the trip to London as a shortcut to gain priority in the question of discovery. Now the arguments took on a more strident turn, with John Keill (1671–1721) taking on the defense of Newton, "with more zeal than judgment," in the words of Cajori [1]. The issue seemed to shift away from the question of who had the ideas first—it was pretty clearly Newton—but one of whether Leibniz came up with the ideas independently. By the time of the publication of the third edition of Newton's *Principia* (1726) Newton had removed Leibniz's name. Newton died the following year but the wrangling continued. One of the first attacks on calculus was by a philosopher-metaphysician, George Berkeley (1685–1753), Bishop of Cloyne, who wrote *The Analyst*, published in 1734, after Newton's death. It was Berkeley who found the idea of finite ratios of absolutely evanescent things absurd—calling them "the ghosts of departed quantities." Others joining the fray around the time of Newton's death were Brook Taylor (1685–1731), Abraham de Moivre (1667–1754), Roger Cotes (1682–1716), and Colin Maclaurin (1698–1746). Though Taylor, de Moivre, and Maclaurin have basic formulas or theorems in elementary calculus named for them, Cotes may be the most interesting person on this list, but he died young, at age 33.

Of one thing we can be sure: the controversy stimulated a lot of early 18th century publishing. Every one of these characters produced a book or two or three, using (sometimes contrary to good judgement) methods best left to historians to quibble over. The ultimate victor had to be Leibniz because of his choice of notation that is stimulating and easy to use, thus pushing along progress in various branches of mathematics and its applications. In a translation of part of Newton's *Principia*, John Colson (1680–1760) analysed a calculus problem using Newton's fluxions (derivatives) and fluents (integrals). We reprint some of the text here, partly to justify including the evocative frontispiece as the cover image for this issue (see Figure 1). As Colson says [2],

> This I shall here attempt, to perform, in a familiar way, by the instance of a Fowler, who is aiming to shoot two Birds at once, as is represented in the Frontispiece. Let us suppose the right Line AB to be parallel to the Horizon, or level with the Ground, in which a Bird is now flying at G, which was lately at F, and a little before at E. And let this Bird be conceived to fly, not with an equable or uniform swiftness, but with a swiftness that always increases, (or with a Velocity that is continually accelerated,) according to some known rate. Let there also be another right Line CD, parallel to the former, at the same or any other convenient distance from the Ground, in which another Bird is now flying at K, which was lately

> at I, and a little before at H; just at the same points of time as the first Bird was at G, F, E, respectively. But to fix our Ideas, and to make our Conceptions the more simple and easy, let us imagine this second Bird to fly equably, or always to describe equal parts of the Line CD in equal times. Then may the equable Velocity of this Bird be used as a known measure, or standard, to which we may always compare the inequable Velocity of the first Bird. Let us now suppose the right Line EH to be drawn, and continued to the point L, so that the proportion (or ratio) of the two Lines EL and HL may be the same as that of the Velocities of the two Birds, when they were at E and H respectively. And let us farther suppose, that the Eye of a Fowler was at the same time at the point L, so that the proportion (or ratio) of the two Lines EL and HL, may be the same as that of the Velocities of the two Birds, when they were at E and H respectively. And let us farther suppose, that the Eye of a Fowler was at the same time at the point L, and that he directed his Gun, or Fowling-piece, according to the right Line LHE, in hopes to shoot both the Birds at once. But not thinking himself then to be sufficiently near, he forbears to discharge his Piece, but still pointing it at the two Birds, he continually advances towards them according to the direction of his Piece, till his Eye is presently at M, and the Birds at the same time in F and I, in the same right Line FIM. And not being yet near enough, we may suppose him to advance farther in the same manner, his Piece being always directed or level'd at the two Birds, while he himself walks forward according to the direction of his Piece, till his Eye is now at N, and the Birds in the same right Line with his Eye, at K and G.

It goes on and on and on. Surely it is an example of "calculus made difficult." Leibniz's notation and economy of language would be a welcome relief from all of these words.

The full Newtonian solution to the above problem can be found in Colson's book. This was not Colson's only contribution to calculus; he is also known as the translator of Gaetana Maria Agnesi's book on calculus, translated from Italian into English. Colson died before it could be published in 1799 as *Analytic Institutions in Four Books*. Of course, when considering the frontispiece of Colson's books on Newton, one cannot avoid the fact that bird lovers will be offended by the example. At any rate the followers of Newton continued to write about fluents and fluxions, but mathematics flourished on the Continent. But with Newton's appointment at the Royal Mint, Newton went on to be rich and famous, showered with honors as only the British can confer them. Of course Newton's place in history is no doubt well deserved [5]. There's the lavish tomb in Westminster Abbey to prove it. Leibniz, on the other hand, died poor, and his demise was little noticed, with, it was reported, a handful of friends showing up for his funeral. And the location of his grave is no longer known. One could claim that it is better to die rich. But at that point, does it really matter?

In Ludovici's encomium [4] we encounter the first Leibniz bibliography (544 items). It also includes an account of all of his known poetry written in Greek, Latin, French, and German. Here we reproduce a folding plate from the book showing

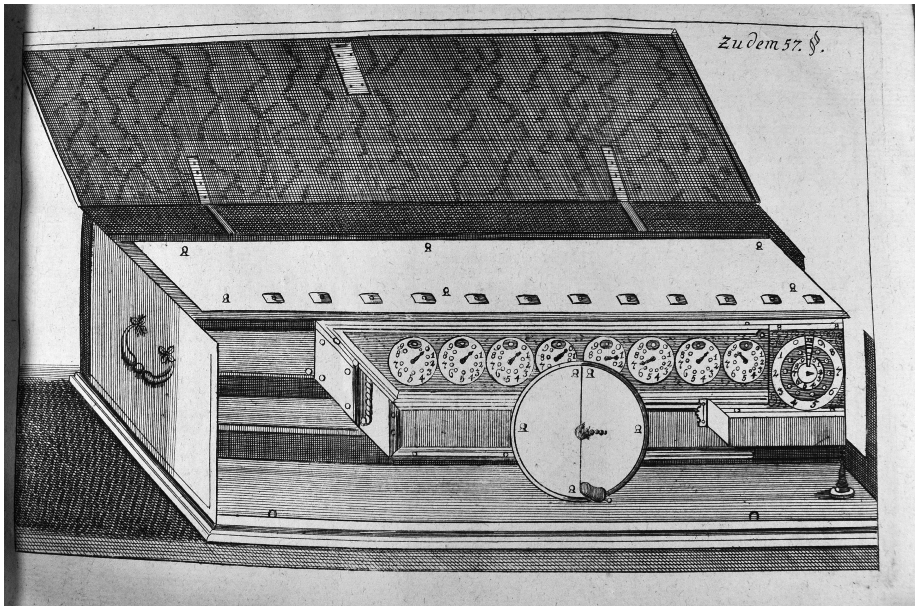

FIGURE 2. A plate showing Leibniz's calculating machine from *Historie der Leibnitzischen Philosophie* by Carl Günther Ludovici, 1737

an engraving of Leibniz's famous calculating machine; other plates show binomial expressions and a barometer. For his calculating machine Leibniz is revered by the computer science community. A tiny volume of his, the *Ars Combinatoria* [3], was acquired from a Berkeley dealer in 1973 for the modest price of $150. This 82-page volume, only a second edition at that, was surprisingly appraised a few years back by a London expert as worth $60,000. Oh that one could count on that kind of appreciation with one's whole collection.

References

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[2] John Colson, *The Method of Fluxions and Infinite Series, with its Application to the Geometry of Curve-lines. By the Inventor Sir Isaac Newton, Kt. Late President of the Royal Society. Translated from the Author's Latin Original not yet made publick. To which is subjoin'd A Perpetual Comment upon the whole Work, Consisting of Annotations, Illustrations, and Supplements, in order to make this Treatise A compleat Institution for the use of Learners*, Woodfall, London, 1736.

[3] Gottfried Leibniz, *Ars Combinatoria*, 2nd edition, Croker, Frankfurt, 1690.

[4] Carl Günther Ludovici, *Ausführlicher Entwurft einer vollständigen Historie der Leibnitzischen Philosophie*, Verlegts Johann Georg Löwe, Leipzig, 1737.

[5] Frank E. Manuel, *A Portrait of Isaac Newton*, Belknap Press (of Harvard University Press), Cambridge, MA, 1968.

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