

INTRODUCTION TO THE ATIYAH ISSUE

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Sir Michael Atiyah (1929–2019) was a giant figure in twentieth century mathematics. His publications, beginning in 1952, span nearly 70 years and he made fundamental contributions to the development of many fields, both directly through his own work and through his wider influence, inspiration, and leadership.

Atiyah began his career in Cambridge, England, but spent a formative year (1955–56) at the Institute for Advanced Study, Princeton. There he met, among others, his long-time collaborators Bott, Hirzebruch, and Singer (making up “the gang of four” as Yau has affectionately referred to them). A few years later Atiyah began his monumental work developing the sister subjects of K -theory (with Hirzebruch) and index theory (with Singer). While the focus of this work was in algebraic topology and global analysis, many of the ideas were rooted in algebraic geometry, the area in which Atiyah had begun his research and to which he made important contributions. During the bulk of this period Atiyah held the Savilian chair in Oxford before another spell 1971–73 in the Institute for Advanced Study, this time as a permanent member. A few years after his return to Oxford, Atiyah’s research took a new direction in which, partly with Singer, he was a pioneer and leader in the development of the interaction between modern geometry and physics, a development which has had a profound influence, reaching into nearly all areas of mathematics, over the half century since.

Many tributes and personal recollections of Atiyah have been published since his death. These include the article of Connes and Kounieher [7] and a collection of recollections [10] both of which appear in the *Notices of the AMS*. An article of Hitchin [11] gives an inside account of the interaction between geometry and physics in Atiyah’s work. All these convey the exceptional quality of Atiyah’s character and talents, both as a mathematician and more broadly. As a mathematician, Atiyah was driven by a search for beauty, depth of understanding and connections between ideas—not particularly interested in “problem solving”. One example is his multiple treatments of the Bott periodicity theorem (see the article of Segal in this issue) including one of his most beautiful papers [1]. The theorem drops out from a general construction (the index of families) and a wonderfully simple but subtle observation, somewhat reminiscent of the way one evaluates a one-dimensional Gaussian integral by going to two dimensions. (In brief, the question involves a relation between two spaces X and $X \times \mathbf{R}^2$ and the trick is to take the product with an extra factor of \mathbf{R}^2 to get $X \times \mathbf{R}^2 \times \mathbf{R}^2$ and then exploit a symmetry between the last two factors.)

Atiyah filled many prominent positions and was an outstanding advocate and spokesman for mathematics and science. In 1990 he curtailed his research activity for a while to take up three simultaneous positions: President of the Royal Society in London, Master of Trinity College, and founding Director of the Isaac Newton Institute in Cambridge. Before that he had served as President of the London Mathematical Society and later, after his nominal retirement to Edinburgh, he was President of the Royal Society of Edinburgh.

Perhaps Atiyah's most exceptional quality was as a communicator. He liked to develop his mathematical ideas in conversations, for which he had enormous stamina, and—talking to everybody—he was able to make seminal connections between previously unrelated fields. His lectures were an inspiration; frequently developing some simple idea in a beautiful and surprising direction in which, as if by magic, all difficulties disappeared—under his guidance the audience would see the whole picture. His writing, both in mathematical papers and for more general readership, had the same qualities and is gathered together in the seven volumes of *Collected works* published by Oxford University Press. (Within these Atiyah explains the depth of preparation that would go into his magical lectures.) Last, but far from least, Atiyah was an extremely kind man, thoughtful for others, as many of the recollections in [10] attest.

This issue of the *Bulletin* contains three articles on Atiyah's work. Graeme Segal focuses on his work in algebraic topology [12], Dan Freed on index theory [9], and my own contribution on his early work in algebraic geometry and later work influenced by the gauge theories coming from physics [8]. As the reader will see, we have taken slightly different approaches to the task and there are some overlaps between the accounts which we have not tried to remove—all of these topics fitted together within Atiyah's overall vision. On the other hand we have not covered everything: there are very substantial contributions we do not discuss. These include work on representation theory [4], contributions to analysis of distributions [2], hyperbolic partial differential equations [3], and, later, to knot and manifold invariants such as [5] and configuration spaces [6].

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