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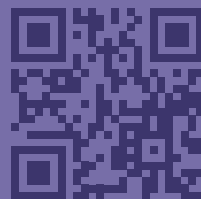
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Articles

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Book Reviews

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BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY
CONTENTS

Vol. 61, No. 4

October 2024

ARTICLES

Mohammed Abouzaid and Nathaniel Bottman, Functoriality in categorical symplectic geometry	525
Alin Bostan, Xavier Caruso, and Julien Roques, Algebraic solutions of linear differential equations: An arithmetic approach	609
Roland Donninger, Spectral theory and self-similar blowup in wave equations	659

BOOK REVIEWS

János Kollár (Reviewer), Complex algebraic threefolds , by M. Kawakita .	687
Todor Tsankov (Reviewer), Coarse geometry of topological groups , by C. Rosendal	693

INDEX TO VOLUME 61 (2024)

BULLETIN ARTICLES

- Abouzaid, Mohammed, and Nathaniel Bottman. *Functoriality in categorical symplectic geometry*, 525
- Adem, Alejandro. *A note from the chief editor*, 199
- Avigad, Jeremy. *Mathematics and the formal turn*, 225
- Bengio, Yoshua, and Nikolay Malkin. *Machine learning and information theory concepts towards an AI mathematician*, 457
- Bostan, Alin, Xavier Caruso, and Julien Roques. *Algebraic solutions of linear differential equations: An arithmetic approach*, 609
- Bottman, Nathaniel. *See* Abouzaid, Mohammed
- Buzzard, Kevin. *Mathematical reasoning and the computer*, 211
- Caruso, Xavier. *See* Bostan, Alin
- Cheng, Eugenia. *How machines can make mathematics more progressive*, 305
- Cohn, Henry. *From sphere packing to Fourier interpolation*, 3
- Commelin, Johan, and Adam Topaz. *Abstraction boundaries and spec driven development in pure mathematics*, 241
- Davies, Alex. *Working with machines in mathematics*, 387
- Davis, Ernest. *Mathematics, word problems, common sense, and artificial intelligence*, 287
- De Toffoli, Silvia. *Proofs for a price: Tomorrow's ultra-rigorous mathematical culture*, 395
- DeDeo, Simon. *AlephZero and mathematical experience*, 375
- Donninger, Roland. *Spectral theory and self-similar blowup in wave equations*, 659
- Eur, Christopher. *Essence of independence: Hodge theory of matroids since June Huh*, 73
- Fraser, Jonathan M., and Liam Stuart. *A new perspective on the Sullivan dictionary via Assouad type dimensions and spectra*, 103
- Fraser, Maia. *See* Poggio, Tomaso
- Fraser, Maia, Andrew Granville, Michael H. Harris, Colin McLarty, Emily Riehl, and Akshay Venkatesh. *Will machines change mathematics?*, 201
- . *Will machines change mathematics?*, 373
- Friedlander, Susan. *Commentary*, 1
- Granville, Andrew. *Missing digits and good approximations*, 23
- . *Proof in the time of machines*, 317
- . *See* Fraser, Maia
- Hairer, Martin. *A stroll around the critical Potts model*, 55
- Harris, Michael. *Automation compels mathematicians to reflect on our values*, 331
- Harris, Michael H. *See* Fraser, Maia
- Malkin, Nikolay. *See* Bengio, Yoshua
- McLarty, Colin. *Poincaré on the value of reasoning machines*, 411
- . *See* Fraser, Maia
- Ochigame, Rodrigo. *Automated mathematics and the reconfiguration of proof and labor*, 423
- Poggio, Tomaso, and Maia Fraser. *Compositional sparsity of learnable functions*, 438
- Riehl, Emily. *See* Fraser, Maia
- Roques, Julien. *See* Bostan, Alin
- Şavk, Oğuz. *A survey of the homology cobordism group*, 119
- Shulman, Michael. *Strange new universes: Proof assistants and synthetic foundations*, 257
- Stuart, Liam. *See* Fraser, Jonathan M.
- Topaz, Adam. *See* Commelin, Johan
- Venkatesh, Akshay. *Some thoughts on automation and mathematical research*, 203
- . *See* Fraser, Maia
- Williamson, Geordie. *Is deep learning a useful tool for the pure mathematician?*, 271

MATHEMATICAL PERSPECTIVES

- Chen, Gui-Qiang G. *Morawetz's contributions to the mathematical theory of transonic flows, shock waves, and partial differential equations of mixed type*, 159
- Friedlander, Susan. *Cathleen Morawetz*, 159
- Romeo, Giuliano. *Continued fractions in the field of p -adic numbers*, 343
- Ronney, Paul. *See* Xin, Jack

INDEX TO VOLUME 61 (2024)

Xin, Jack, Yifeng Yu, and Paul Ronney. *Lagrangian, game theoretic, and PDE methods for averaging G-equations in turbulent combustion: existence and beyond*, 470
Yu, Yifeng. *See* Xin, Jack

BOOK REVIEWS

Fernández-Real, X., and X. Ros-Oton. *Regularity theory for elliptic PDE*, reviewed by Connor Mooney, 515
Kawakita, M. *Complex algebraic threefolds*, reviewed by János Kollár, 687
Michalek, Mateusz, and Bernd Sturmfels. *Invitation to nonlinear algebra*, reviewed by Alicia Dickenstein and Giorgio Ottaviani, 185
Rosendal, C. *Coarse geometry of topological groups*, reviewed by Todor Tsankov, 693
Ros-Oton, X. *See* Fernández-Real, X.
Sturmfels, Bernd. *See* Michalek, Mateusz

BULLETIN

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VOLUME 61 | 2024

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BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY
CONTENTS

Vol. 61, No. 1

January 2024

Articles

Susan Friedlander, Commentary	1
Henry Cohn, From sphere packing to Fourier interpolation	3
Andrew Granville, Missing digits and good approximations	23
Martin Hairer, A stroll around the critical Potts model	55
Christopher Eur, Essence of independence: Hodge theory of matroids since June Huh	73
Jonathan M. Fraser and Liam Stuart, A new perspective on the Sullivan dictionary via Assouad type dimensions and spectra	103
Oğuz Şavk, A survey of the homology cobordism group	119

Mathematical Perspectives

Susan Friedlander, Cathleen Morawetz	159
Gui-Qiang G. Chen, Morawetz's contributions to the mathematical theory of transonic flows, shock waves, and partial differential equations of mixed type	159
Selected Mathematical Reviews	171

Book Reviews

Alicia Dickenstein and Giorgio Ottaviani (Reviewers), Invitation to nonlinear algebra , by Mateusz Michałek and Bernd Sturmfels	185
---	-----

Vol. 61, No. 2

April 2024

Articles

Alejandro Adem, A note from the chief editor	199
Maia Fraser, Andrew Granville, Michael H. Harris, Colin McLarty, Emily Riehl, and Akshay Venkatesh, Will machines change mathematics?	201
Akshay Venkatesh, Some thoughts on automation and mathematical research	203
Kevin Buzzard, Mathematical reasoning and the computer	211
Jeremy Avigad, Mathematics and the formal turn	225
Johan Commelin and Adam Topaz, Abstraction boundaries and spec driven development in pure mathematics	241
Michael Shulman, Strange new universes: Proof assistants and synthetic foundations	257
Geordie Williamson, Is deep learning a useful tool for the pure mathematician?	271

Ernest Davis , Mathematics, word problems, common sense, and artificial intelligence	287
Eugenia Cheng , How machines can make mathematics more congressive .	305
Andrew Granville , Proof in the time of machines	317
Michael Harris , Automation compels mathematicians to reflect on our values	331

Mathematical Perspectives

Giuliano Romeo , Continued fractions in the field of p-adic numbers	343
---	-----

Vol. 61, No. 3

July 2024

Articles

Maia Fraser , Andrew Granville , Michael H. Harris , Colin McLarty , Emily Riehl , and Akshay Venkatesh , Will machines change mathematics?	373
Simon DeDeo , AlephZero and mathematical experience	375
Alex Davies , Working with machines in mathematics	387
Silvia De Toffoli , Proofs for a price: Tomorrow’s ultra-rigorous mathematical culture	395
Colin McLarty , Poincaré on the value of reasoning machines	411
Rodrigo Ochigame , Automated mathematics and the reconfiguration of proof and labor	423
Tomaso Poggio and Maia Fraser , Compositional sparsity of learnable functions	438
Yoshua Bengio and Nikolay Malkin , Machine learning and information theory concepts towards an AI mathematician	457

Mathematical Perspectives

Jack Xin , Yifeng Yu , and Paul Ronney , Lagrangian, game theoretic, and PDE methods for averaging G-equations in turbulent combustion: existence and beyond	470
---	-----

Book Reviews

Connor Mooney (Reviewer), Regularity theory for elliptic PDE , by X. Fernández-Real and X. Ros-Oton	515
--	-----

Vol. 61, No. 4

October 2024

Articles

Mohammed Abouzaid and Nathaniel Bottman , Functoriality in categorical symplectic geometry	525
--	-----

Alin Bostan, Xavier Caruso, and Julien Roques , Algebraic solutions of linear differential equations: An arithmetic approach	609
Roland Donninger , Spectral theory and self-similar blowup in wave equations	659

Book Reviews

János Kollár (Reviewer), Complex algebraic threefolds, by M. Kawakita .	687
Todor Tsankov (Reviewer), Coarse geometry of topological groups, by C. Rosendal	693

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CONTENTS

Vol. 61, No. 4

October 2024

ARTICLES

- Mohammed Abouzaid and Nathaniel Bottman**, Functoriality in categorical symplectic geometry 525
- Alin Bostan, Xavier Caruso, and Julien Roques**, Algebraic solutions of linear differential equations: An arithmetic approach 609
- Roland Donninger**, Spectral theory and self-similar blowup in wave equations 659

BOOK REVIEWS

- János Kollár** (Reviewer), Complex algebraic threefolds, by M. Kawakita . 687
- Todor Tsankov** (Reviewer), Coarse geometry of topological groups, by C. Rosendal 693

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