

BULLETIN of the

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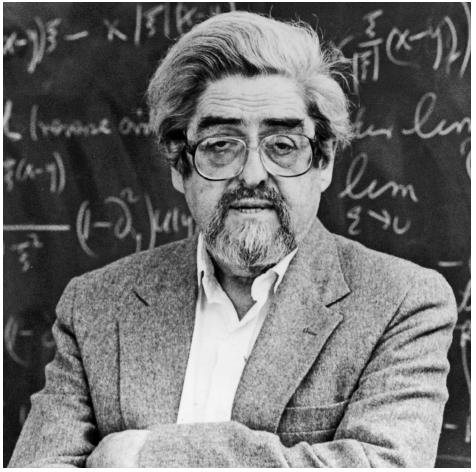
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Louis Nirenberg (1925-2020)

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