MATHEMATICAL PERSPECTIVES

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 53, Number 4, October 2016, Pages 673–679
http://dx.doi.org/10.1090/bull/1541
Article electronically published on June 30, 2016

ABOUT THE COVER: RAMANUJAN IN BRONZE

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Srinivasa Ramanujan's name is one of the best known in the history of mathematics. The romantic story of someone born into a family of modest means, raised in a small city in southern india, and who goes on to become one of the greatest mathematicians in all of recorded history has been told again and again. So beyond giving a memory-refreshing outline of the story I shall try to avoid repeating too many well-known facts.

The young Ramanujan learned some mathematics by reading a few available but long out-of-date mid-19th century English books, one by G. S. Carr (*A Synopsis of Elementary Results in Pure and Applied Mathematics*), a collection of formulas and statements of theorems, seldom with any proofs. He then proceeded to collect in notes amazing conjectures while learning, largely on his own, whole parts of number theory and analysis. He did not survive long in college because he was not interested in most of the required curriculum. He only cared about mathematics. In 1911–1912 he and other Indian mathematicians started sending his discoveries to prominent mathematicians in England, but these were essentially ignored until he sent some of his work to the greatest English mathematician of the time, G. H. Hardy, who recognized that Ramanujan had rediscovered some known but significant results, had guessed wrong in some cases, but had also come up with conjectures that were startlingly deep, important, and unknown up to that time [13]. Hardy arranged to have Ramanujan come to England, which he did in 1914, against the wishes of his mother, who on religious (and probably personal) grounds did not want him to leave India. At Cambridge he worked with Hardy, J. E. Littlewood, G. N. Watson, and others until his health failed for reasons that are not entirely clear. Tuberculosis has been mentioned, along with the foul winter weather in England, which contrasted with the warm climate of southern India. Being a Brahmin, he was a vegetarian and therefore had great difficulty in finding agreeable food that neither contained meat nor animal fats used in the preparation. There are various other more exotic

conjectures, but in the end we do not know for sure what all contributed to his early death in 1920, only months after he returned to India.

One of Ramanujan's best-known discoveries was an exact formula for calculating the number of partitions, $p(n)$, of a positive integer n. This problem involves calculating the number of ways of writing n as a sum of positive integers (including n itself) and ignoring the order in which the numbers appear. For example, it is easy to see that; $p(5) = 7$. In the mid-18th century Euler had found a recursion formula to calculate values of $p(n)$, but until Ramanujan there was no direct formula known. Hardy and Ramanujan, using their famous "circle method", found the exact formula for $p(n)$, "an achievement undertaken and mostly completed by G. H. Hardy and S. Ramanujan and fully completed and perfected by H. Rademacher. . . . This unbelievable identity [which we will not quote here] wherein the left-hand side is the humble arithmetic function $p(n)$ and the right-hand side is an infinite series involving π, square roots, complex roots of unity, and derivatives of hyperbolic functions, provides not only a theoretical formula for $p(n)$ but also a formula which admits relatively rapid computation" [4, pp. 68–70]. It's a stunning formula, providing, for example, the value of $p(200) : 3972999029388$.

Ramanujan's notes were filled with extraordinary formulas and observations that often went far beyond what contemporaries had observed, let alone proved. People have claimed, certainly correctly, that he was almost without equal as a mathematical genius, and some have gone on to say that in some ways he went beyond Einstein—everybody's idea of what a mathematical genius should be (and look like). But there was a difference. Einstein was basing his startling observations on a solid background in mathematics and physics at first-class places, Zurich's Polytechnic (ETH) and the University in Berlin. Ramanujan, by contrast, was largely self-educated.

The narrative, beyond a few stories published in India during Ramanujan's early years, essentially begins with G. H. Hardy's obituary essay in the *Proceedings of the London Mathematical Society* and the *Proceedings of the Royal Society*, reprinted in [8, pp. xl–lviii], where Hardy tells of Ramanujan's arrival in England where his work dazzled the mathematical community. (My own copy of this book is the one Hardy gave to Pólya at the time of publication.) Within a few years Ramanujan was being honored with membership in the Royal Society of London and election as Fellow of Trinity College, Cambridge, both in 1918. In 1940 Hardy wrote about him again in [9], an essay on his life and his work in England.

In 1991 Robert Kanigel published a splendid biography, *The Man Who Knew Infinity* [10], which probably remains the best source of information on Ramanujan's life. A movie with the same name as the Kanigel book was released in April 2016 in the U.S. and received widespread and positive reviews. The cast was star studded—Hardy is played by Oscar-winning actor Jeremy Irons, Ramanujan by Dev Patel, and lesser roles by Stephen Fry and Jeremy Northam. It was largely filmed at Cambridge and in India. An earlier film on Ramanujan, *The Man Who Loved Numbers*, was shown on public television in the United States in 1988 as part of the *NOVA* series. It was a beautiful film and contained a memorable interview with Ramanujan's wife, Janaki Ammal, who was still alive when the film was made. She had married Ramanujan at the age of nine in a marriage arranged by his mother. The couple lived together for a rather short time before he left for England, so they had little time together until he came back to India and worked on a manuscript

known later as the "lost notebooks" in the months remaining before his death. In the film Janaki is interviewed and this tiny, frail woman, then in her 90s, said "All I can tell you is that day and night he worked on sums. He didn't do anything else. He wasn't interested in anything else. He wouldn't stop work even to eat. We had to make rice balls for him and placed them in the palm of his hand. Isn't that extraordinary?" Through history there must have been many spouses of mathematicians who would sympathize with this woman as they wondered about how their husbands or wives spent their time, "doing their sums".

As recently as 2007 David Leavitt, a writer who had earlier ventured into mathematical biography with his *The Man Who Knew Too Much: Alan Turing and the Invention of the Computer* (W. W. Norton, 2006), published a novel about Ramanujan, *The Indian Clerk: A Novel* [12]. Though it was awarded a favorable front-page review in the *New York Times Sunday Book Review* section, it was not to everyone's taste. The great librarian and book expert, Lawrence Clark Powell, once remarked: "I believe a good work of fiction about a place is a better guide than a bad work of fact" [11, p. 16]. That might in some cases be true, but if one substitutes "person" for "place", it's risky. Leavitt had gotten into trouble with an earlier work, *While England Sleeps*, a novel that the English poet Stephen Spender claimed was based on his memoirs and he charged that Leavitt misconstrued the facts. So Spender sued. Leavitt made the mistake of publishing his book while Spender was still alive. The case was settled out of court, but the book had to be revised and reissued by the publisher. In the case of *The Indian Clerk*, the cast members were no longer alive and able to object. It raises a serious question about the wisdom of writing historical fiction. It can so easily change into fictional history and spread misinformation. I did not finish reading Leavitt's book because I feared that in time I would come to assume that conversations reported in the book actually took place, when in fact they almost certainly did not.

On the other hand, there was quite a splendid play about Ramanujan, *Partition*, written by Ira Hauptman and produced at the Aurora Theatre in Berkeley in 2003. The principal characters were Ramanujan, Hardy, a fictional classicist named Billington, the Hindu Goddess Namagiri of Namakkal (in the play clad in a colorful and elegant sari), and Pierre de Fermat. Now that's clearly fiction because Fermat lived in a different century, but it made sense to include Fermat in a piece of theatre because the story worked in a fictional interest on the part of Ramanujan in solving Fermat's Last Theorem [12]. Ramanujan told people that he received some of his ideas from the family deity, the Goddess Namagiri, who came to him in his sleep and "would write equations on his tongue" [10, p. 36]. When people tell me this, I assure them that I have been a witness to it. I saw it happen on a stage in Berkeley.

Krishnaswami Alladi tells in his review of the play in *The Hindu* [2] that when he and George Andrews arrived in San Francisco for meetings of the American Mathematical Society that year and found that *Partition* was playing, they decided they had to see it, but seats were scarce. (They would have been, of course. It was a play about mathematics!) Miraculously, two seats were available for a Saturday night performance. Alladi credits this good fortune to the intervention of the Goddess Namagiri! A wide-ranging review of the play by Kenneth A. Ribet appeared in the *Notices of the American Mathematical Society* [14]. The story of

Ramanujan had reached the musical world even earlier: an opera, *Ramanujan*, by Sandeep Bhagwati, was premiered in Munich, April 21, 1998 [5].

More recently, a multimedia presentation by Complicité (earlier known as the Théâtre de Complicité), *A Disappearing Number*, by Simon McBumey (music by Nitin Sawhney), opened in Plymouth, England, in 2007, and later played at various theatre festivals in Holland, Germany, and Austria, as well as at the National Theatre in London. Eventually, it was broadcast to cinemas worldwide via *National Theatre Live*. The plot ran over two time periods, a historical section in Cambridge when Ramanujan and Hardy met, as well as a contemporary and fictional account of a mathematician and her husband, paralleling in a general way the earlier story. The playwright was inspired by his reading Hardy's *A Mathematician's Apology*. The play enjoyed considerable success, receiving several theatre awards, including the prestigious 2008 Laurence Olivier Award for Best Play. Unfortunately, it did not play widely in the United States (only in 2010 in Ann Arbor, Michigan, and at the Lincoln Center Festival in New York), but it did appear in many venues abroad, in Milan, Barcelona, Paris, Sydney, and, not surprisingly, in Mumbai and Hyderabad, where it played during the International Congress of Mathematicians in the summer of 2010.

Ramanujan left a legacy of provocative formulas that have prompted generations of mathematicians to try to understand his conjectures and eventually provide proofs. Three American mathematicians have been at the forefront in continuing these investigations: George Andrews of the Pennsylvania State University, Richard Askey at University of Wisconsin–Madison, and Bruce Bemdt at University of Illinois at Urbana–Champaign. Bemdt has described a pilgrimage in India in [6] and publishd two volumes on Ramanujan's notebooks [7]. The first is a touching account Berndt gives of a trip to India to visit Ramanujan's home and schools and to meet members of his family. An earlier article by Andrews appeared in the *American Mathematical Monthly* in 1979 [3] and included accounts of some of Ramanujan's later mathematical work to complement those provided earlier by Hardy, who had no access to the "lost notebooks" that Andrews discovered in some papers of another mathematician at Trinity College, Cambridge, in 1976: roughly 100 pages of densely packed formulas written in Ramanujan's hand. Periodically, there appear summaries of progress on proving the conjectures in the notebooks. And the work goes on.

With so much having been written about Ramanujan over the many years since his death, one might wonder why someone as relatively unfamiliar with the subject as I am, would attempt to add to the literature about Ramanujan, other than for the pleasure I have had in reviewing the elegant pieces written by Hardy and others, including Hardy's short summary of his own life, his *A Mathematician's Apology*. One should never pass up an opportunity to read that small book again, something that Atle Selberg referred to as "a great piece of literature" in a conversation we had in 1999 [1, p. 266].

And now to the curious event that prompted this note. In August 2011, I received an email from the noted geometer and Escher expert, Doris Schattschneider, informing me that a Bronze bust of Ramanujan was coming up for auction in Philadelphia at Freeman's (the oldest active American auction house). She wondered whether I knew of anyone who would be interested. The lot was hidden away at the end of a catalogue of American paintings, drawings, and sculpture, not likely

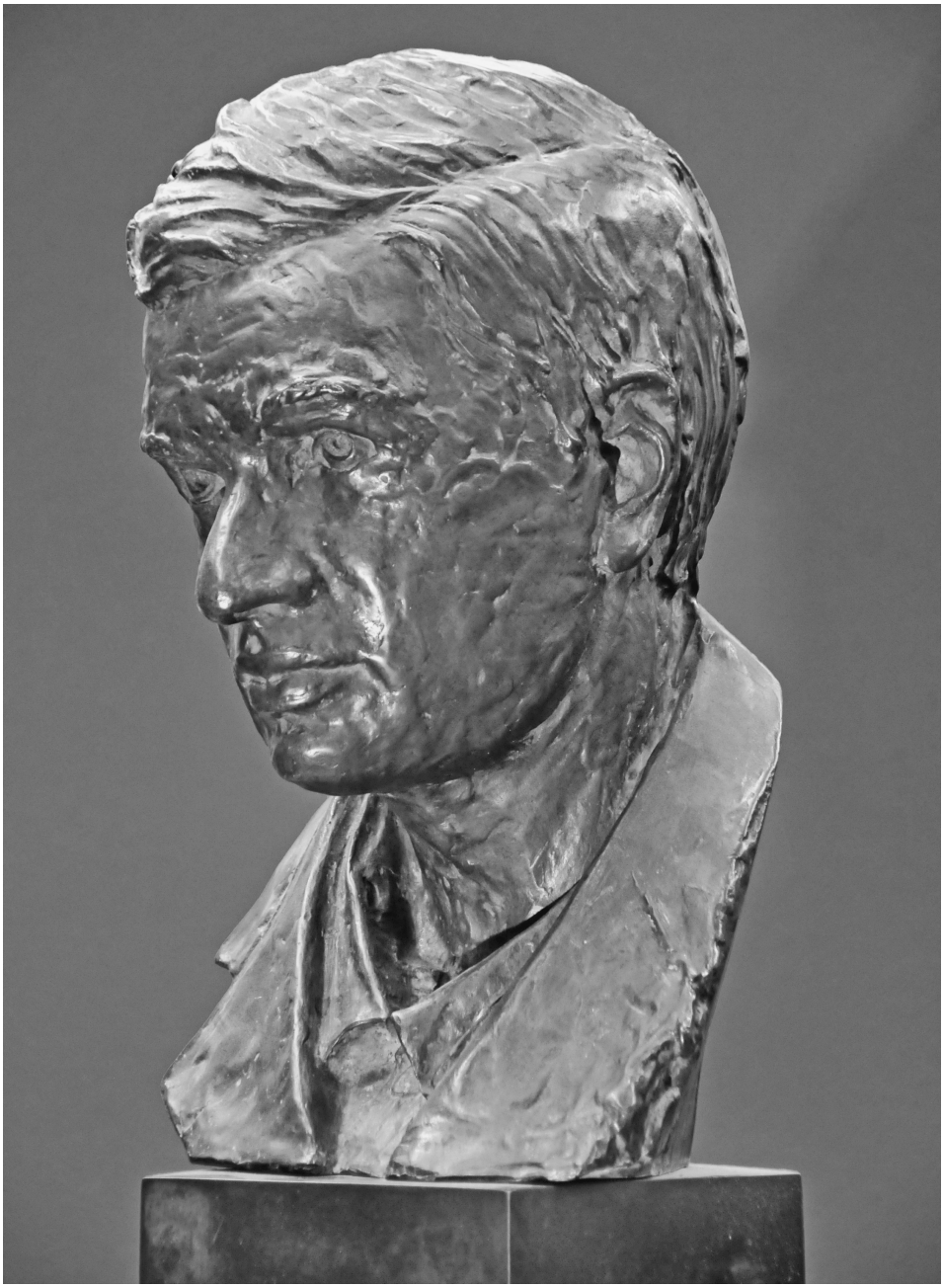

FIGURE 1. CLEAR COPY OF THE COVER. Bronze bust of Ramunajan by Paul T. Granlund

to attract wide attention in the mathematical world. Well, the answer was obvious. I bid on it and the bust is now in my office. But the story is complicated. Kanigel mentioned in his biography that Ramanujan's wife Janika had raised the question with Richard Askey of why, after her being promised that there would be a statue

of Ramanujan in his home town, Kumbakonam (he was born nearby in Erode), she was still waiting for a statue. Bemdt calls Kumbakonam a town, though it has a population of roughly 150,000. Askey responded by commissioning the American sculptor. Paul T. Granlund, to produce a bust using the passport photograph taken when Ramanujan left for England. Granlund was a prolific artist, perhaps best known for his sculpture of Charles Lindbergh at Le Bourget outside Paris, the airfield where Lindbergh landed after his 1927 solo flight over the Atlantic. Other casts of this can be seen on the grounds of the State Capitol in St. Paul, Minnesota, and at the terminal at Lindbergh Field in San Diego. I contacted Askey to find out where my copy of the sculpture might have come from, and in his response he outlined the history of the work [5]. There were ten copies cast in 1983 (plus an artist's proof). One was given to Ramanujan's wife, Janika, and is now in the Ramanujan Institute of Mathematical Sciences in Madras (Chennai). Four others are in India, at research institutes in Delhi, Poona, Bangalore, and Mumbai. One that was originally acquired by S. Chandrasekhar is in London at the headquarters of the Royal Society, another in a building near the Isaac Newton Institute at Cambridge. The other three are in the United States, one owned by Askey, one by George Andrews, and the one in my office. (The artist's proof is at Gustavus Adolphus College in Minnesota, where Granlund was artist-in-residence.) My copy is almost certainly the one that was once owned by James Vaughn, the Texas oilman and philanthropist who was for many years generous to various mathematical organizations. He died in 2007.

So, this is the explanation for the picture of the Granlund bronze bust on the cover of this issue

Endnote: All right. How much did I pay for it? Someone is sure to ask. The auction house estimate in the sale catalogue was already low—US$1,000–1,500—and I was ready to bid far more than that. But when the lot came up, the auctioneer at the sale announced that the opening bid for that lot would have to be $500. I bid that, and there were no further bids. Of course, I paid considerably more than that since there was a 25% premium on the hammer price as well as an even more alarming bill for putting it in a crate and shipping it to California. Nevertheless, it was something of a steal.

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