

MATHEMATICAL PERSPECTIVES

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 59, Number 4, October 2022, Pages 607–609
<https://doi.org/10.1090/bull/1778>
Article electronically published on June 30, 2022

ABOUT THE COVER: EDUARD HELLY

HERWIG HAUSER

Eduard Helly, born in Vienna in 1884, is known to every other math student through his famous theorem in discrete geometry:

If in a collection of $k \geq n + 1$ convex sets in \mathbb{R}^n every $n + 1$ sets have a common intersection, then all sets meet in a common point.

This striking result was discovered by Helly around 1913, though a written proof of it was provided by him only in 1923 [He1, DGK]. Already in 1921, Johann Radon published a proof of this theorem, using his own result that any set of $n + 2$ points in \mathbb{R}^n can be partitioned into two sets whose convex hulls intersect. He gives full credit to Helly. Another proof was found independently by Denes König.

It is less well known that if Helly had received due credit early in his career for having proved other ground breaking results, Helly's name would also be coupled with the Hahn–Banach and the Banach–Steinhaus theorems.

To wit, in a spectacular paper entitled “On linear functional operators” [He2] from 1912, Helly formulated and proved, among other results, a predecessor of the Hahn–Banach theorem, in the version of characterizing finitely many values c_i such that there exists a bounded linear functional on the space $\mathcal{C}[a, b]$ of continuous functions on a real interval having the prescribed values at given functions f_i . Helly's ideas were used by Hahn and Banach later on, without, however, mentioning his name. Also, the concept of Banach space is already designed in his paper of 1912, long before Banach's dissertation in 1920. See the article by Harry Hochstadt for more mathematical details [Ho].

Brilliant as a student of Wilhelm Wirtinger and Franz Mertens, Helly got a one year fellowship to Göttingen after his dissertation in 1907. From 1908 on Helly lectured in Viennese high schools and *Volksbildungseinrichtungen*, a kind of workers' university. Heavily wounded in World War I (his heart became dislocated by a gunshot) and captured by the Russian army, he spent four years in Siberian internment camps. There he met and taught Tibor Radó, who, after escaping from the camp and struggling his way back to Hungary on his own, solved in 1930 the Plateau problem (as also Jesse Douglas did).



FIGURE 1. About the cover: Eduard Helly. Reprinted by permission from Copyright Clearance Center: Springer Nature, The Mathematical Intelligencer, *Eduard Helly, father of the Hahn–Banach theorem*, Harry Hochstadt. 1980

Helly was released only in 1920 and able to return to Vienna. There he married the mathematician Elise Bloch, who was his long-time fiancée. At the age of 37, he just had two publications, though excellent ones. A next paper “On systems of linear equations with infinitely many unknowns” was immediately recognized by the mathematical community and accepted by *Monatshefte* [He3]. Despite having the reputation as an outstanding mind and enjoying the support of several senior mathematicians, Helly was not hired in academia. Instead, he found positions at insurance companies, culminating in his role as actuary at the second largest company in Europe of that time, Phoenix Life Insurance. This company crashed in 1936 amid a huge swamp of corruption. The employees, including Helly, found themselves on the streets. Helly and his wife supported themselves by giving private lessons.

In this period, he regularly attended the Mathematical Seminar at Boltzmann-gasse 5 in Vienna’s ninth district, and the famous Café Central, where chess playing

became one of his other passions. Among others, he became a close friend of Richard Réti, the Czechoslovakian chess grandmaster.

In 1938 the situation in Vienna changed dramatically with the *Anschluss*. Helly and his family, who were Jewish, were forced to emigrate to the United States. Again they earned their living through teaching school children. Finally, Albert Einstein and Hermann Weyl were able to provide a position for Helly and his wife at the College of Paterson, New Jersey, in 1939. In 1943, and after a few other positions as instructor, Helly was appointed as a full professor at the Illinois Institute of Technology in Chicago. At long last Helly had a stable academic position. However, sadly, his ill-fortunes continued and a heart attack led to his death in November 1943 at the age of 59.

Photographs of Helly rarely show him smiling (see Figure 1). But he was known as a cheerful, sympathetic person, gentle in personal communication, and, overall, as a lucid and long-sighted mathematician.

ACKNOWLEDGMENTS

We are indebted to Peter Michor, Wolfgang Ruppert, and Karl Sigmund for valuable help in the preparation of this note and for providing access to their biographic texts [RM] and [Si].

REFERENCES

- [DGK] L. Danzer, B. Grünbaum, and V. Klee, *Helly's theorem and its relatives*, Proc. Sympos. Pure Math., Vol. VII, Amer. Math. Soc., Providence, R.I., 1963, pp. 101–180. MR0157289
- [He1] E. Helly, *Über Mengen konvexer Körper mit gemeinschaftlichen Punkten*, Jahresberichte der Deutschen Mathematischen Vereinigung, **32** (1923), 175–176.
- [He2] E. Helly, *On linear functional operators*, Sitzungsberichte der Akademie der Wissenschaften Wien, **121** (1912), 265–297.
- [He3] E. Helly, *Über Systeme linearer Gleichungen mit unendlich vielen Unbekannten* (German), Monatsh. Math. Phys. **31** (1921), no. 1, 60–91, DOI 10.1007/BF01702713. MR1549097
- [Ho] H. Hochstadt, *Eduard Helly, father of the Hahn–Banach theorem*, Math. Intelligencer **2** (1979/80), no. 3, 123–125, DOI 10.1007/BF03023052. MR595079
- [RM] W. Ruppert and P. Michor, *Mathematik in Österreich und die NS-Zeit*, Springer (to appear).
- [Si] K. Sigmund, *Failing Phoenix: Tauber, Helly, and Viennese Life Insurance*, Math. Intelligencer **26** (2004), no. 2, 21–33, DOI 10.1007/BF02985648. MR2067894