CONFORMAL GEOMETRY AND DYNAMICS
An Electronic Journal of the American Mathematical Society
Volume 13, Pages 136–138 (May 26, 2009)
S 1088-4173(09)00193-3

CORRIGENDUM TO "FREE SUBGROUPS OF SURFACE MAPPING CLASS GROUPS"

JAMES W. ANDERSON, JAVIER ARAMAYONA, AND KENNETH J. SHACKLETON

Abstract. We provide a corrigendum to the results of Conform. Geom. Dyn. **11** (2007), 44–55, pointing out an error in the proofs of Propositions 4.3 and 5.4 and providing corrected statements.

The proofs of Propositions 4.3 and 5.4 in [1] are incorrect: controlling the length of the path q does not imply that q is a quasi-geodesic, and thus part (3) of Theorem 4.2 cannot be applied here. Consequently the estimate for the constant R given in the statements of Proposition 5.4, Corollary 5.5, Corollary 5.6, and Theorem 5.7 does not apply.

In Proposition A below, we offer a corrected version of Proposition 5.4, for a constant R_0 that still depends only on the translation distances of the pseudo-Anosovs and the distance between their axes. However, we add that the method of proof does not allow us to obtain an explicit estimate for R_0, unlike that quoted in Proposition 5.4.

Proposition A also recovers Proposition 4.3 sufficiently to deduce the generation of free subgroups, treating the pertinent case of geodesic axes for pseudo-Anosov mapping classes. This said, the result on generation of free subgroups we are able to obtain is now a somewhat straightforward consequence of the work of Ivanov. It seems to be unknown whether Proposition A holds for all precompact Teichmüller geodesics.

Subsequent to the publication of [1] more general results regarding the generation of free subgroups of the mapping class groups were obtained by Fujiwara [2] and Mangahas [4].

Proposition A. *Given a surface* Σ *and constants* $L, D > 0$, *there exists* $R_0 = R_0(L, D, \Sigma) \geq 0$ *such that the following holds: Let* ϕ, ψ *be independent pseudo-Anosov mapping classes of translation distance at most* L, *whose axes are at distance at most* D. *Let* $c, c' : \mathbb{R} \to T(\Sigma)$ *be arc-length parametrizations of their respective axes so that* $d_T(O, O') = d_T(c(\mathbb{R}), c'(\mathbb{R}))$, *where* $O = c(0)$ *and* $O' = c'(0)$. *Then, for all* $x \in c(\mathbb{R}) \setminus B_{R_0}(O)$ *and all* $y \in c'(\mathbb{R}) \setminus B_{R_0}(O')$,

$$d_T(x, y) \geq \ max\ \{d_T(O, x), d_T(O', y)\}.$$

Proof. Let $\lambda \in$ Fix (ϕ) and $\mu \in$ Fix (ψ). Since ϕ and ψ are independent, λ and μ are distinct and together fill Σ. There exists a unique biinfinite Teichmüller geodesic from λ to μ, which we denote by $[\lambda, \mu]$. Let $M = M(\phi, \psi) > 0$ be such that

Received by the editors January 13, 2009.
2000 *Mathematics Subject Classification.* Primary 20F65; Secondary 57M50.

$d_T([\lambda,\mu],[O,O']) \leq M$, where $[O,O']$ denotes the Teichmüller geodesic segment from O to O'.

By Proposition 5.1 of [3], if (x_n) and (y_n) are sequences in $T(\Sigma)$ converging to λ and μ, respectively, the Teichmüller geodesic segments $[x_n,y_n]$ from x_n to y_n accumulate on $[\lambda,\mu]$ as $n\to\infty$. Using this, and since c and c' are both bi-infinite geodesics, we may take $R_0 = R_0(M) > 0$ such that for all $x \in c(\mathbb{R}) \setminus B_{R_0}(O)$ and for all $y \in c'(\mathbb{R}) \setminus B_{R_0}(O')$, we have

$$d_T(x,O), d_T(y,O') \geq 2(2M+D),$$

and

$$d_T([x,y],[O,O']) \leq 2M.$$

Let p be any point on $[x,y]$ at distance at most $2M$ from $[O,O']$. By the triangle inequality, and observing that $d_T(p,O), d_T(p,O') \leq 2M+D$, we have

$$d_T(x,p) \geq d_T(x,O) - (2M+D)$$

and

$$d_T(y,p) \geq d_T(y,O') - (2M+D).$$

Since $[x,y]$ is a geodesic, we see that

$$\begin{aligned} d_T(x,y) = d_T(x,p) + d_T(y,p) &\geq d_T(x,O) + d_T(y,O') - 2(2M+D) \\ &\geq \max\{d_T(x,O), d_T(y,O')\}. \end{aligned}$$

In the argument just given, the value of R_0 depends on the choice of ϕ and ψ. However, up to conjugation, there are only finitely many pairs of independent pseudo-Anosov mapping classes of translation distance at most L and whose axes are at distance at most D, by a result of Ivanov (stated as Theorem 2.1 in [1]) and the discreteness of MCG(Σ). Therefore, we may choose a $R_0 = R_0(L,D,\Sigma)$ that works for any two independent pseudo-Anosov mapping classes of translation distance at most L, whose axes are at distance at most D, as desired. □

One then obtains correct versions of Corollaries 4.4, 4.5, 5.5 and 5.6, and Theorem 5.7 of [1] by replacing R with R_0 and considering, instead of ϵ-precompact geodesics, axes of two pseudo-Anosov mapping classes whose translation distances are bounded above by a fixed constant and whose axes are within a fixed bounded distance of one another. We remark that the corrected version of Theorem 5.7 follows from Corollary 4.7 and the fact that there are only finitely many pairs of independent pseudo-Anosov mapping classes of translation distance at most L and whose axes are at distance at most D.

Acknowledgements

We thank Dick Canary for pointing out the error in [1]. We also thank Dick Canary and Juan Souto for conversations.

References

[1] J. W. Anderson, J. Aramayona, K. J. Shackleton, Free subgroups of mapping class groups, *Conformal Geometry and Dynamics* **11** (2007), 44–55. MR2295997 (2008a:20068)

[2] K. Fujiwara, Subgroups generated by two pseudo-Anosov elements in a mapping class group. I. Uniform exponential growth. In *Groups of Diffeomorphisms*, 283-296, ASPM 52, 2008, Mathematical Society of Japan.

[3] E. Klarreich, The boundary at infinity of the curve complex and the relative Teichmüller space, *Stony Brook preprint*, 1999.

[4] J. Mangahas, Uniform uniform exponential growth of subgroups of the mapping class group. Preprint, `arXiv:0805.0133`

School of Mathematics, University of Southampton, Southampton SO17 1BJ, England

Department of Mathematics, National University of Ireland, Galway, Ireland

University of Tokyo (IPMU), 5-1-5 Kashiwa-no-ha, Kashiwa-shi, Chiba, 277-8568 Japan