The Work of John Tate

J.S. Milne

Tate helped shape the great reformulation of arithmetic and geometry which has taken place since the 1950s.

Andrew Wiles.1

This is an exposition of Tate’s work, written on the occasion of the award to him of the Abel prize. True to the epigraph, I have attempted to explain it in the context of the “great reformulation”.

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