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CLASSIFICATION OF ADMISSIBLE NILPOTENT ORBITS IN SIMPLE EXCEPTIONAL REAL LIE ALGEBRAS OF INNER TYPE

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Abstract. In this paper we give a classification of admissible nilpotent orbits of the noncompact simple exceptional real Lie groups of inner type. We use a lemma of Takuya Ohta and some information from the work of Dragomir Djoković to construct a simple algorithm which allows us to decide the admissiblity of a given orbit.

Introduction

The notion of admissible coadjoint orbits was first introduced by M. Duflo twenty years ago [D]. He also provided a bijection between such orbits and unitary representations of nilpotent and type I solvable groups. This is an instance of the "Orbit method" introduced by A. A. Kirillov in 1962 for nilpotent Lie groups. The Orbit method/philosophy proposes that questions about representations of a Lie group can be understood in terms of data attached to coadjoint orbits in the dual of the Lie algebra of the group. The method works perfectly for nilpotent Lie groups [K]. In the case of a simply connected type I solvable Lie group, all orbits are admissible; the correspondence with representations is due to Auslander and Kostant [A-K]. For real reductive Lie groups there is substantial evidence linking admissible orbits and unitary representations. For example, standard methods such as parabolic induction can be used to associate unitary representations to admissible semisimple orbits [V]. However, the theory is not well developed for nilpotent orbits. For example we do not have a clear strategy to attach a representation to a general admissible orbit. In many cases Vogan and Barbash have proven that the set of irreducible representations obtained from the admissible nilpotent orbits or the unipotent representations are the building blocks for all unitary representations of the group [V]. In the case of real semisimple Lie groups the admissible nilpotent orbits seem to be very good candidates for which a general technique may be developed. They are known for the classical simple Lie groups. This paper describes them for all exceptional non-compact real simple Lie groups of inner type. Results for the other two real simple exceptional Lie groups may be found in [No2], which is appearing concurrently with this paper.

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Summary of known results

Let G be a real semisimple Lie group with Lie algebra $\mathfrak{g}$ and recall the *coadjoint representation* of G on $\mathfrak{g}^*$, the dual space of $\mathfrak{g}$, defined as follows:

$$Ad_g^*(f)(E) = f(Ad_{g^{-1}}(E)) \qquad g \in G \quad \text{ and } \quad E \in \mathfrak{g}.$$

The differential of the above representation on $\mathfrak{g}$ is

$$ad_X^*(f)(E) = -f([X, E]) \qquad X \in \mathfrak{g}.$$

Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ be a Cartan decomposition of $\mathfrak{g}$. It is known that there exists a G-invariant non-degenerate bilinear form $\langle , \rangle$ on $\mathfrak{g}$ which is negative definite on $\mathfrak{k}$ and positive definite on $\mathfrak{p}$. We will make use of such a form to identify coadjoint orbits in $\mathfrak{g}^*$ with adjoint orbits in $\mathfrak{g}$. Therefore we shall determine admissible nilpotent orbits of G in $\mathfrak{g}$. We will consider the cases where G is adjoint or simply connected.

For $\lambda \in \mathfrak{g}^*$ define $G^\lambda = \{g \in G : g(\lambda) = \lambda\}$ and $\mathfrak{g}^\lambda = \{X \in \mathfrak{g} : \lambda([X, \mathfrak{g}]) = 0\}$. Then there is a non-degenerate symplectic form ω_λ on $\mathfrak{g}/\mathfrak{g}^\lambda$ given by $\omega_\lambda(X+\mathfrak{g}^\lambda, Y+\mathfrak{g}^\lambda) = \lambda([X, Y])$. Moreover, G^λ preserves ω_λ. Denote by $Sp(\omega_\lambda)$ the symplectic group defined by ω_λ. Define the group

$$\tilde{G}^\lambda = \{(g, m) \in G^\lambda \times M(\omega_\lambda) : \psi(g) = \pi(m)\}$$

where $M(\omega_\lambda)$ is the *metaplectic group* associated with $Sp(\omega_\lambda)$, ψ is a natural homomorphism from G^λ to $Sp(\omega_\lambda)$ and π is defined as follows:

$$1 \longrightarrow \{1, \xi\} \longrightarrow M(\omega_\lambda) \xrightarrow{\pi} Sp(\omega_\lambda) \longrightarrow 1,$$

$M(\omega_\lambda)$ is a two-fold covering of $Sp(\omega_\lambda)$ which can be pulled back to give $\tilde{G}^\lambda$ as a double cover of G^λ.

Definition 1 [D]**.** A representation (ρ, V) of $\tilde{G}^\lambda$ is called admissible if $\rho(\xi) = -1_V$ and $d\rho(E) = \sqrt{-1}\lambda(E)1_V$ for all $E \in \mathfrak{g}^\lambda$. The linear functional λ is said to be admissible if $\tilde{G}^\lambda$ has at least one admissible representation.

An element E of $\mathfrak{g}$ is admissible if and only if its image λ_E under $\langle , \rangle$ is admissible in $\mathfrak{g}^*$.

We note that in the case of the trivial nilpotent orbit the covering $\tilde{G}^\lambda$ is trivial. Hence the trivial nilpotent orbit is always admissible. This is the orbit to which one would want to attach the trivial representation in the Orbit Method scheme. From now on we will concern ourselves with non-zero nilpotent orbits.

J. Schwartz [Sch] translated the admissibility of real nilpotent orbits into the admissiblity of nilpotent elements in complex symmetric spaces via the so-called Kostant-Sekiguchi [Se] correspondence. T. Ohta [O] used this technique to determine admissibility of nilpotent orbits in the classical Lie algebras. We should also point out that Schwartz has classified nilpotent admissible orbits of several classical Lie groups.

The Kostant-Sekiguchi correpondence. Let $\mathfrak{g}$ be a real semisimple Lie algebra with adjoint group G and $\mathfrak{g}_{\mathbb{C}}$ its complexification. Also let $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ be a Cartan decomposition of $\mathfrak{g}$. Finally, let θ be the corresponding Cartan involution of $\mathfrak{g}$ and σ be the conjugation of $\mathfrak{g}_{\mathbb{C}}$ with regard to $\mathfrak{g}$. Then $\mathfrak{g}_{\mathbb{C}} = \mathfrak{k}_{\mathbb{C}} \oplus \mathfrak{p}_{\mathbb{C}}$ where $\mathfrak{k}_{\mathbb{C}}$ and $\mathfrak{p}_{\mathbb{C}}$ are obtained by complexifying $\mathfrak{k}$ and $\mathfrak{p}$ respectively. Denote by $K_{\mathbb{C}}$ the connected subgroup of the adjoint group $G_{\mathbb{C}}$ of $\mathfrak{g}_{\mathbb{C}}$, with Lie algebra $\mathfrak{k}_{\mathbb{C}}$.

A triple (x, e, f) in $\mathfrak{g}_{\mathbb{C}}$ is called a standard triple if $[x, e] = 2e$, $[x, f] = -2f$ and $[e, f] = x$. If $x \in \mathfrak{k}_{\mathbb{C}}$, e and $f \in \mathfrak{p}_{\mathbb{C}}$, then (x, e, f) is a normal triple. It is a result of

Kostant and Rallis [K-R] that any nilpotent e of $\mathfrak{p}_{\mathbb{C}}$ can be embedded in a standard normal triple (x, e, f). Moreover, e is $K_{\mathbb{C}}$-conjugate to a nilpotent e' inside of a normal triple (x', e', f') with $\sigma(e') = f'$ [Se]. The triple (x', e', f') will be called a *Kostant-Sekiguchi* or KS-triple.

Every nilpotent E' in $\mathfrak{g}$ is G-conjugate to the element E of a triple (H, E, F) in $\mathfrak{g}$ with the property that $\theta(H) = -H$ and $\theta(E) = -F$ [Se]. Such a triple will be called a KS-triple also.

Define a map c from the set of KS-triples of $\mathfrak{g}$ to the set of normal triples of $\mathfrak{g}_{\mathbb{C}}$ as follows:

$$\begin{aligned} x = c(H) &= i(E - F), \\ e = c(E) &= \frac{1}{2}(H - i(E + F)), \\ f = c(F) &= \frac{1}{2}(H + i(E + F)). \end{aligned}$$

The triple (x, e, f) is called the Cayley transform of (H, E, F). It is easy to verify that the triple (x, e, f) is a KS-triple and that $x \in i\mathfrak{k}$. The Kostant-Sekiguchi correspondence [Se] gives a one to one map between the set of G-conjugacy classes of nilpotents in $\mathfrak{g}$ and the $K_{\mathbb{C}}$-conjugacy classes of nilpotents in $\mathfrak{p}_{\mathbb{C}}$. This correspondence sends the zero orbit to the zero orbit and the orbit through the nilpositive element of a KS-triple to the one through the nilpositive element of its Cayley transform.

Let e be nonzero nilpotent in $\mathfrak{p}_{\mathbb{C}}$. Then $K_{\mathbb{C}}^e$ acts $\mathfrak{k}_{\mathbb{C}}/\ \mathfrak{k}_{\mathbb{C}}^e$ and $(\mathfrak{k}_{\mathbb{C}}/\mathfrak{k}_{\mathbb{C}}^e)^*$. Define the character δ_e of $K_{\mathbb{C}}^e$ as follows:

$$\delta_e(g) = (det(g|_{\mathfrak{k}_{\mathbb{C}}/\mathfrak{k}_{\mathbb{C}}^e}))^{-1} \quad g \in K_{\mathbb{C}}^e.$$

Using δ_e and the homomorphism $s : \mathbb{C}^\times \to \mathbb{C}^\times$, with $s(z) = z^2$ we obtain the following double cover of $K_{\mathbb{C}}^e$:

$$\tilde{K}_{\mathbb{C}}^e = \{(g, z) \in K_{\mathbb{C}}^e \times \mathbb{C}^\times : \delta_e(g) = z^2\}.$$

It turns out (see [V1]) that this covering is precisely the one induced on K^E, when $E = c(e)$ and λ is identified with E by the metaplectic covering $M(\omega^\lambda)$. Schwartz used such a covering to show that admissibility of real nilpotent orbits is equivalent to admissibility of nilpotent orbits in complex symmetric spaces [V1].

Definitition 2 [V1]**.** A representation χ of $K_{\mathbb{C}}^e$ is said to be admissible if its differential is half the differential of δ_e. The nilpotent e is admissible if $K_{\mathbb{C}}^e$ has at least one admissible representation.

The following theorem is due to J. Schwartz [Sch].

Theorem [Schwartz]. *There is a natural bijection between the equivalence classes of nilpotent admissible G-orbits and the equivalence classes of nilpotent admissible $K_{\mathbb{C}}$-orbits.*

Proof. See [V1], Lemma 7.8, Theorem 7.11, and Theorem 7.14. □

In fact, the question of admissibility of nilpotent orbits can be translated into a question on the representation of the identity component $(K_{\mathbb{C}}^e)_\circ$ of the group $K_{\mathbb{C}}^e$. See [V].

As Vogan pointed out the character δ_e is difficult to compute explicitly from the above description. However, Takuya Ohta [O] has found an explicit description of

δ_e which we shall discuss below. We will need some notation. Let (x, e, f) to be a KS-triple with $x \in i\mathfrak{k}$. From the representation theory of $\mathfrak{sl}_2$, $\mathfrak{g}_{\mathbb{C}}$ has the following eigenspace decomposition:

$$\mathfrak{g}_{\mathbb{C}} = \bigoplus_{j\in\mathbb{Z}} \mathfrak{g}_{\mathbb{C}}^{(j)} \qquad \text{where } \mathfrak{g}_{\mathbb{C}}^{(j)} = \{z \in \mathfrak{g}_{\mathbb{C}} | [x, z] = jz\}.$$

Similarly we have

$$\mathfrak{k}_{\mathbb{C}} = \bigoplus_{j\in\mathbb{Z}} \mathfrak{k}_{\mathbb{C}}^{(j)} \qquad \text{where } \mathfrak{k}_{\mathbb{C}}^{(j)} = \{z \in \mathfrak{k}_{\mathbb{C}} | [x, z] = jz\}$$

and

$$\mathfrak{p}_{\mathbb{C}} = \bigoplus_{j\in\mathbb{Z}} \mathfrak{p}_{\mathbb{C}}^{(j)} \qquad \text{where } \mathfrak{p}_{\mathbb{C}}^{(j)} = \{z \in \mathfrak{p}_{\mathbb{C}} | [x, z] = jz\}.$$

Moreover, the centralizers of e in $\mathfrak{k}_{\mathbb{C}}$ and $\mathfrak{p}_{\mathbb{C}}$ decompose as

$$\mathfrak{k}_{\mathbb{C}}^e = \bigoplus_{j\in\mathbb{Z}} (\mathfrak{k}_{\mathbb{C}}^e \cap \mathfrak{k}_{\mathbb{C}}^j) = \bigoplus_{j\geq 0} (\mathfrak{k}_{\mathbb{C}}^e \cap \mathfrak{k}_{\mathbb{C}}^j)$$

and

$$\mathfrak{p}_{\mathbb{C}}^e = \bigoplus_{j\in\mathbb{Z}} (\mathfrak{p}_{\mathbb{C}}^e \cap \mathfrak{p}_{\mathbb{C}}^j) = \bigoplus_{j\geq 0} (\mathfrak{p}_{\mathbb{C}}^e \cap \mathfrak{p}_{\mathbb{C}}^j).$$

It is a known fact that $\mathfrak{k}_{\mathbb{C}}^e = \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \oplus u_e$, where $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{k}_{\mathbb{C}}^e \cap \mathfrak{k}_{\mathbb{C}}^0$ is a reductive subalgebra of $\mathfrak{k}_{\mathbb{C}}^e$ and $u_e = \bigoplus_{j>0} (\mathfrak{k}_{\mathbb{C}}^e \cap \mathfrak{k}_{\mathbb{C}}^j)$ consists of nilpotent elements [S-S]. Denote by $d\delta_e$ the differential of δ_e. Then we have:

Lemma 1 [Ohta]. *The differential of δ_e is trivial on u_e. Suppose that $\mathfrak{t}_{\mathbb{C}}$ is a toral subalgebra of $\mathfrak{k}_{\mathbb{C}}$ containing x. Then the centralizer $\mathfrak{t}_{\mathbb{C}}^e$ of e in $\mathfrak{t}_{\mathbb{C}}$ is a toral subalgebra of $\mathfrak{t}_{\mathbb{C}}^{(x,e,f)}$ and $d\delta_e$ on $\mathfrak{t}_{\mathbb{C}}^e$ is given by*

$$d\delta_e(t) = \sum_{i\geq 1} tr(ad(t)|_{\mathfrak{k}_{\mathbb{C}}^i}) - \sum_{i\geq 2} tr(ad(t)|_{\mathfrak{p}_{\mathbb{C}}^i}) = -\sum_{i\geq 1} tr(ad(t)|_{\mathfrak{p}_{\mathbb{C}}^i}) + \sum_{i\geq 2} tr(ad(t)|_{\mathfrak{k}_{\mathbb{C}}^i}).$$

Proof. See Ohta [O]. □

Remark. If $\mathfrak{g}$ is itself a complex reductive Lie algebra, then one can show that δ_e is trivial. Hence all nilpotent elements of a complex reductive Lie algebra are admissible. See [O].

The preceding lemma suggests that admissibility is a question on tori of $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$. This is indeed the case as we shall see below.

Lemma 2 [Ohta]. *Let $\mathfrak{t}_1$ be a Cartan subalgebra of $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$ and T_1 the corresponding connected subgroup of $(K_{\mathbb{C}}^{(x,e,f)})_\circ$ the identity component of $K_{\mathbb{C}}^{(x,e,f)}$. Then e is admissible if and only if there exists a character, χ, of T_1 such that $\delta_e(g) = (\chi(g))^2$ for all $g \in T_1$.*

Proof. This follows easily from Definition 2 and Lemma 1. □

Corollary 1 [Ohta]. *Suppose that $T_1 \simeq (\mathbb{C}^\times)^r$. Then $d\delta_e|_{\mathfrak{t}_1}$ is a linear map $\mathbb{C}^r \longrightarrow \mathbb{C}$. From the previous lemma e is admissible if and only if each coefficient of z_i in the linear combination of $(z_1, z_2, \ldots z_r)$ is an even integer.*

Proof. Obvious from Lemma 2. □

Maintaining the above notation, recall that a nilpotent element $e \in \mathfrak{p}_{\mathbb{C}}$ is *even* if ad_x has only even eigenvalues. Schwartz (see also Vogan [V1], Corollary 7.28) has shown:

Theorem [Schwartz]. *Every even nilpotent element is admissible.*

The only exceptional real group for which admissibility has been completely determined is the non-compact real form of G_2. We refer the reader to M. Nevins [Ne] for a proof of the following theorem:

Theorem [Nevins]. *Let G be a real Lie group of type G_2 over $\mathbb{R}$. If G is simply connected, then all nilpotent coadjoint orbits are admissible. If G is adjoint, then only the 8-dimensional orbit fails to be admissible.*

New results

Noticed nilpotent elements of $\mathfrak{p}_{\mathbb{C}}$ are admissible. Maintaining the above notation we shall say that a nilpotent element $e \in \mathfrak{p}_{\mathbb{C}}$ is noticed if and only if $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} = 0$. The noticed nilpotents of $\mathfrak{p}_{\mathbb{C}}$ were used in [No] to classify nilpotent orbits of complex symmetric spaces. We deduce from Lemma 2:

Theorem A. *Every noticed nilpotent element of $\mathfrak{p}_{\mathbb{C}}$ is admissible.*

Proof. For such an element the $d\delta_e$ is trivial. □

Admissible nilpotent orbits of exceptional inner-type real Lie algebras. Our main result is the list of admissible nilpotent $K_{\mathbb{C}}$-orbits on $\mathfrak{p}_{\mathbb{C}}$ in the case where $\mathfrak{g}$ is an exceptional real simple Lie algebra of inner type that is $rank(\mathfrak{k}_{\mathbb{C}}) = rank(\mathfrak{g}_{\mathbb{C}}) = l$. We shall use the Bourbaki [Bo] system of roots for $\mathfrak{g}_{\mathbb{C}}$. Also our orbit numbering follows that of Djoković. From our previous remarks it is enough to consider non-noticed and non-even orbits of real forms of types E and F only. Consequently, we will consider the following groups: FI, FII, EII, EIII, EV, EVI, EVII, EVIII and EIX. First we will concern ourselves with the case where G is the adjoint Lie group of $\mathfrak{g}$, and then we will study admissibility under the simply connected cover G_{sc}. We observe that admissibility under the adjoint group implies admissibility under the simply connected cover.

From Lemma 2, it is clear that an explicit knowledge of the torus T_1 would be very helpful in determining admissible nilpotent orbits. However, such information may be hard to obtain from the data usually available. The fact that $\mathfrak{t}_{\mathbb{C}}$ is a Cartan subalgebra of $\mathfrak{k}_{\mathbb{C}}$ does not necessarily imply that $\mathfrak{t}_{\mathbb{C}}^{(x,e,f)}$ is a Cartan subalgebra of $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$. As an example let $\mathfrak{g}_{\mathbb{C}} = \mathfrak{sp}_2(\mathbb{C})$ and $\mathfrak{k}_{\mathbb{C}} = \mathfrak{u}_2$ and $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} = \mathfrak{so}_2(\mathbb{C})$. Following Bourbaki the compact roots of $\mathfrak{g}_{\mathbb{C}}$ are $\pm(\epsilon_1 - \epsilon_2)$ and the non-compact roots are $\pm(\epsilon_1+\epsilon_2)$, $\pm 2\epsilon_1$ and $\pm 2\epsilon_2$. Consider the normal triple (x, e, f) with $x = H_{2\epsilon_1}+H_{2\epsilon_2}$, $e = X_{2\epsilon_1} + X_{2\epsilon_2}$ and $f = X_{-2\epsilon_1} + X_{-2\epsilon_2}$. A computation shows that $\mathfrak{t}_{\mathbb{C}}^e = 0$ while $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$ has a one-dimensional Cartan subalgebra. We will show that in the case of the exceptional groups under consideration we will always be able to explicitly compute a maximal torus of $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$ for all the non-even and non-noticed nilpotent orbits.

In his classification of $K_{\mathbb{C}}$-nilpotent elements of $\mathfrak{p}_{\mathbb{C}}$ D. Djoković [D1] labelled each $K_{\mathbb{C}}$-orbit by the values of the simple roots of $\mathfrak{k}_{\mathbb{C}}$ on the neutral element x of the normal triple (x, e, f) associated to the orbit. He also gave a list of minimal regular semisimple subalgebras containing the nilpotent e up to $K_{\mathbb{C}}$-conjugacy classes and the type of $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$. Regular semisimple minimal algebras are used in the Dynkin classification. We refer the reader to [Dy] and [Dy1] for more information on such algebras.

Using information from Djoković and Ohta we design the following algorithm to determine admissibility under the adjoint group:

Algorithm for G adjoint group of $\mathfrak{g}$.
Begin

Let $\mathfrak{g}$ be one of the simple real Lie algebras under consideration.
1. Choose a set of simple roots for $\mathfrak{k}_{\mathbb{C}}$.
2. **For** each $K_{\mathbb{C}}$-nilpotent orbits in $\mathfrak{p}_{\mathbb{C}}$ **do**
   3. Compute a representative normal triple (x, e, f)
4. Compute all positive eigenspaces of ad_x on $\mathfrak{k}_{\mathbb{C}}$ and $\mathfrak{p}_{\mathbb{C}}$
5. Compute a maximal torus $\mathfrak{t}_{\mathbb{C}}^1 \subseteq \mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$
6. Compute $d\delta_e(t)$ where t is a generic element of $\mathfrak{t}_{\mathbb{C}}^1$
7. **If** the coefficient of all the terms in $d\delta_e(t)$ is even **Then**
   8. e is admissible
9. **Else**
   10. e is not admissible under the adjoint group G
11. **EndFor**

End.

Implementation of the algorithm. To implement step 1 we choose the system of simple roots given in Knapp [Kn]. Using the label given by Djoković we can compute the neutral element $x \in \mathfrak{k}_{\mathbb{C}}$.This is done by solving a simple system of linear equations.

An analysis of Djoković's tables reveals that each nilpotent in $\mathfrak{p}_{\mathbb{C}}$ sits in a minimal regular subalgebra $\mathfrak{s}$ such that rank $(\mathfrak{s})$ + rank $(\mathfrak{k}_{\mathbb{C}}^{(x,e,f)})$ = rank $(\mathfrak{g}_{\mathbb{C}})$. Therefore we always choose e to be a linear combination of root vectors generating $\mathfrak{s}$. This allows us to find a basis for $\mathfrak{t}_{\mathbb{C}}^1$.

Remark. In all the cases that we consider, a basis vector $H \in \mathfrak{t}_{\mathbb{C}}^1$ is always an integer linear combination of the vectors $H_{\alpha_1}, \ldots, H_{\alpha_l}$ corresponding to the Bourbaki simple roots $\Delta = \{\alpha_1, \ldots, \alpha_l\}$ of $\mathfrak{g}_{\mathbb{C}}$. We always choose $H = \sum\limits_{i=1}^{l} a_i H_i$ so that the nonzero $a_i's$ do not have a nontrivial common divisor. In other words, H is not a multiple of another vector. This is important to be sure that the value of $d\delta_e$ on H is interpreted correctly.

In addition to our own programs, we used the software LiE to compute the eigenspaces of ad_x. Mathematica 4 was also used to solve the system of equations. The computations were carried out on a powerbook Macintosh G3.

FII.

Theorem B. *Let $\mathfrak{g}$ be a real Lie algebra of type FII. Then all nilpotent orbits of G in $\mathfrak{g}$ are admissible.*

Proof. Since $\mathfrak{g}$ has only two nonzero nilpotent orbits of which one is even, hence admissible by Schwartz [Sch], we shall prove that the remaining non-even orbit is also admissible.

The previous theorem of Schwartz tells us that admissbility of nilpotent orbits of G in $\mathfrak{g}$ is equivalent to that of nilpotent orbits of $K_{\mathbb{C}}$ in $\mathfrak{p}_{\mathbb{C}}$. Therefore let $\Delta = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ be the Bourbaki system of simple roots of $\mathfrak{g}_{\mathbb{C}}$. Then $\Delta_k = \{\beta_1, \beta_2, \beta_3, \beta_4 : \beta_1 = \alpha_2 + 2\alpha_3 + 2\alpha_4, \beta_2 = \alpha_1, \beta_3 = \alpha_2, \beta_4 = \alpha_3\}$ is a system of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_4(\mathbb{C})$. The label for this orbit in [D] is $[0,0,0,1]$. We can compute the neutral element of a normal triple $\{x, e, f\}$ associated with the orbit. This is done by solving the system of equation $\beta_1(x) = 0$, $\beta_2(x) = 0$, $\beta_3(x) = 0$, $\beta_4(x) = 1$. We find $x = 2H_{\alpha 1} + 4H_{\alpha 2} + 3H_{\alpha 3} + H_{\alpha 4}$ where the $H_{\alpha i}$'s are elements of the Cartan subalgebra $\mathfrak{h}_{\mathbb{C}}$ defined by the roots α_i's. We choose $e = X_{\alpha_1+2\alpha_2+3\alpha_3+\alpha_4}$ where $X_{\alpha_1+2\alpha_2+3\alpha_3+\alpha_4}$ is a basis element of the root space $\mathfrak{g}_{\mathbb{C}}^{(\alpha_1+2\alpha_2+3\alpha_3+\alpha_4)}$. Consequently, $f = X_{-\alpha_1-2\alpha_2-3\alpha_3-\alpha_4}$. The following space $\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha 1} \oplus \mathbb{C}H_{\alpha 2} \oplus \mathbb{C}(H_{\alpha 3} + H_{\alpha 4})$ is a maximal torus of $\mathfrak{t}_{\mathbb{C}}^{(x,e,f)}$. Next we find the ad_x-eigenspaces needed to compute $d\delta_e$. We have

$$\begin{aligned}
\mathfrak{p}^1_{\mathbb{C}} &= \mathbb{C}X_{\alpha_2+2\alpha_3+\alpha_4} \oplus \mathbb{C}X_{\alpha_1+\alpha_2+2\alpha_3+\alpha_4} \oplus \mathbb{C}X_{\alpha_1+2\alpha_2+2\alpha_3+\alpha_4} \oplus \mathbb{C}X_{-\alpha_4},\\
\mathfrak{p}^2_{\mathbb{C}} &= \mathbb{C}X_{\alpha_1+2\alpha_2+3\alpha_3+\alpha_4},\\
\mathfrak{k}^2_{\mathbb{C}} &= \mathbb{C}X_{\alpha_2+2\alpha_3} \oplus \mathbb{C}X_{\alpha_1+\alpha_2+2\alpha_3} \oplus \mathbb{C}X_{\alpha_1+2\alpha_2+2\alpha_3} \oplus \mathbb{C}X_{\alpha_1+2\alpha_2+4\alpha_3+2\alpha_4}\\
&\quad \oplus \mathbb{C}X_{\alpha_1+3\alpha_2+4\alpha_3+2\alpha_4} \oplus \mathbb{C}X_{2\alpha_1+3\alpha_2+4\alpha_3+2\alpha_4},\\
\mathfrak{k}^i_{\mathbb{C}} &= \mathfrak{p}^i_{\mathbb{C}} = 0 \quad \text{for } i > 2.
\end{aligned}$$

Let $z = z_1 H_{\alpha_1} + z_2 H_{\alpha_2} + z_3(H_{\alpha_3} + H_{\alpha_4})$ be a generic element of $\mathfrak{t}^1_{\mathbb{C}}$. Then

$$d\delta_e(z) = (3\alpha_1 + 6\alpha_2 + 9\alpha_3 + 3\alpha_4)(z) = 0.$$

Hence e is admissible from Ohta's lemma. □

The proof of the above theorem is really an implementation of our algorithm. This is the technique that we shall use to determine admissibility under the adjoint group G. However, the details will be omitted as we present the result for each non-even non-noticed orbit of the simple algebras under consideration.

Admissibility under the simply connected group of $\mathfrak{g}$. Let G_{sc} be the simply connected group with lie algebra $\mathfrak{g}$. Then admissibility under the adjoint group G implies admissibility under G_{sc} but not vice versa.

If a nilpotent orbit O_e of $K_{\mathbb{C}}$ in $\mathfrak{p}_{\mathbb{C}}$ is admissible under the adjoint group, we say that it is **admissible**. If O_e is admissible under the simply connected cover but not the adjoint group, then we say that it is **sc-admissible**. If O_e is not admissible under the simply connected cover, then we say that it is **not admissible**.

As an example we consider the non-compact real form of G_2. Let $\Delta = \{\alpha_1, \alpha_2\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \beta_2\}$, where $\beta_1 = \alpha_1$ and $\beta_2 = 3\alpha_1 + 2\alpha_2$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sl}_2(\mathbb{C}) \oplus \mathfrak{sl}_2(\mathbb{C})$. The $K_{\mathbb{C}}$-orbit of $e = X_{2\alpha_1+\alpha_2}$ is not admissible under the adjoint group. Using the normal triple $(H_{2\alpha_1+\alpha_2}, X_{2\alpha_1+\alpha_2}, X_{-2\alpha_1-\alpha_2})$ and the above algorithm, one shows that $\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha 2}$ is a maximal torus of $\mathfrak{k}_{\mathbb{C}}^{(H_{2\alpha_1+\alpha_2},, X_{2\alpha_1+\alpha_2}, X_{-2\alpha_1-\alpha_2})}$ and $d\delta_e(z_1 H_{\alpha_2}) = z_1$.

However, the situation changes when we work under the simply connected group. The characters of G_{sc} are integer linear combinations of the fundamental weights $\lambda_1 = \beta_1/2$ and $\lambda_2 = \beta_2/2$. If we choose the character $\chi = \lambda_1$, then $\chi(z_1 H_{\alpha_2}) = \frac{1}{2}z_1$, which shows that δ_e has a square root as a character of the maximal torus corresponding to $\mathfrak{t}^1_{\mathbb{C}}$. Hence the orbit is *sc*-admissible.

The above method will be used to study admissibility under a simply connected real Lie group of inner type.

Another computational scheme for deciding admissibility. The above algorithm and the preceding method allow us to test for admissibility. They are used in the computations which produce the tables below. However, one may also use a less intensive computational scheme to achieve the same goals. This alternative was brought to our attention by David Vogan.

First we state the following results of Schwartz which Is Corollary 7.27 of [V].

Theorem [Schwartz]. *Maintaining the above notation, let H be a maximal compact subgroup of $K^e_{\mathbb{C}}$. Then e is admissible if and only if the determinant of the action of $H_\circ$ on the -1 eigenspace of x on $\mathfrak{p}_{\mathbb{C}}$ is the square of another character of $H_\circ$.*

Suppose $\mathfrak{g}_{\mathbb{C}} = \mathfrak{k}_{\mathbb{C}} \oplus \mathfrak{p}_{\mathbb{C}}$ and $\mathfrak{t}_{\mathbb{C}}$ is a maximal torus in $\mathfrak{k}_{\mathbb{C}}$, corresponding to a subgroup T of $K_{\mathbb{C}}$. Fix a nilpotent orbit O_e of $K_{\mathbb{C}}$ on $\mathfrak{p}_{\mathbb{C}}$; say that the reductive part of the stabilizer in $K_{\mathbb{C}}$ has rank $n-r$ (with n the dimension of T). Then there is a representative of the nilpotent orbit of the form $e = \sum\limits_{\beta\in S} c_\beta X_\beta$ where $c_\beta \in \mathbb{C}$ and S is a set of roots of $\mathfrak{t}_{\mathbb{C}}$ in $\mathfrak{p}_{\mathbb{C}}$, and the span of S has dimension r. We can arrange also that the semisimple part x of the triple (x, e, f) belong to $\mathfrak{t}_{\mathbb{C}}$. Then we can take, for a maximal torus in $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)}$, $\mathfrak{t}^1_{\mathbb{C}} = \{z \in \mathfrak{t}_{\mathbb{C}} | \beta(z) = 0, \text{ for all } \beta \in S\}$.

Define $R = \{\text{weights } \gamma \text{ of } \mathfrak{t}_{\mathbb{C}} \text{ on } \mathfrak{p}_{\mathbb{C}} | \gamma(x) = 1\}$ and $\rho(R) =$ half the sum of the weights in R. Then from the previous theorem the orbit O_e is admissible if and only if some combination $\rho(R) + \sum\limits_{\beta\in S} q_\beta \beta$, where $q_\beta \in \mathbb{C}$, is integral; that is, exponentiates to a character of T.

The value of this formulation is that one only needs to know x (which determines R) and the set of roots in S; everything else is a roots and weights calculation.

We conclude this section with the following theorems whose proofs consist of the results given in tables I–VIII.

Theorem C. *Let $\mathfrak{g}_{\mathbb{C}}$ be an exceptional simple complex Lie algebra. If $\mathfrak{g}$ is a non-split real form of $\mathfrak{g}_{\mathbb{C}}$ of inner type other than EII, the real rank 4 form of E_6, then all the nilpotent orbits of the simply connected group G_{sc} on $\mathfrak{g}$ are admissible.*

Remark 1. In fact, very few nilpotent orbits fail to be admissible. In EVIII, for example, only 9 orbits out of 115 are not admissible and only 5 out of the 94 orbits of EV are not admissible. There is exactly one non-admissible orbit in FI. And EII has 4 non-admissible orbits. Here we mean admissibility under the simply connected Lie group.

Connections with special orbits. A complex nilpotent special orbit is one associated to a special representation of the Weyl group via the Springer correspondence [L]. They are one of the important tools in the classification of primitive ideals of the universal enveloping algebra $\mathfrak{U}(\mathfrak{g}_{\mathbb{C}})$. A real nilpotent orbit is special if it comes from a special complex nilpotent orbit.

There is a connection between special nilpotent orbits and admissible nilpotent orbits (see [Ne]). For example, it is known that for the classical split groups and many other cases the special orbits are admissible. In the case of the exceptional simple real Lie algebras of inner type we have:

Theorem D. *If $\mathfrak{g}$ is an exceptional real Lie algebra of type EIII, EVI, EVII, EIX, FI, FII and G_2, then all special nilpotent orbits are admissible under G_{sc}.*

Remark 2. It was conjectured that if G is a real, simply connected, split form of an exceptional complex Lie group, then all special nilpotent orbits of G are admissible.

The above conjecture fails for the split forms of E_7 and E_8. Curiously it happens for two special orbits in EV and one in $EVIII$; all are contained in a minimal algebra of type $A_4 \oplus A_1$. However, in all cases there is an admissible real orbit coming from the same special complex orbit.

In $G_{\mathbb{C}}$ irreducible characters of the Weyl group W correspond to rational Hecke algebra representations [C, L]. It would be interesting to see if there is a connection to the fact that only characters associated with orbits $A_4 \oplus A_1$ in E_7, $A_4 \oplus A_1$ and $E_6(a_1) \oplus A_1$ in E_8 do not correspond to rational Hecke algebra representations.

The following tables consist of the results of the algorithm described above applied to non-even and non-noticed nilpotent orbits of the remaining exceptional simple real Lie algebras of inner type. We group $K_{\mathbb{C}}$-orbits on $\mathfrak{p}_{\mathbb{C}}$ coming from the same $G_{\mathbb{C}}$-orbits on $\mathfrak{g}_{\mathbb{C}}$ together. In the tables, when the nilpotent e is semiregular in the minimal regular subalgebra $\mathfrak{s}$, then e is given in terms of parameters u and v which can be determined by solving the system $x = [e, f]$. For more information on semiregular nilpotents in $\mathfrak{p}_{\mathbb{C}}$ the reader may consult Djoković [D2].

Finally, we should add that recently, Monica Nevins from The University of Ottawa, in a preprint, has computed the admissible nilpotents of the split forms of F_4, E_6 and E_7 using techniques for p-adic groups.

FI. Let $\mathfrak{g} = FI$, the other real form of $\mathfrak{g}_{\mathbb{C}} = F_4$ and $\Delta = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \ldots, \beta_4\}$, where $\beta_1 = \alpha_4$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_2$ and $\beta_4 = 2\alpha_1 + 3\alpha_2 + 4\alpha_3 + 2\alpha_4$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sp}_3(\mathbb{C}) \oplus \mathfrak{sl}_2(\mathbb{C})$. For each non-even and non-noticed orbit of $\mathfrak{g}$ we give the Djoković's label, the normal triple (x, e, f); we do not need to write f down because we can deduce it from e by replacing every root vector X_β by $X_{-\beta}$. The maximal torus $\mathfrak{t}^1_{\mathbb{C}}$ and the value of $d\delta_e(z) = \sum_{i=1}^{r} a_i z_i$, where r = rank of $K_{\mathbb{C}}^{(x,e,f)}$, are also given. The fundamental weights of $\mathfrak{k}_{\mathbb{C}}$ are $\lambda_1 = \beta_1 + \beta_2 + \beta_3/2$, $\lambda_2 = \beta_1 + 2\beta_2 + \beta_3$, $\lambda_3 = \beta_1 + 2\beta_2 + 3\beta_3/2$ and $\lambda_4 = \beta_4/2$. The character χ is the differential of a square root of δ_e.

Table I

1. 001 1 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_1$
 $x = H_{\alpha_1} + 3H_{\alpha_2} + 2H_{\alpha_3} + H_{\alpha_4}$, $e = X_{\alpha_1+3\alpha_2+4\alpha_3+2\alpha_4}$
 $\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2})$
 $d\delta_e(z) = -z_3$ **sc-admissible** $\chi = \lambda_3$ Not special

2. 100 2 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$
 $x = 2H_{\alpha_1} + 4H_{\alpha_2} + 3H_{\alpha_3} + 2H_{\alpha_4}$, $e = X_{\alpha_1+2\alpha_2+3\alpha_3+2\alpha_4}$
 $\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3}) \oplus \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + H_{\alpha_3})$
 $d\delta_e(z) = -4z_3$ **admissible** special

3. 010 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$
 $x = 2H_{\alpha_2} + 2H_{\alpha_3} + H_{\alpha_4}$, $e = X_{\alpha_1+3\alpha_2+4\alpha_3+2\alpha4} + X_{-\alpha_1-\alpha_2}$
 $\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3})$
 $d\delta_e(z) = 0$ **admissible** special

4. 001 3 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_1$
$x = 3H_{\alpha_1} + 6H_{\alpha_2} + 4H_{\alpha_3} + 2H_{\alpha_4}$, $e = X_{\alpha_1+2\alpha_2+3\alpha_3+2\alpha 4} + X_{\alpha_1+2\alpha_2+2\alpha_3}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3}$
$d\delta_e(z) = -4z_2$ **admissible** special

5. 101 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq +T_2$
$x = H_{\alpha_1} + 4H_{\alpha_2} + 3H_{\alpha_3} + 2H_{\alpha_4}$, $e = X_{\alpha_1+2\alpha_2+3\alpha_3+2\alpha 4} + X_{-\alpha_1}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_2})$
$d\delta_e(z) = 0$ **admissible** special

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10. 110 2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 2H_{\alpha_1} + 6H_{\alpha_2} + 5H_{\alpha_3} + 3H_{\alpha_4}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha 4} + X_{-\alpha_1}) + X_{\alpha_1+2\alpha_2+3\alpha_3+\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(2H_{\alpha_1} + 4H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4})$
$d\delta_e(z) = -3z_1$ **Not admissible** Not special

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11. 102 4 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$
$x = 4H_{\alpha_1} + 10H_{\alpha_2} + 7H_{\alpha_3} + 4H_{\alpha_4}$
$e = \sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3} + X_{-\alpha_1}) + 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3}$
$d\delta_e(z) = 0$ **admissible** Not special

12. 012 2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_2$
$x = 2H_{\alpha_1} + 8H_{\alpha_2} + 6H_{\alpha_3} + 3H_{\alpha_4}$
$e = \sqrt{3}X_{\alpha_1+2\alpha_2+2\alpha_3+\alpha_4} + 2X_{-\alpha_1-\alpha_2}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3})$
$d\delta_e(z) = -z_2$ **sc-admissible** $\chi = \lambda_1$ Not special

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13. 111 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = H_{\alpha_1} + 6H_{\alpha_2} + 5H_{\alpha_3} + 3H_{\alpha_4}$
$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+3\alpha_3+\alpha_4} + X_{-\alpha_1-\alpha_2-\alpha_3}) + X_{\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + H_{\alpha_4})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

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14. 103 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = H_{\alpha_1} + 7H_{\alpha_2} + 5H_{\alpha_3} + 3H_{\alpha_4}$
$e = \sqrt{3}X_{\alpha_1+2\alpha_2+2\alpha_3+\alpha_4} + 2X_{-\alpha_1-\alpha_2} + X_{-\alpha_1-\alpha_2-2\alpha_3}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

15. 111 3 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 3H_{\alpha_1} + 9H_{\alpha_2} + 7H_{\alpha_3} + 4H_{\alpha_4}$
$e = \sqrt{3}X_{\alpha_1+2\alpha_2+2\alpha_3+\alpha_4} + 2X_{-\alpha_1-\alpha_2} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = \lambda_1$ Not special

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21. 131 3 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$ $x = 3H_{\alpha_1} + 13H_{\alpha_2} + 11H_{\alpha_3} + 6H_{\alpha_4}$
$e = \sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+\alpha_4} + 2\sqrt{2}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + 3X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3})$
$d\delta_e(z) = 0$ **admissible** special

EII. Let $\mathfrak{g} = EII$ a real form of $\mathfrak{g}_{\mathbb{C}} = E_6$ and $\Delta = \{\alpha_1, \alpha_2, \dots, \alpha_6\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \dots, \beta_6\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_4$, $\beta_4 = \alpha_5$, $\beta_5 = \alpha_6$ and $\beta_6 = \alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sl}_6(\mathbb{C}) \oplus \mathfrak{sl}_2(\mathbb{C})$. The fundamental weights of $\mathfrak{k}_{\mathbb{C}}$ are $\lambda_1 = 1/6(5\beta_1+4\beta_2+3\beta_3+2\beta_4+\beta_5)$, $\lambda_2 = 1/6(4\beta_1+8\beta_2+6\beta_3+4\beta_4+2\beta_5)$, $\lambda_3 = 1/6(3\beta_1+6\beta_2+9\beta_3+6\beta_4+3\beta_5)$, $\lambda_4 = 1/6(2\beta_1+4\beta_2+6\beta_3+8\beta_4+4\beta_5)$, $\lambda_5 = 1/6(\beta_1+2\beta_2+3\beta_3+4\beta_4+5\beta_5)$ and $\lambda_6 = \beta_6/2$. The character χ is a square root of δ_e.

Table II

1. 00100 1 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_2 + T_1$
   $x = H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}$, $e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
   $\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_1}+3H_{\alpha_2}+2H_{\alpha_3}+3H_{\alpha_4}+2H_{\alpha_5}+H_{\alpha_6})$
   $d\delta_e(z) = -6z_5$ **admissible** special

2. 10001 2 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_2 + T_2$
   $x = 2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6}$
   $e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
   $\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5}) \oplus \mathbb{C}(-H_{\alpha_1} + H_{\alpha_5} + H_{\alpha_6})$
   $d\delta_e(z) = -4z_1$ **admissible** special

3. 01010 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 3A_1 + T_1$
   $x = H_{\alpha_1} + 2H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}$
   $e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2}$
   $\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5})$
   $d\delta_e(z) = 0$ **admissible** special

4. 00100 3 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_1$
   $x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 4H_{\alpha_5} + 2H_{\alpha_6}$
   $e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
   $\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_5})$
   $d\delta_e(z) = -7z_1$ **sc-admissible** $\chi = 7\lambda_6$ Not special

5. 10101 1 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_2$
   $x = 2H_{\alpha_1} + H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6}$
   $e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2}$
   $\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} - H_{\alpha_6})$
   $d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_6$ Not special

9. 21001 1 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_2$
   $x = 3H_{\alpha_1} + H_{\alpha_2} + 4H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6}$
   $e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2})$
   $\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_4}) \oplus \mathbb{C}(-H_{\alpha_1} + H_{\alpha_3} + H_{\alpha_6})$
   $d\delta_e(z) = -3z_2 + 3z_3$ **Not admissible** special

10. 10012 1 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_2$
   $x = 2H_{\alpha_1} + H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 4H_{\alpha_5} + 3H_{\alpha_6}$
   $e = X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2})$
   $\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_5} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_6})$
   $d\delta_e(z) = 3z_2 - 3z_3$ **Not admissible** special

12. 30100 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_1$
   $x = 3H_{\alpha_1} + 3H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}$

$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\quad +\sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_2-\alpha_4})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(3H_{\alpha_1}+2H_{\alpha_2}-H_{\alpha_3}-3H_{\alpha_4}+H_{\alpha_6})\oplus\mathbb{C}(-2H_{\alpha_1}-H_{\alpha_2}+H_{\alpha_3}+3H_{\alpha_4}+H_{\alpha_5})$
$d\delta_e(z) = -12z_1 + 9z_2$ **sc-admissible** $\chi = \lambda_3$ special

13. 00103 0 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = H_{\alpha_1} + 2H_{\alpha_3} + 3H_{\alpha_4} + 3H_{\alpha_5} + 3H_{\alpha_6}$
$e = X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\quad +\sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_4})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 3H_{\alpha_3} + 3H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -3z_1 + 3z_2$ **sc-admissible** $\chi = \lambda_3$ special

14. 11011 2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 3H_{\alpha_1} + 2H_{\alpha_2} + 5H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 3H_{\alpha_6}$
$e = X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$
$\quad +\sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -3z_1 - 3z_2$ **sc-admissible** $\chi = -3\lambda_3$ special

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15. 10201 4 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_1$
$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 7H_{\alpha_3} + 10H_{\alpha_4} + 7H_{\alpha_5} + 4H_{\alpha_6}$
$e = 2X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{3}(X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_2})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_1} - H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

16. 01210 2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_2$
$x = 3H_{\alpha_1} + 2H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 3H_{\alpha_6}$
$e = 2X_{-\alpha_2-\alpha_4} + \sqrt{3}(X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + H_{\alpha_4})$
$d\delta_e(z) = -2z_3$ **admissible** special

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17. 11111 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 3H_{\alpha_1} + H_{\alpha_2} + 5H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 3H_{\alpha_6}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6})$
$\quad +X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{2}(X_{-\alpha_2-\alpha_3-\alpha_4} + X_{-\alpha_2-\alpha_4-\alpha_5})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

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18. 10301 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 3H_{\alpha_1} + H_{\alpha_2} + 5H_{\alpha_3} + 7H_{\alpha_4} + 5H_{\alpha_5} + 3H_{\alpha_6}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6})$
$\quad +2X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + X_{-\alpha_2-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + H_{\alpha_4} + 2H_{\alpha_6}) \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_4} + 2H_{\alpha_5})$
$d\delta_e(z) = -z_1 - z_2$ **sc-admissible** $\chi = \lambda_3$ Not special

19. 11111 3 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 4H_{\alpha_1} + 3H_{\alpha_2} + 7H_{\alpha_3} + 9H_{\alpha_4} + 7H_{\alpha_5} + 4H_{\alpha_6}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$
$\quad +X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + 2X_{-\alpha_2-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(-H_{\alpha_1} - 2H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -3z_1$ **sc-admissible** $\chi = 3\lambda_3$ Not special

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27. 12113 1 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 4H_{\alpha_1} + H_{\alpha_2} + 7H_{\alpha_3} + 8H_{\alpha_4} + 7H_{\alpha_5} + 5H_{\alpha_6}$
$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}) + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$\quad +\sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4})$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(2H_{\alpha_1} + 3H_{\alpha_2} + H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

28. 31121 1 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 5H_{\alpha_1} + H_{\alpha_2} + 7H_{\alpha_3} + 8H_{\alpha_4} + 7H_{\alpha_5} + 4H_{\alpha_6}$
$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}) + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$
$\quad +\sqrt{6}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6})$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + 3H_{\alpha_2} + 5H_{\alpha_3} + 6H_{\alpha_4} + H_{\alpha_5} + 2H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

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29. 31310 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 7H_{\alpha_1} + 4H_{\alpha_2} + 11H_{\alpha_3} + 14H_{\alpha_4} + 10H_{\alpha_5} + 5H_{\alpha_6}$
$e = \sqrt{1-\sqrt{6}}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{1+\sqrt{6}}X_{-\alpha_2-\alpha_4-\alpha_5}$
$\quad +\sqrt{10}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{6-\sqrt{6}}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$
$\quad +\sqrt{6+\sqrt{6}}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + \sqrt{6}X_{-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(-H_{\alpha_1} + H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = 3z_1$ **Not admissible** special

30. 01313 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 5H_{\alpha_1} + 4H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 11H_{\alpha_5} + 7H_{\alpha_6}$
$e = \sqrt{1-\sqrt{6}}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{1+\sqrt{6}}X_{-\alpha_2-\alpha_3-\alpha_4}$
$\quad +\sqrt{6+\sqrt{6}}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}$
$\quad +\sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{6}X_{-\alpha_2-\alpha_4-\alpha_5} + \sqrt{6-\sqrt{6}}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(-H_{\alpha_1} - 2H_{\alpha_3} - H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -3z_1$ **Not admissible** special

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31. 13131 3 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 6H_{\alpha_1} + 3H_{\alpha_2} + 11H_{\alpha_3} + 13H_{\alpha_4} + 11H_{\alpha_5} + 6H_{\alpha_6}$
$e = 3X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{8}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + \sqrt{5}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$\quad +\sqrt{5}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{8}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = \lambda_3$ Not special

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EIII. Let $\mathfrak{g} = EIII$ a real form of $\mathfrak{g}_{\mathbb{C}} = E_6$ and $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_6\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \ldots, \beta_6\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_4$, $\beta_4 = \alpha_2$, $\beta_5 = \alpha_5$ and $\beta_6 = -\alpha_1 - 2\alpha_2 - 2\alpha_3 - 3\alpha_4 - 2\alpha_5 - \alpha_6$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_{10}(\mathbb{C}) \oplus \mathbb{C}$. The fundamental weights of $\mathfrak{k}_{\mathbb{C}}$ are $\lambda_1 = \beta_1 + \beta_2 + \beta_3 + 1/2(\beta_4 + \beta_5)$, $\lambda_2 = \beta_1 + 2\beta_2 + 2\beta_3 + \beta_4 + \beta_5$, $\lambda_3 = \beta_1 + 2\beta_2 + 3\beta_3 + 3/2(\beta_4 + \beta_5)$, $\lambda_4 = 1/2(\beta_1 + 2\beta_2 + 3\beta_3 + 5/2\beta_4 + 3/2\beta_5)$, $\lambda_5 = 1/2(\beta_1 + 2\beta_2 + 3\beta_3 + 3/2\beta_4 + 5/2\beta_5)$ and $\lambda_6 = -\beta_6$. The character χ is a square root of δ_e.

TABLE III

1. 00001 0 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_4 + T_1$
$x = -H_{\alpha_6}$, $e = X_{-\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(2H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = 6z_5$ **admissible** special

2. 00010 -2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_4 + T_1$
$x = H_{\alpha_1} + 2H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}$, $e = X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$
$d\delta_e(z) = -6z_5$ **admissible** special

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3. 10000 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_3 + T_1$
$x = -H_{\alpha_2} - H_{\alpha_3} - 2H_{\alpha_4} - 2H_{\alpha_5} - 2H_{\alpha_6}$,
$e = X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + X_{-\alpha_4-\alpha_5-\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4}) \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_3})$
$d\delta_e(z) = 12z_3$ **admissible** special

4. 10000 -2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_3 + T_1$
$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$
$d\delta_e(z) = 12z_3$ **admissible** special

5. 00011 -2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3 + T_1$
$x = H_{\alpha_1} + 2H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_5}$
$e = X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(2H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}H_{\alpha_4})$
$d\delta_e(z) = 0$ **admissible** special

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7. 11010 -2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_1$
$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 3H_{\alpha_4} + H_{\alpha_5} - H_{\alpha_6}$
$e = \sqrt{2}(X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}) + X_{-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_3} + H_{\alpha_6})$
$d\delta_e(z) = 3z_3$ **sc-admissible** $\chi = \lambda_1$ special

8. 11001 -3 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_1$
$x = 3H_{\alpha_1} + 3H_{\alpha_2} + 5H_{\alpha_3} + 6H_{\alpha_4} + 4H_{\alpha_5} + H_{\alpha_6}$
$e = \sqrt{2}(X_{-\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}) + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(-H_{\alpha_1} + H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -3z_3$ **sc-admissible** $\chi = \lambda_1$ special

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10. 00013 -2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_2 + T_1$
$x = H_{\alpha_1} + 2H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_5} - 2H_{\alpha_6}$
$e = \sqrt{3}(X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6})$
$+2X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = 6z_3$ **admissible** special

11. 00031 -6 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_2 + T_1$
$x = 3H_{\alpha_1} + 6H_{\alpha_2} + 6H_{\alpha_3} + 9H_{\alpha_4} + 6H_{\alpha_5} + 2H_{\alpha_6}$
$e = 2X_{-\alpha_6} + \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6})$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -6z_3$ **admissible** special

EV. Let $\mathfrak{g} = EV$, a real form of $\mathfrak{g}_{\mathbb{C}} = E_7$, and $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_7\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \ldots, \beta_7\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_4$, $\beta_4 = \alpha_5$, $\beta_5 = \alpha_6$, $\beta_6 = \alpha_7$ and $\beta_7 = \alpha_1 + 2\alpha_2 + 2\alpha_3 + 3\alpha_4 + 2\alpha_5 + \alpha_6$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sl}_8(\mathbb{C})$. The fundamental weights of $\mathfrak{k}_{\mathbb{C}}$ are $\lambda_1 = 1/8(7\beta_1 + 6\beta_2 + 5\beta_3 + 4\beta_4 + 3\beta_5 + 2\beta_6 + \beta_7)$, $\lambda_2 = 1/8(6\beta_1 + 12\beta_2 + 10\beta_3 + 8\beta_4 + 6\beta_5 + 4\beta_6 + 2\beta_7)$, $\lambda_3 = 1/8(5\beta_1 + 10\beta_2 + 15\beta_3 + 12\beta_4 + 9\beta_5 + 6\beta_6 + 3\beta_7)$, $\lambda_4 = 1/8(4\beta_1 + 8\beta_2 + 12\beta_3 + 16\beta_4 + 12\beta_5 + 8\beta_6 + 4\beta_7)$, $\lambda_5 = 1/8(3\beta_1 + 6\beta_2 + 9\beta_3 + 12\beta_4 + 15\beta_5 + 10\beta_6 + 5\beta_7)$ $\lambda_6 = 1/8(2\beta_1 + 4\beta_2 + 6\beta_3 + 8\beta_4 + 10\beta_5 + 12\beta_6 + 6\beta_7)$ and $\lambda_7 = 1/8(\beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 + 5\beta_5 + 6\beta_6 + 7\beta_7)$. The character χ is a square root of δ_e.

Table IV

1. 0001000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_3$
$x = H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_5})$
$d\delta_e(z) = 0$ **admissible** special

2. 0100010 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_2 + 2A_1 + T_1$
$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 4H_{\alpha_3} + 5H_{\alpha_4} + 4H_{\alpha_5} + 3H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

5. 1001001 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_2$
$x = 3H_{\alpha_1} + 3H_{\alpha_2} + 5H_{\alpha_3} + 7H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$\quad + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} - H_{\alpha_5}) \oplus \mathbb{C}(-H_{\alpha_1} + H_{\alpha_5} + H_{\alpha_6})$
$\quad \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_7})$
$d\delta_e(z) = -z_1 + z_2$ **Not admissible** Not special

8. 1100100 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_1$
$x = 3H_{\alpha_1} + 2H_{\alpha_2} + 5H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\quad + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(-H_{\alpha_1} + H_{\alpha_5} + H_{\alpha_6})$
$\quad \oplus \mathbb{C}(-H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = -z_1 + 2z_2$ **sc-admissible** $\chi = -\lambda_1$ Not special

9. 0010011 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_1$
$x = 3H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 9H_{\alpha_4} + 7H_{\alpha_5} + 5H_{\alpha_6} + 3H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\quad + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$
$d\delta_e(z) = -z_2$ **sc-admissible** $\chi = \lambda_3$ Not special

10. 2010001 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_2$
$x = 4H_{\alpha_1} + 3H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}) + X_{-\alpha_2}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = 2z_2$ **admissible** special

11. 1000102 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_2$
$x = 4H_{\alpha_1} + 5H_{\alpha_2} + 7H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6})$
$+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_5} + \oplus\mathbb{C}(H_{\alpha_2} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_7})$
$d\delta_e(z) = -2z_3 - 2z_4$ **admissible** special

12. 0101010 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1$
$x = 3H_{\alpha_1} + 3H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 7H_{\alpha_5} + 5H_{\alpha_6} + 3H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = 0$ **admissible** special

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13. 3000100 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_1$
$x = 4H_{\alpha_1} + 2H_{\alpha_2} + 5H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + X_{-\alpha_2}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 4H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} - H_{\alpha_7})$
$d\delta_e(z) = 4z_1$ **admissible** special

14. 0010003 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_1$
$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6})$
$+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(-H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -4z_1$ **admissible** special

15. 1010101 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_1$
$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 7H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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20. 0102010 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 3A_1 + T_1$
$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 8H_{\alpha_3} + 11H_{\alpha_4} + 10H_{\alpha_5} + 7H_{\alpha_6} + 4H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4})$
$+2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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24. 1101011 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 5H_{\alpha_1} + 5H_{\alpha_2} + 9H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 7H_{\alpha_6} + 4H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$
$+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2}) + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

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25. 1011101 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_3$
$x = 5H_{\alpha_1} + 5H_{\alpha_2} + 9H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 7H_{\alpha_6} + 4H_{\alpha_7}$
$e = 2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$\quad +\sqrt{3}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_4}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_1} - H_{\alpha_6})$
$\quad \oplus\mathbb{C}(H_{\alpha_1} - H_{\alpha_3} - H_{\alpha_4} + H_{\alpha_7})$
$d\delta_e(z) = -z_1 - z_2 - z_3$ **Not admissible** Not special

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28. 1111010 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_2$
$x = 5H_{\alpha_1} + 4H_{\alpha_2} + 9H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 7H_{\alpha_6} + 4H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_2-\alpha_4-\alpha_5}$
$\quad +\sqrt{3}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_3-\alpha_4})$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_4} - H_{\alpha_7})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

29. 0101111 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_2$
$x = 5H_{\alpha_1} + 6H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 12H_{\alpha_5} + 9H_{\alpha_6} + 5H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + 2X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$\quad +X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2})$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(-H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} - H_{\alpha_6})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_3$ Not special

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31. 2101101 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$
$x = 6H_{\alpha_1} + 5H_{\alpha_2} + 10H_{\alpha_3} + 13H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_2-\alpha_4-\alpha_5}$
$\quad +\sqrt{3}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_3-\alpha_4}) + X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_4} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

32. 1011012 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$
$x = 6H_{\alpha_1} + 7H_{\alpha_2} + 11H_{\alpha_3} + 16H_{\alpha_4} + 13H_{\alpha_5} + 9H_{\alpha_6} + 5H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$\quad +\sqrt{3}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2}) + 2X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_3} - H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

33. 0120101 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_2$
$x = 5H_{\alpha_1} + 5H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{3-\sqrt{3}}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + 2X_{-\alpha_2-\alpha_4-\alpha_5}$
$\quad +\sqrt{3+\sqrt{3}}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{1+\sqrt{3}}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$\quad +\sqrt{1-\sqrt{3}}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

34. 1010210 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_2$
$x = 5H_{\alpha_1} + 5H_{\alpha_2} + 9H_{\alpha_3} + 13H_{\alpha_4} + 11H_{\alpha_5} + 9H_{\alpha_6} + 5H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{3+\sqrt{3}}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$\quad +2X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{3-\sqrt{3}}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\quad +\sqrt{1-\sqrt{3}}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}\sqrt{1+\sqrt{3}}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + H_{\alpha_4} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

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35. 1030010 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 5H_{\alpha_1} + 4H_{\alpha_2} + 9H_{\alpha_3} + 13H_{\alpha_4} + 10H_{\alpha_5} + 7H_{\alpha_6} + 4H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+} + X_{-\alpha_2-\alpha_4-\alpha_5}) + 2X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} - H_{\alpha_4} - H_{\alpha_6} + 2H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + 2H_{\alpha_6} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

36. 0100301 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 5H_{\alpha_1} + 6H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 12H_{\alpha_5} + 10H_{\alpha_6} + 5H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6})$
$+2X_{-\alpha_2} + \sqrt{2}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_5} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

37. 1110111 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 6H_{\alpha_1} + 6H_{\alpha_2} + 11H_{\alpha_3} + 15H_{\alpha_4} + 12H_{\alpha_5} + 9H_{\alpha_6} + 5H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{3-\sqrt{3}}X_{-\alpha_2-\alpha_4}$
$+2X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{1-\sqrt{3}}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{3+\sqrt{3}}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{1+\sqrt{3}}X_{-\alpha_2-\alpha_4-\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

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46. 2103101 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 8H_{\alpha_1} + 7H_{\alpha_2} + 14H_{\alpha_3} + 19H_{\alpha_4} + 17H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$
$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$
$+X_{\alpha_1\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7})$
$+X_{-\alpha_2-\alpha_4-\alpha_5} + \sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(-H_{\alpha_4} + H_{\alpha_7})$
$d\delta_e(z) = z_1$ **sc-admissible** $\chi = \lambda_7$ Not special

47. 1013012 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 8H_{\alpha_1} + 9H_{\alpha_2} + 15H_{\alpha_3} + 22H_{\alpha_4} + 19H_{\alpha_5} + 13H_{\alpha_6} + 7H_{\alpha_7}$
$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5}) + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_6})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_7$ Not special

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48. 3101021 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 8H_{\alpha_1} + 7H_{\alpha_2} + 13H_{\alpha_3} + 17H_{\alpha_4} + 14H_{\alpha_5} + 10H_{\alpha_6} + 6H_{\alpha_7}$
$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$
$+\sqrt{6}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_4-\alpha_5}) + X_{-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} - H_{\alpha_3} - H_{\alpha_4} + 2H_{\alpha_6} + 2H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} - H_{\alpha_7})$
$d\delta_e(z) = z_1 + z_2$ **Not admissible** special

49. 1201013 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 8H_{\alpha_1} + 9H_{\alpha_2} + 15H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 11H_{\alpha_6} + 6H_{\alpha_7}$
$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$
$+\sqrt{6}(+X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2}) + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} - H_{\alpha_6}) \oplus \mathbb{C}(-H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -3z_1 + z_2$ **Not admissible** special

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51. 3013010 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 8H_{\alpha_1} + 6H_{\alpha_2} + 13H_{\alpha_3} + 18H_{\alpha_4} + 16H_{\alpha_5} + 11H_{\alpha_6} + 6H_{\alpha_7}$
$e = \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{5-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{2+u}X_{-\alpha_2-\alpha_4-\alpha_5} + \sqrt{10-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + \sqrt{7}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$
$+\sqrt{u}X_{+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$ $:u = \frac{1}{4}(13+\sqrt{249})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_3} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = 2z_2$ **admissible** special

52. 0103013 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 8H_{\alpha_1} + 10H_{\alpha_2} + 16H_{\alpha_3} + 23H_{\alpha_4} + 20H_{\alpha_5} + 14H_{\alpha_6} + 7H_{\alpha_7}$
$e = \sqrt{7-u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$
$+\sqrt{7}X_{+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{5+u}X_{-\alpha_2-\alpha_3-\alpha_4} + \sqrt{5-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$+\sqrt{u}X_{+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$ $:u = \frac{1}{4}(7+\sqrt{249})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -2z_2$ **admissible** special

53. 1112111 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 8H_{\alpha_1} + 8H_{\alpha_2} + 15H_{\alpha_3} + 21H_{\alpha_4} + 18H_{\alpha_5} + 13H_{\alpha_6} + 7H_{\alpha_7}$
$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$
$+X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$
$+X_{+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_4-\alpha_5} + \sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

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59. 1211121 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 9H_{\alpha_1} + 9H_{\alpha_2} + 17H_{\alpha_3} + 23H_{\alpha_4} + 19H_{\alpha_5} + 14H_{\alpha_6} + 8H_{\alpha_7}$
$e = 3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{8}(X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6})$
$+\sqrt{5}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = -z_1$ **Not admissible** Not special

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60. 1311111 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 9H_{\alpha_1} + 8H_{\alpha_2} + 17H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 13H_{\alpha_6} + 7H_{\alpha_7}$
$e = 3X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{8}(X_{\alpha_1\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7})$
$+\sqrt{5}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4}) + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} - H_{\alpha_6})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

61. 1111131 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 9H_{\alpha_1} + 10H_{\alpha_2} + 17H_{\alpha_3} + 24H_{\alpha_4} + 20H_{\alpha_5} + 15H_{\alpha_6} + 9H_{\alpha_7}$
$e = 3X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{8}(X_{-\alpha_2-\alpha_4-\alpha_5} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5})$
$+\sqrt{5}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4}) + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

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64. 1310301 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 9H_{\alpha_1} + 8H_{\alpha_2} + 17H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 14H_{\alpha_6} + 7H_{\alpha_7}$
$e = \sqrt{9-u}X_{-\alpha_2-\alpha_3-\alpha_4} + \sqrt{7+u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$

$+\sqrt{2-u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}+\sqrt{4+v}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{v}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}+\sqrt{10-v}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{4+v}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}+\sqrt{u}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_3}+2H_{\alpha_4}+H_{\alpha_5})$
$d\delta_e(z)=-z_1$ **sc-admissible** $\chi=\lambda_5$ Not special

65. 1030131 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_1$
$x=9H_{\alpha_1}+10H_{\alpha_2}+17H_{\alpha_3}+25H_{\alpha_4}+20H_{\alpha_5}+15H_{\alpha_6}+9H_{\alpha_7}$
$e=\sqrt{2+u+v}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}+\sqrt{10-v}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5}+\sqrt{-4+v}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{u}X_{-\alpha_2-\alpha_3-\alpha_4}+\sqrt{14-u-v}X_{-\alpha_2-\alpha_4}$
$\sqrt{7-u-v}X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6+\alpha_7}+\sqrt{v}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_3}+H_{\alpha_4}+H_{\alpha_5}+H_{\alpha_6})$
$d\delta_e(z)=-z_1$ **sc-admissible** $\chi=\lambda_3$ Not special

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72. 3013131 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_1$
$x=12H_{\alpha_1}+12H_{\alpha_2}+21H_{\alpha_3}+30H_{\alpha_4}+26H_{\alpha_5}+19H_{\alpha_6}+11H_{\alpha_7}$
$e=\sqrt{u}X_{-\alpha_2-\alpha_4-\alpha_5}+\sqrt{2-u}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}+\sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{9+u}X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6+\alpha_7}+\sqrt{14}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}+\sqrt{11-u}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{18}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$ $: u=(1-\sqrt{10})$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_2}+H_{\alpha_3}+2H_{\alpha_4}+H_{\alpha_5}+H_{\alpha_6})$
$d\delta_e(z)=0$ **admissible** special

73. 1313103 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_1$
$x=12H_{\alpha_1}+12H_{\alpha_2}+23H_{\alpha_3}+31H_{\alpha_4}+26H_{\alpha_5}+18H_{\alpha_6}+9H_{\alpha_7}$
$e=\sqrt{11-u}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7}+\sqrt{9-u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{11}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}+\sqrt{9+u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{14}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}+\sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$
$+\sqrt{u}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$ $: u=\frac{1}{4}(11-\sqrt{769})$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_2}+H_{\alpha_3}+H_{\alpha_4}+H_{\alpha_5})$
$d\delta_e(z)=0$ **admissible** special

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74. 3113121 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_1$
$x=12H_{\alpha_1}+11H_{\alpha_2}+21H_{\alpha_3}+29H_{\alpha_4}+25H_{\alpha_5}+18H_{\alpha_6}+10H_{\alpha_7}$
$e=\sqrt{10}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}+\sqrt{14}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}+\sqrt{10}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}+X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}+\sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_3}+H_{\alpha_4}-H_{\alpha_7})$
$d\delta_e(z)=0$ **admissible** special

75. 1213113 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_1$
$x=12H_{\alpha_1}+13H_{\alpha_2}+23H_{\alpha_3}+32H_{\alpha_4}+27H_{\alpha_5}+19H_{\alpha_6}+10H_{\alpha_7}$
$e=\sqrt{10}X_{-\alpha_2-\alpha_4-\alpha_5}+\sqrt{14}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}+\sqrt{10}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}+X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}+\sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(-H_{\alpha_3}+H_{\alpha_6}+H_{\alpha_7})$
$d\delta_e(z)=0$ **admissible** special

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82. 1413131 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_1$
$x=16H_{\alpha_1}+14H_{\alpha_2}+29H_{\alpha_3}+38H_{\alpha_4}+32H_{\alpha_5}+23H_{\alpha_6}+13H_{\alpha_7}$
$e=\sqrt{15}(X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}+X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7})+\sqrt{28}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}+\sqrt{18}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}+\sqrt{24}X_{\alpha_1\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_2}+H_{\alpha_3}+2H_{\alpha_4}+H_{\alpha_5}+H_{\alpha_6})$
$d\delta_e(z)=-z_1$ **sc-admissible** $\chi=\lambda_1$ Not special

83. 1313143 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 16H_{\alpha_1} + 18H_{\alpha_2} + 31H_{\alpha_3} + 43H_{\alpha_4} + 36H_{\alpha_5} + 26H_{\alpha_6} + 15H_{\alpha_7}$
$e = \sqrt{28}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + \sqrt{18}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + \sqrt{15}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{24}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{15}X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{10}X_{\alpha_1\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -z1$ **sc-admissible** $\chi = \lambda_1$ Not special

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EVI. Let $\mathfrak{g} = EVI$, a real form of $\mathfrak{g}_{\mathbb{C}} = E_7$, and $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_7\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \ldots, \beta_7\}$, where $\beta_1 = \alpha_7$, $\beta_2 = \alpha_6$, $\beta_3 = \alpha_5$, $\beta_4 = \alpha_4$, $\beta_5 = \alpha_3$, $\beta_6 = \alpha_2$ and $\beta_7 = 2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 3\alpha_5 + 2\alpha_6 + \alpha_7$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_{12}(\mathbb{C}) \oplus \mathfrak{sl}_2(\mathbb{C})$. The fundamental weights of $\mathfrak{k}_{\mathbb{C}}$ are $\lambda_1 = \beta_1 + \beta_2 + \beta_3 + \beta_4 + 1/2(\beta_5 + \beta_6)$, $\lambda_2 = \beta_1 + 2\beta_2 + 2\beta_3 + 2\beta_4 + \beta_5 + \beta_6$, $\lambda_3 = \beta_1 + 2\beta_2 + 3\beta_3 + 3\beta_4 + 3/2(\beta_5 + \beta_6)$, $\lambda_4 = \beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 + 2(\beta_5 + \beta_6)$, $\lambda_5 = 1/2(\beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 + 3\beta_5 + 2\beta_6)$ $\lambda_6 = 1/2(\beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 + 2\beta_5 + 3\beta_6)$ and $\lambda_7 = \beta_2/2$.

TABLE V

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1. 000010 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_5 + T_1$
$x = H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7}$
$e = X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_3}) \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_7}$
$d\delta_e(z) = -4z_1$ **admissible** special

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2. 010000 2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_3 + A_1 + T_1$
$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_6})$
$d\delta_e(z) = -8z_1$ **admissible** special

3. 000100 0 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3 + 2A_1$
$x = 2H_{\alpha_2} + 2H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7}$
$e = X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + 2H_{\alpha_4})$
$d\delta_e(z) = 0$ **admissible** special

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4. 000010 3 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq C_3 + T_1$
$x = 3H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7}$
$d\delta_e(z) = -13z_1$ **sc-admissible** $\chi = -13\lambda_7$ Not special

5. 010010 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_2 + A_1 + T_1$
$x = H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + 2H_{\alpha_4})$
$d\delta_e(z) = -3z_4$ **sc-admissible** $\chi = -3\lambda_7$ Not special

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9. 110001 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T_2$
$x = H_{\alpha_1} + 4H_{\alpha_2} + 4H_{\alpha_3} + 7H_{\alpha_4} + 6H_{\alpha_5} + 5H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{2}X_{-\alpha_1-\alpha_3}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_6})$
$d\delta_e(z) = 2z_2 - 2z_4$ **admissible** special

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10. 200100 0 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$
$x = 3H_{\alpha_2} + 3H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{2}X_{-\alpha_1-\alpha_3-\alpha_4} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_4} + 2H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} - H_{\alpha_7})$
$d\delta_e(z) = -2z_2 + 4z_3$ **admissible** special

11. 010100 2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$
$x = 2H_{\alpha_1} + 5H_{\alpha_2} + 6H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{2}X_{-\alpha_1} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_6})$
$d\delta_e(z) = -6z_3$ **admissible** special

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12. 010020 4 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3 + A_1$
$x = 4H_{\alpha_1} + 7H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_1}) + 2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7}$
$d\delta_e(z) = 0$ **admissible** special

13. 000120 2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_2 + A_1 + T_1$
$x = 2H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}) + 2X_{-\alpha_1-\alpha_3}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_7}) \oplus C(H_{\alpha_1} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -4z_4$ **admissible** special

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16. 010110 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_1$
$x = H_{\alpha_1} + 5H_{\alpha_2} + 6H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7})$
$+\sqrt{2}(X_{-\alpha_1-\alpha_3-\alpha_4} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}) + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} - H_{\alpha_5} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + 2H_{\alpha_7})$
$d\delta_e(z) = -2z_2$ **admissible** Not special

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17. 010030 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$
$x = H_{\alpha_1} + 5H_{\alpha_2} + 7H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+X_{-\alpha_1-\alpha_3} + 2X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_6})$
$d\delta_e(z) = -z_3$ **sc-admissible** $\chi = -\lambda_7$ Not special

18. 010110 3 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$
$x = 3H_{\alpha_1} + 7H_{\alpha_2} + 9H_{\alpha_3} + 14H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+2X_{-\alpha_1-\alpha_3} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_5} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_7})$
$d\delta_e(z) = -7z_3$ **sc-admissible** $\chi = -7\lambda_7$ Not special

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24. 201011 2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 2H_{\alpha_1} + 7H_{\alpha_2} + 8H_{\alpha_3} + 13H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 5H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6})$
$\quad +2X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + \sqrt{2}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = -8z_2$ **admissible** special

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27. 111110 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = H_{\alpha_1} + 7H_{\alpha_2} + 8H_{\alpha_3} + 14H_{\alpha_4} + 12H_{\alpha_5} + 9H_{\alpha_6} + 5H_{\alpha_7}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + 2(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$\quad +X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}) + \sqrt{6}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$\quad +X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} - H_{\alpha_5} + H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -2z_1 - z_2$ **sc-admissible** $\chi = \lambda_3$ special

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28. 201031 4 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 4H_{\alpha_1} + 11H_{\alpha_2} + 14H_{\alpha_3} + 21H_{\alpha_4} + 17H_{\alpha_5} + 12H_{\alpha_6} + 7H_{\alpha_7}$
$e = \sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{6+\sqrt{6}}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\quad +\sqrt{1-\sqrt{6}}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{6}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\quad +\sqrt{1+\sqrt{6}}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{6-\sqrt{6}}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_5} - H_{\alpha_7})$
$d\delta_e(z) = 2z_2$ **admissible** special

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30. 010310 3 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 3H_{\alpha_1} + 11H_{\alpha_2} + 13H_{\alpha_3} + 22H_{\alpha_4} + 17H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$
$e = \sqrt{5}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$\quad +3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{8}(X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$\quad +X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = -3z_1$ **sc-admissible** $\chi = -3\lambda_7$ Not special

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EVII. Let $\mathfrak{g} = EVII$, a real form of $\mathfrak{g}_{\mathbb{C}} = E_7$, and $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_7\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \ldots, \beta_7\}$, where $\beta_1 = \alpha_6$, $\beta_2 = \alpha_2$, $\beta_3 = \alpha_5$, $\beta_4 = \alpha_4$, $\beta_5 = \alpha_3$, $\beta_6 = \alpha_1$ and $\beta_7 = -2\alpha_1 - 2\alpha_2 - 3\alpha_3 - 4\alpha_4 - 3\alpha_5 - 2\alpha_6 - \alpha_7$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = E_6 \oplus (\mathbb{C})$.

Table VI

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1. 100000 0 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq D_5 + T_1$
$x = -H_{\alpha_7}$
$e = X_{-\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 8z_6$ **admissible** special

2. 000001 -2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq D_5 + T_1$
$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7}$
$e = X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_7}$
$d\delta_e(z) = 0$ **admissible** special

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3. 000001 0 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_4 + T_1$
$x = -H_{\alpha_2} - H_{\alpha_3} - 2H_{\alpha_4} - 2H_{\alpha_5} - 2H_{\alpha_6} - 2H_{\alpha_7}$
$e = X_{-\alpha_7} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 16z_5$ **admissible** special

4. 100000 -2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_4 + T_1$
$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7}$
$d\delta_e(z) = -16z_5$ **admissible** special

5. 100001 -2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq D_4 + T_1$
$x = -2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6}$
$e = +X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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11. 010010 -2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_3 + T_1$
$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 5H_{\alpha_4} + 3H_{\alpha_5} + H_{\alpha_6} - H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7})$
$+X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + H_{\alpha_7})$
$d\delta_e(z) = 6z_4$ **admissible** special

12. 011000 -3 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_3 + T_1$
$x = 3H_{\alpha_1} + 5H_{\alpha_2} + 6H_{\alpha_3} + 9H_{\alpha_4} + 7H_{\alpha_5} + 4H_{\alpha_6} + H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_6-\alpha_7})$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_5} + H_{\alpha_7})$
$d\delta_e(z) = -6z_4$ **admissible** special

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13. 300001 -2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_3 + T_1$
$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} - 2H_{\alpha_7}$
$e = 2X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + \sqrt{3}(X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} + 2H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 8z_4$ **admissible** special

14. 100003 -6 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_3 + T_1$
$x = 6H_{\alpha_1} + 6H_{\alpha_2} + 9H_{\alpha_3} + 12H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 2H_{\alpha_7}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}) + 2X_{-\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_4} + 2H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -8z_4$ **admissible** special

EVIII. Let $\mathfrak{g} = EVIII$, a real form of $\mathfrak{g}_{\mathbb{C}} = E_8$, and $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_8\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \ldots, \beta_8\}$, where $\beta_1 = 2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 3\alpha_5 + 2\alpha_6 + \alpha_7$, $\beta_2 = \alpha_8$, $\beta_3 = \alpha_7$, $\beta_4 = \alpha_6$, $\beta_5 = \alpha_5$, $\beta_6 = \alpha_4$, $\beta_7 = \alpha_2$ and $\beta_8 = \alpha_3$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_{16}(\mathbb{C})$. The fundamental weights of $\mathfrak{k}_{\mathbb{C}}$ are $\lambda_1 = \beta_1+\beta_2+\beta_3+\beta_4+\beta_5+\beta_6+1/2(\beta_7+\beta_8)$, $\lambda_2 = \beta_1+2\beta_2+2\beta_3+2\beta_4+2\beta_5+2\beta_6+\beta_7+\beta_8$, $\lambda_3 = (\beta_1+2\beta_2+3\beta_3+3\beta_4+3\beta_5+3\beta_6+3/2(\beta_7+\beta_8)$, $\lambda_4 = \beta_1+2\beta_2+3\beta_3+4\beta_4+4\beta_5+4\beta_6+2(\beta_7+\beta_8)$, $\lambda_5 = \beta_1+2\beta_2+3\beta_3+4\beta_4+5\beta_5+5\beta_6+5/2(\beta_7+\beta_8)$, $\lambda_6 = \beta_1+2\beta_2+3\beta_3+4\beta_4+4\beta_5+6\beta_6+3(\beta_7+\beta_8)$, $\lambda_7 = 1/2(\beta_1+2\beta_2+3\beta_3+4\beta_4+5\beta_5+6\beta_6+4\beta_7+3\beta_8)$ and $\lambda_8 = 1/2(\beta_1+2\beta_2+3\beta_3+4\beta_4+5\beta_5+6\beta_6+3\beta_7+4\beta_8)$.

Table VII

1. 00000010 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_7$
$x = H_{\alpha_1}+3H_{\alpha_2}+3H_{\alpha_3}+5H_{\alpha_4}+4H_{\alpha_5}+3H_{\alpha_6}+2H_{\alpha_7}+H_{\alpha_8}$
$e = X_{\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1}+H_{\alpha_2})\oplus\mathbb{C}H_{\alpha_3}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}H_{\alpha_5}\oplus\mathbb{C}H_{\alpha_6}\oplus CH_{\alpha_7}\oplus\mathbb{C}H_{\alpha_8}$
$d\delta_e(z) = 0$ **admissible** special

2. 00010000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3+B_3$
$x = 2H_{\alpha_1}+4H_{\alpha_2}+5H_{\alpha_3}+8H_{\alpha_4}+7H_{\alpha_5}+6H_{\alpha_6}+4H_{\alpha_7}+2H_{\alpha_8}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}+X_{\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}H_{\alpha_5}\oplus\mathbb{C}H_{\alpha_7}\oplus CH_{\alpha_8}\oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

3. 01000010 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq C_3+A_1+T_1$
$x = 3H_{\alpha_1}+6H_{\alpha_2}+7H_{\alpha_3}+11H_{\alpha_4}+9H_{\alpha_5}+7H_{\alpha_6}+5H_{\alpha_7}+3H_{\alpha_8}$
$e = X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\quad +X_{\alpha_1+2\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3}\oplus\mathbb{C}H_{\alpha_5}\oplus\mathbb{C}H_{\alpha_7}\oplus\mathbb{C}(H_{\alpha_2}+H_{\alpha_4}-H_{\alpha_8})\oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_4}+H_{\alpha_6}+H_{\alpha_8})$
$d\delta_e(z) = z_4-z_5$ **Not admissible** Not special

6. 10001000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3+T_1$
$x = 4H_{\alpha_1}+7H_{\alpha_2}+9H_{\alpha_3}+14H_{\alpha_4}+12H_{\alpha_5}+9H_{\alpha_6}+6H_{\alpha_7}+3H_{\alpha_8}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$\quad +X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}+X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2}-H_{\alpha_8})\oplus\mathbb{C}(H_{\alpha_4}-H_{\alpha_7})\oplus\mathbb{C}(H_{\alpha_3}-H_{\alpha_6})$
$\quad \oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+2H_{\alpha_3}+2H_{\alpha_4}+H_{\alpha_5})$
$d\delta_e(z) = -z_4$ **sc-admissible** $\chi = -\lambda_1$ Not special

7. 11000001 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_2+T_1$
$x = 5H_{\alpha_1}+8H_{\alpha_2}+11H_{\alpha_3}+16H_{\alpha_4}+13H_{\alpha_5}+10H_{\alpha_6}+7H_{\alpha_7}+4H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8})$
$\quad +X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}H_{\alpha_6}\oplus\mathbb{C}H_{\alpha_7}\oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_3}+H_{\alpha_5})$
$d\delta_e(z) = -2z_4$ **admissible** special

8. 00010010 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3$
$x = 3H_{\alpha_1}+7H_{\alpha_2}+8H_{\alpha_3}+13H_{\alpha_4}+11H_{\alpha_5}+9H_{\alpha_6}+6H_{\alpha_7}+3H_{\alpha_8}$
$e = X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}+X_{-\alpha_1}$
$\quad +X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_4}-H_{\alpha_7})\oplus\mathbb{C}(H_{\alpha_5}-H_{\alpha_8})$
$\quad \oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+2H_{\alpha_3}+3H_{\alpha_4}+2H_{\alpha_5}+H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

9. 20010000 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_2 + A_1 + T_1$
$x = 6H_{\alpha_1} + 9H_{\alpha_2} + 12H_{\alpha_3} + 18H_{\alpha_4} + 15H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_8}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -4z_1$ **admissible** special

10. 01000100 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3 + T_1$
$x = 4H_{\alpha_1} + 8H_{\alpha_2} + 10H_{\alpha_3} + 16H_{\alpha_4} + 13H_{\alpha_5} + 10H_{\alpha_6} + 7H_{\alpha_7} + 4H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_6} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + 2H_{\alpha_4})$
$d\delta_e(z) = 0$ **admissible** special

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11. 00010020 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3 + B_2$
$x = 4H_{\alpha_1} + 10H_{\alpha_2} + 11H_{\alpha_3} + 18H_{\alpha_4} + 15H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$
$e = \sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6})$
$+2X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_8} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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12. 30000001 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2 + T_1$
$x = 7H_{\alpha_1} + 10H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_8}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -7z_1$ **sc-admissible** $\chi = -7\lambda_1$ Not special

13. 10010001 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$
$x = 5H_{\alpha_1} + 9H_{\alpha_2} + 12H_{\alpha_3} + 18H_{\alpha_4} + 15H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+X_{-\alpha_1}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_8}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

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17. 01010010 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$
$x = 5H_{\alpha_1} + 10H_{\alpha_2} + 12H_{\alpha_3} + 19H_{\alpha_4} + 16H_{\alpha_5} + 13H_{\alpha_6} + 9H_{\alpha_7} + 5H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3})$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} - H_{\alpha_3} - H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5})$
$\oplus\mathbb{C}(H_{\alpha_1} - H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -2z_3$ **admissible** Not special

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18. 01000110 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 3A_1 + T_1$
$x = 5H_{\alpha_1} + 11H_{\alpha_2} + 13H_{\alpha_3} + 21H_{\alpha_4} + 17H_{\alpha_5} + 13H_{\alpha_6} + 9H_{\alpha_7} + 5H_{\alpha_8}$
$e = \sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3-\alpha_4})$
$+2X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_8}) \oplus \mathbb{C}(H_{\alpha_6} + H_{\alpha_7})$
$\quad \oplus\mathbb{C}(-H_{\alpha_4} - H_{\alpha_5} + H_{\alpha_7} + H_{\alpha_8}) \oplus \mathbb{C}H_{\alpha_3}$
$d\delta_e(z) = -z_1 - z_3$ **Not admissible** Not special

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22. 10100100 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 6H_{\alpha_1} + 11H_{\alpha_2} + 14H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 14H_{\alpha_6} + 10H_{\alpha_7} + 5H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\quad +X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3})$
$\quad +X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_6} + H_{\alpha_7}) \oplus \mathbb{C}(-H_{\alpha_2} + 2H_{\alpha_5} + H_{\alpha_8})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

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23. 10010011 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_2$
$x = 6H_{\alpha_1} + 12H_{\alpha_2} + 15H_{\alpha_3} + 23H_{\alpha_4} + 19H_{\alpha_5} + 15H_{\alpha_6} + 10H_{\alpha_7} + 5H_{\alpha_8}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\quad +\sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_3})$
$\quad +2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_5} - H_{\alpha_7})$
$\quad \oplus\mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + H_{\alpha_5} - H_{\alpha_8})$
$d\delta_e(z) = -z_1 - z_3$ **sc-admissible** $\chi = -\lambda_1$ Not special

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24. 11001010 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 7H_{\alpha_1} + 13H_{\alpha_2} + 16H_{\alpha_3} + 25H_{\alpha_4} + 21H_{\alpha_5} + 16H_{\alpha_6} + 11H_{\alpha_7} + 6H_{\alpha_8}$
$e = 2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\quad +\sqrt{3}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1}$
$\quad +X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_7})$
$d\delta_e(z) = -2z_1$ **admissible** special

25. 00100101 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_3$
$x = 5H_{\alpha_1} + 11H_{\alpha_2} + 14H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 14H_{\alpha_6} + 10H_{\alpha_7} + 5H_{\alpha_8}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\quad +\sqrt{2+u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$\quad +\sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$\quad +\sqrt{4-u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + 2X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$\quad +\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$ $: u = 1 - \sqrt{3}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+2H_{\alpha_3}+2H_{\alpha_4}+H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5}+H_{\alpha_8}) \oplus \mathbb{C}(-H_{\alpha_3}+H_{\alpha_6}+H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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26. 20100011 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_1$
$x = 8H_{\alpha_1} + 14H_{\alpha_2} + 18H_{\alpha_3} + 27H_{\alpha_4} + 22H_{\alpha_5} + 17H_{\alpha_6} + 12H_{\alpha_7} + 6H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$\quad +2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_8})$
$d\delta_e(z) = 0$ **admissible** special

27. 10001002 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_2$
$x = 6H_{\alpha_1} + 12H_{\alpha_2} + 16H_{\alpha_3} + 24H_{\alpha_4} + 20H_{\alpha_5} + 15H_{\alpha_6} + 10H_{\alpha_7} + 5H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$\quad +\sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$\quad +2X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_6} - H_{\alpha_8})$
$\oplus\mathbb{C}(H_{\alpha_4} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

28. 01010100 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 6H_{\alpha_1} + 12H_{\alpha_2} + 15H_{\alpha_3} + 24H_{\alpha_4} + 20H_{\alpha_5} + 16H_{\alpha_6} + 11H_{\alpha_7} + 6H_{\alpha_8}$
$e = X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{2+u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{4-u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + 2X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$ $: u = 1-\sqrt{3}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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31. 00100003 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 5H_{\alpha_1} + 11H_{\alpha_2} + 15H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 14H_{\alpha_6} + 10H_{\alpha_7} + 5H_{\alpha_8}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7})$
$+2X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + X_{-\alpha_1-\alpha_3-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} - H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + 3H_{\alpha_5} + 3H_{\alpha_6} + 2H_{\alpha_7})$
$\oplus\mathbb{C}(H_{\alpha_1} - 3H_{\alpha_2} + H_{\alpha_3} + 4H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7} - H_{\alpha_8})$
$d\delta_e(z) = z_1 + z_2$ **sc-admissible** $\chi = \lambda_1$ Not special

32. 10101001 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 7H_{\alpha_1} + 13H_{\alpha_2} + 17H_{\alpha_3} + 26H_{\alpha_4} + 22H_{\alpha_5} + 17H_{\alpha_6} + 12H_{\alpha_7} + 6H_{\alpha_8}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7})$
$+2X_{-\alpha_1-\alpha_3-\alpha_4} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} - H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_8})$
$\oplus\mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -z_1 - z_2$ **sc-admissible** $\chi = -\lambda_1$ Not special

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33. 11001030 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2 + T_1$
$x = 9H_{\alpha_1} + 19H_{\alpha_2} + 22H_{\alpha_3} + 35H_{\alpha_4} + 29H_{\alpha_5} + 22H_{\alpha_6} + 15H_{\alpha_7} + 8H_{\alpha_8}$
$e = \sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+\sqrt{10}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_5-\alpha_6-\alpha_7-\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_6})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

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37. 11110010 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_2$
$x = 9H_{\alpha_1} + 16H_{\alpha_2} + 20H_{\alpha_3} + 31H_{\alpha_4} + 26H_{\alpha_5} + 21H_{\alpha_6} + 15H_{\alpha_7} + 8H_{\alpha_8}$
$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1}) + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+2(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_5})$
$d\delta_e(z) = -2z_1 + z_2$ **Not admissible** special

38. 01010110 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$
$x = 7H_{\alpha_1} + 15H_{\alpha_2} + 18H_{\alpha_3} + 29H_{\alpha_4} + 24H_{\alpha_5} + 19H_{\alpha_6} + 13H_{\alpha_7} + 7H_{\alpha_8}$
$e = 2(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6})$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4})$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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39. 10110100 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 8H_{\alpha_1} + 15H_{\alpha_2} + 19H_{\alpha_3} + 30H_{\alpha_4} + 25H_{\alpha_5} + 20H_{\alpha_6} + 14H_{\alpha_7} + 7H_{\alpha_8}$
$e = 2(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + X_{-\alpha_1-\alpha_3-\alpha_4})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} - H_{\alpha_5} - H_{\alpha_8}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_5} - H_{\alpha_7})$
$d\delta_e(z) = -z_2$ **Not admissible** Not special

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40. 20100031 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_1$
$x = 10H_{\alpha_1} + 20H_{\alpha_2} + 24H_{\alpha_3} + 37H_{\alpha_4} + 30H_{\alpha_5} + 23H_{\alpha_6} + 16H_{\alpha_7} + 8H_{\alpha_8}$
$e = \sqrt{7-u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{7}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{5-u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{7+u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$ $: u = \frac{1}{4}(7-\sqrt{249})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_6})$
$d\delta_e(z) = -2z_2$ **admissible** special

41. 01010120 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1$
$x = 8H_{\alpha_1} + 18H_{\alpha_2} + 21H_{\alpha_3} + 34H_{\alpha_4} + 28H_{\alpha_5} + 22H_{\alpha_6} + 15H_{\alpha_7} + 8H_{\alpha_8}$
$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}) + \sqrt{10}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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42. 21010100 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 10H_{\alpha_1} + 17H_{\alpha_2} + 22H_{\alpha_3} + 34H_{\alpha_4} + 28H_{\alpha_5} + 22H_{\alpha_6} + 15H_{\alpha_7} + 8H_{\alpha_8}$
$e = X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+2(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_7})$
$d\delta_e(z) = -5z_1$ **sc-admissible** $\chi = -5\lambda_1$ special

43. 01200100 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 8H_{\alpha_1} + 15H_{\alpha_2} + 19H_{\alpha_3} + 30H_{\alpha_4} + 25H_{\alpha_5} + 20H_{\alpha_6} + 15H_{\alpha_7} + 8H_{\alpha_8}$
$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6})$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_7}) \oplus \mathbb{C}(2H_{\alpha_1} - H_{\alpha_2} + 3H_{\alpha_3} - H_{\alpha_5} + 3H_{\alpha_7})$
$d\delta_e(z) = -z_1 - 4z_2$ **sc-admissible** $\chi = -2\lambda_1 + \lambda_7$ special

44. 10101011 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 8H_{\alpha_1} + 16H_{\alpha_2} + 20H_{\alpha_3} + 31H_{\alpha_4} + 26H_{\alpha_5} + 20H_{\alpha_6} + 14H_{\alpha_7} + 7H_{\alpha_8}$
$e = 2(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7})$
$+\sqrt{2+u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{4-u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4})$
$+\sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5}$ $: u = 1-\sqrt{3}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} + H_{\alpha_8})$
$d\delta_e(z) = 0$ **admissible** special

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47. 01020110 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_1$
$x = 9H_{\alpha_1} + 19H_{\alpha_2} + 23H_{\alpha_3} + 37H_{\alpha_4} + 31H_{\alpha_5} + 25H_{\alpha_6} + 17H_{\alpha_7} + 9H_{\alpha_8}$
$e = \sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+3X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{5}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{8}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6} + H_{\alpha_7})$
$\oplus\mathbb{C}(H_{\alpha_3} + H_{\alpha_4} - H_{\alpha_8})$
$d\delta_e(z) = z_3$ **Not admissible** Not special

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48. 30001030 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$
$x = 11H_{\alpha_1} + 21H_{\alpha_2} + 25H_{\alpha_3} + 39H_{\alpha_4} + 32H_{\alpha_5} + 24H_{\alpha_6} + 16H_{\alpha_7} + 8H_{\alpha_8}$
$e = \sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{7}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{7-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{5+u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5} + \sqrt{5-u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$ $: u = \frac{1}{4}(7 - \sqrt{249})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_8})$
$d\delta_e(z) = -4z_1$ **admissible** special

49. 10101021 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 9H_{\alpha_1} + 19H_{\alpha_2} + 23H_{\alpha_3} + 36H_{\alpha_4} + 30H_{\alpha_5} + 23H_{\alpha_6} + 16H_{\alpha_7} + 8H_{\alpha_8}$
$e = \sqrt{7}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{7-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$\sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$+\sqrt{5+u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{5-u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$ $: u = \frac{1}{4}(7 - \sqrt{249})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + 2H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_6})$
$d\delta_e(z) = 0$ **admissible** special

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50. 11010101 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 9H_{\alpha_1} + 17H_{\alpha_2} + 22H_{\alpha_3} + 34H_{\alpha_4} + 28H_{\alpha_5} + 22H_{\alpha_6} + 15H_{\alpha_7} + 8H_{\alpha_8}$
$e = \sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1-\alpha_3-\alpha_4})$
$+2(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+\sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5})$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + 3H_{\alpha_2} + 2H_{\alpha_4} + 2H_{\alpha_6} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

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57. 11101011 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 10H_{\alpha_1} + 19H_{\alpha_2} + 24H_{\alpha_3} + 37H_{\alpha_4} + 31H_{\alpha_5} + 24H_{\alpha_6} + 17H_{\alpha_7} + 9H_{\alpha_8}$
$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6})$
$+2(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4})$
$+\sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(2H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 5H_{\alpha_4} + 3H_{\alpha_5} + 4H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -4z_1$ **admissible** Not special

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58. 10111011 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 10H_{\alpha_1} + 20H_{\alpha_2} + 25H_{\alpha_3} + 39H_{\alpha_4} + 33H_{\alpha_5} + 26H_{\alpha_6} + 18H_{\alpha_7} + 9H_{\alpha_8}$
$e = 3X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$

$+\sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{8}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{5}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$\oplus\mathbb{C}(H_{\alpha_3} - H_{\alpha_4} - H_{\alpha_5} + H_{\alpha_8})$
$d\delta_e(z) = -z_1 - z_2$ **Not admissible** Not special

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59. 11010111 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 10H_{\alpha_1} + 20H_{\alpha_2} + 25H_{\alpha_3} + 39H_{\alpha_4} + 32H_{\alpha_5} + 25H_{\alpha_6} + 17H_{\alpha_7} + 9H_{\alpha_8}$
$e = \sqrt{7}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{7-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{2}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{5-u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{5+u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$ $: u = \frac{1}{4}(7 - \sqrt{249})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 3H_{\alpha_4} + H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -3z_1$ **Not admissible** Not special

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60. 21011011 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$
$x = 12H_{\alpha_1} + 22H_{\alpha_2} + 28H_{\alpha_3} + 43H_{\alpha_4} + 36H_{\alpha_5} + 28H_{\alpha_6} + 19H_{\alpha_7} + 10H_{\alpha_8}$
$e = \sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{3}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{3}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\sqrt{10}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** Not special

61. 10102100 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 10H_{\alpha_1} + 20H_{\alpha_2} + 25H_{\alpha_3} + 40H_{\alpha_4} + 34H_{\alpha_5} + 26H_{\alpha_6} + 18H_{\alpha_7} + 9H_{\alpha_8}$
$e = 3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{-4+u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{10-u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{-7+u+v}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{16-u-v}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{14-u-v}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-} + \sqrt{v}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_8})$
$d\delta_e(z) = -z1$ **sc-admissible** $\chi = -\lambda_1$ Not special

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62. 11110110 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 11H_{\alpha_1} + 21H_{\alpha_2} + 26H_{\alpha_3} + 41H_{\alpha_4} + 34H_{\alpha_5} + 27H_{\alpha_6} + 19H_{\alpha_7} + 10H_{\alpha_8}$
$e = 3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{8}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$\sqrt{5}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

63. 01011101 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 9H_{\alpha_1} + 19H_{\alpha_2} + 24H_{\alpha_3} + 38H_{\alpha_4} + 32H_{\alpha_5} + 25H_{\alpha_6} + 17H_{\alpha_7} + 9H_{\alpha_8}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + 3X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{5}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

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64. 01003001 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 9H_{\alpha_1} + 19H_{\alpha_2} + 24H_{\alpha_3} + 38H_{\alpha_4} + 33H_{\alpha_5} + 25H_{\alpha_6} + 17H_{\alpha_7} + 9H_{\alpha_8}$
$e = \sqrt{5}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + 3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{8}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}) + \sqrt{2}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+\sqrt{5}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

65. 11101101 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 11H_{\alpha_1} + 21H_{\alpha_2} + 27H_{\alpha_3} + 42H_{\alpha_4} + 35H_{\alpha_5} + 27H_{\alpha_6} + 19H_{\alpha_7} + 10H_{\alpha_8}$
$e = \sqrt{10-u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{4-u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + 3X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{9-v}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + \sqrt{-2-u+v}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{16-v}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{v}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

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66. 11110130 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 13H_{\alpha_1} + 27H_{\alpha_2} + 32H_{\alpha_3} + 51H_{\alpha_4} + 42H_{\alpha_5} + 33H_{\alpha_6} + 23H_{\alpha_7} + 12H_{\alpha_8}$
$e = \sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + 4X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} - H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6} + 2H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} - H_{\alpha_7})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

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72. 21011031 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$
$x = 14H_{\alpha_1} + 28H_{\alpha_2} + 34H_{\alpha_3} + 53H_{\alpha_4} + 44H_{\alpha_5} + 34H_{\alpha_6} + 23H_{\alpha_7} + 12H_{\alpha_8}$
$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{18}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$+\sqrt{14}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

73. 01201031 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$
$x = 12H_{\alpha_1} + 26H_{\alpha_2} + 31H_{\alpha_3} + 49H_{\alpha_4} + 41H_{\alpha_5} + 32H_{\alpha_6} + 23H_{\alpha_7} + 12H_{\alpha_8}$
$e = \sqrt{11-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{9+u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{9-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{11}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{14}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$ $: u = \frac{1}{4}(11 - \sqrt{769})$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_6} + 2H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} - H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

---

74. 11111101 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$
$x = 13H_{\alpha_1} + 25H_{\alpha_2} + 32H_{\alpha_3} + 50H_{\alpha_4} + 42H_{\alpha_5} + 33H_{\alpha_6} + 23H_{\alpha_7} + 12H_{\alpha_8}$

$$
\begin{aligned}
e = {} & X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} \\
& +\sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} \\
& +\sqrt{12}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} \\
& +\sqrt{12}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}
\end{aligned}
$$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} - H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6} - H_{\alpha_7})$

$d\delta_e(z) = 0$ **admissible** special

---

75. 11101121 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 13H_{\alpha_1} + 27H_{\alpha_2} + 33H_{\alpha_3} + 52H_{\alpha_4} + 43H_{\alpha_5} + 33H_{\alpha_6} + 23H_{\alpha_7} + 12H_{\alpha_8}$

$$
\begin{aligned}
e = {} & X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{9+u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} \\
& +\sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} \\
& +\sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{11-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} \\
& +\sqrt{14}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8} \\
& +\sqrt{u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5} \qquad : u = 1-\sqrt{10}
\end{aligned}
$$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$

$d\delta_e(z) = 0$ **admissible** special

76. 10300130 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 13H_{\alpha_1} + 27H_{\alpha_2} + 32H_{\alpha_3} + 51H_{\alpha_4} + 42H_{\alpha_5} + 33H_{\alpha_6} + 24H_{\alpha_7} + 12H_{\alpha_8}$

$$
\begin{aligned}
e = {} & \sqrt{9+u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{9-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} \\
& +\sqrt{11-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} \\
& +X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + \sqrt{11}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} \\
& +\sqrt{14}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} \\
& +\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} \qquad : u = \tfrac{1}{4}(11-\sqrt{769})
\end{aligned}
$$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6})$

$d\delta_e(z) = 0$ **admissible** special

---

83. 21031031 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$

$x = 18H_{\alpha_1} + 36H_{\alpha_2} + 44H_{\alpha_3} + 69H_{\alpha_4} + 58H_{\alpha_5} + 46H_{\alpha_6} + 31H_{\alpha_7} + 16H_{\alpha_8}$

$$
\begin{aligned}
e = {} & \sqrt{15}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} \\
& +\sqrt{24}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} \\
& +\sqrt{28}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5} + \sqrt{18}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}
\end{aligned}
$$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_7})$

$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

---

84. 31010211 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 16H_{\alpha_1} + 30H_{\alpha_2} + 38H_{\alpha_3} + 59H_{\alpha_4} + 48H_{\alpha_5} + 37H_{\alpha_6} + 25H_{\alpha_7} + 13H_{\alpha_8}$

$$
\begin{aligned}
e = {} & \sqrt{(4+v)}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{18-v}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} \\
& +\sqrt{2-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} \\
& +\sqrt{11+u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{13-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} \\
& +\sqrt{14-v}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{13-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5} \\
& +\sqrt{v}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}
\end{aligned}
$$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} - H_{\alpha_6} - H_{\alpha_7} - H_{\alpha_8})$

$d\delta_e(z) = 0$ **admissible** special

---

86. 12111111 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 16H_{\alpha_1} + 31H_{\alpha_2} + 39H_{\alpha_3} + 61H_{\alpha_4} + 51H_{\alpha_5} + 40H_{\alpha_6} + 28H_{\alpha_7} + 15H_{\alpha_8}$

$$
\begin{aligned}
e = {} & \sqrt{15}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{7}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} \\
& +4X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{12}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} \\
& +\sqrt{12}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + \sqrt{7}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} \\
& +\sqrt{15}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}
\end{aligned}
$$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} - H_{\alpha_5} - H_{\alpha_8})$
$d\delta_e(z) = z_1$ **Not admissible** Not special

---

87. 13111101 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 17H_{\alpha_1} + 31H_{\alpha_2} + 40H_{\alpha_3} + 62H_{\alpha_4} + 52H_{\alpha_5} + 41H_{\alpha_6} + 29H_{\alpha_7} + 16H_{\alpha_8}$

$e = \sqrt{-14+u}X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{30-u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{22}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{12}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{22-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{12}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$ $: u = \frac{1}{2}(33-\sqrt{249})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -3z_1$ **sc-admissible** $\chi = -3\lambda_1$ special

---

89. 11121121 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 17H_{\alpha_1} + 35H_{\alpha_2} + 43H_{\alpha_3} + 68H_{\alpha_4} + 57H_{\alpha_5} + 45H_{\alpha_6} + 31H_{\alpha_7} + 16H_{\alpha_8}$

$e = \sqrt{15}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+\sqrt{24}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + 2\sqrt{7}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{18}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

90. 30130130 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 19H_{\alpha_1} + 37H_{\alpha_2} + 45H_{\alpha_3} + 71H_{\alpha_4} + 59H_{\alpha_5} + 47H_{\alpha_6} + 32H_{\alpha_7} + 16H_{\alpha_8}$

$e = \sqrt{24}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{15}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{15}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{28}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{18}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -2z_1$ **admissible** special

---

96. 13111141 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 21H_{\alpha_1} + 43H_{\alpha_2} + 52H_{\alpha_3} + 82H_{\alpha_4} + 68H_{\alpha_5} + 53H_{\alpha_6} + 37H_{\alpha_7} + 20H_{\alpha_8}$

$e = \sqrt{42}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{30}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + 4X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+4X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + \sqrt{30}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{22}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -2z_1$ **admissible** Not special

---

97. 13103041 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 21H_{\alpha_1} + 43H_{\alpha_2} + 52H_{\alpha_3} + 82H_{\alpha_4} + 69H_{\alpha_5} + 53H_{\alpha_6} + 37H_{\alpha_7} + 20H_{\alpha_8}$

$e = \sqrt{29+u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{31-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{32-v}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{13-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{15+v}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{47-v}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{22}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{v}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

---

100. 31131211 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 24H_{\alpha_1} + 46H_{\alpha_2} + 58H_{\alpha_3} + 91H_{\alpha_4} + 76H_{\alpha_5} + 60H_{\alpha_6} + 41H_{\alpha_7} + 21H_{\alpha_8}$
$e = \sqrt{40}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{30}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{12}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{21}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{21}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{22}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+6X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_4} - H_{\alpha_7} - H_{\alpha_8})$
$d\delta_e(z) = z_1$ **Not admissible** Not special

103. 13131043 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 25H_{\alpha_1} + 51H_{\alpha_2} + 63H_{\alpha_3} + 98H_{\alpha_4} + 82H_{\alpha_5} + 65H_{\alpha_6} + 45H_{\alpha_7} + 24H_{\alpha_8}$
$e = i\sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{57}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{21}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{56}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+4X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{26}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{40}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5} + \sqrt{21}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$
$d\delta_e(z) = 0$ **admissible** special

108. 34131341 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
$x = 35H_{\alpha_1} + 69H_{\alpha_2} + 85H_{\alpha_3} + 134H_{\alpha_4} + 111H_{\alpha_5} + 87H_{\alpha_6} + 60H_{\alpha_7} + 32H_{\alpha_8}$
$e = \sqrt{66}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{27}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{75}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + 7X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{96}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{34}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{52}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$
$d\delta_e(z) = -z_1$ **sc-admissible** $\chi = -\lambda_1$ Not special

EIX. Let $\mathfrak{g} = EIX$, a real form of $\mathfrak{g}_{\mathbb{C}} = E_8$, and $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_8\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$, then $\Delta_k = \{\beta_1, \ldots, \beta_8\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_2$, $\beta_3 = \alpha_3$, $\beta_4 = \alpha_4$, $\beta_5 = \alpha_5$, $\beta_6 = \alpha_6$, $\beta_7 = \alpha_7$ and $\beta_8 = 2\alpha_1+3\alpha_2+4\alpha_3+6\alpha_4+5\alpha_5+4\alpha_6+3\alpha_7+2\alpha_8$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = E_7 \oplus \mathfrak{sl}_2(\mathbb{C})$. The fundamental weights of $\mathfrak{k}_{\mathbb{C}}$ are $\lambda_1 = 2\beta_1+2\beta_2+3\beta_3+4\beta_4+3\beta_5+2\beta_6+\beta_7$, $\lambda_2 = 1/2(4\beta_1+7\beta_2+8\beta_3+12\beta_4+9\beta_5+6\beta_6+3\beta_7)$, $\lambda_3 = 3\beta_1 + 4\beta_2 + 6\beta_3 + 8\beta_4 + 6\beta_5 + 4\beta_6 + 2\beta_7$, $\lambda_4 = 4\beta_1 + 6\beta_2 + 8\beta_3 + 12\beta_4 + 9\beta_5 + 6\beta_6 + 3\beta_7$, $\lambda_5 = 1/2(6\beta_1 + 9\beta_2 + 12\beta_3 + 18\beta_4 + 15\beta_5 + 10\beta_6 + 5\beta_7)$, $\lambda_6 = 2\beta_1+3\beta_2+4\beta_3+6\beta_4+5\beta_5+4\beta_6+2\beta_7$, $\lambda_7 = 1/2(2\beta_1+3\beta_2+4\beta_3+6\beta_4+5\beta_5+4\beta_6+3\beta_7)$ and $\lambda_8 = \beta_8/2$.

Table VIII

1. 0000001 1 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq E_6 + T_1$
$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 3H_{\alpha_7} + H_{\alpha_8}$
$e = X_{2\alpha_1+3\alpha_2+4\alpha_3+6\alpha_4+5\alpha_5+4\alpha_6+3\alpha_7+\alpha_8}$
$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus CH_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -8z_7$ **admissible** special

2. 1000000 2 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_5 + T_1$
$x = 4H_{\alpha_1} + 5H_{\alpha_2} + 7H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 4H_{\alpha_7} + 2H_{\alpha_8}$
$e = X_{2\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{2\alpha_1+2\alpha_2+4\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}H_{\alpha_8} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3})$
$d\delta_e(z) = -16z_5$ **admissible** special

3. 0000010 0 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq D_5 + T_1$
$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$
$e = X_{2\alpha_1+3\alpha_2+4\alpha_3+6\alpha_4+5\alpha_5+4\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_6} + 2H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = 0$ **admissible** special

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4. 0000001 3 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq F_4 + T_1$
$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 6H_{\alpha_7} + 3H_{\alpha_8}$
$e = X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_8} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$
$d\delta_e(z) = -25z_3$ **sc-admissible** $\chi = -25\lambda_8$ Not special

5. 1000001 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_4 + T_1$
$x = 4H_{\alpha_1} + 5H_{\alpha_2} + 7H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 4H_{\alpha_7} + H_{\alpha_8}$
$e = X_{2\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{2\alpha_1+2\alpha_2+4\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$+X_{-\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(2H_{\alpha_7} + H_{\alpha_8}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3})$
$d\delta_e(z) = -7z_4$ **sc-admissible** $\chi = -7\lambda_8$ Not special

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9. 1100000 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_4 + T_1$
$x = 5H_{\alpha_1} + 7H_{\alpha_2} + 9H_{\alpha_3} + 13H_{\alpha_4} + 10H_{\alpha_5} + 7H_{\alpha_6} + 4H_{\alpha_7} + H_{\alpha_8}$
$e = \sqrt{2}(X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_8})$
$+X_{\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -6z_5$ **admissible** special

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10. 1000010 2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_3 + T_1$
$x = 6H_{\alpha_1} + 8H_{\alpha_2} + 11H_{\alpha_3} + 16H_{\alpha_4} + 13H_{\alpha_5} + 10H_{\alpha_6} + 6H_{\alpha_7} + 2H_{\alpha_8}$
$e = \sqrt{2}(X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_7-\alpha_8})$
$+X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_6} + H_{\alpha_8})$
$d\delta_e(z) = -12z_4$ **admissible** special

11. 0001000 0 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_3 + A_1$
$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$
$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+X_{2\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_2} + 3H_{\alpha_3} + 3H_{\alpha_4} + H_{\alpha_8})$
$d\delta_e(z) = 0$ **admissible** special

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12. 1000002 4 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq D_5$
$x = 8H_{\alpha_1} + 11H_{\alpha_2} + 15H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 14H_{\alpha_6} + 10H_{\alpha_7} + 4H_{\alpha_8}$
$e = \sqrt{2}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_8})$
$+2X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$
$d\delta_e(z) = 0$ **admissible** special

13. 0000012 2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_4 + T_1$
$x = 6H_{\alpha_1} + 9H_{\alpha_2} + 12H_{\alpha_3} + 18H_{\alpha_4} + 15H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 2H_{\alpha_8}$

$e=\sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+2X_{-\alpha_7-\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_1}\oplus\mathbb{C}H_{\alpha_3}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}(H_{\alpha_2}+H_{\alpha_5})\oplus\mathbb{C}(H_{\alpha_5}+2H_{\alpha_6}+H_{\alpha_7}+H_{\alpha_8})$
$d\delta_e(z)=-8z_5$ **admissible** special

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15. 1000011 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq G_2+T_1$
$x=6H_{\alpha_1}+8H_{\alpha_2}+11H_{\alpha_3}+16H_{\alpha_4}+13H_{\alpha_5}+10H_{\alpha_6}+6H_{\alpha_7}+H_{\alpha_8}$
$e=\sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}+X_{-\alpha_6-\alpha_7-\alpha_8})$
$+\sqrt{2}(X_{\alpha_1+2\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}+X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_3}\oplus\mathbb{C}(H_{\alpha_2}+H_{\alpha_4})\oplus\mathbb{C}(2H_{\alpha_1}-2H_{\alpha_2}+3H_{\alpha_5}+2H_{\alpha_6}+H_{\alpha_8})$
$d\delta_e(z)=-2z_3$ **admissible** Not special

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16. 1000011 3 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq B_3+T_1$
$x=8H_{\alpha_1}+11H_{\alpha_2}+15H_{\alpha_3}+22H_{\alpha_4}+18H_{\alpha_5}+14H_{\alpha_6}+9H_{\alpha_7}+3H_{\alpha_8}$
$e=\sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}+X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+2X_{-\alpha_7-\alpha_8}+X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_2}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}H_{\alpha_5}\oplus\mathbb{C}(H_{\alpha_3}+2H_{\alpha_6}+H_{\alpha_7}+H_{\alpha_8})$
$d\delta_e(z)=-15z_4$ **sc-admissible** $\chi=-15\lambda_8$ Not special

17. 1000003 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq B_3+T_1$
$x=6H_{\alpha_1}+8H_{\alpha_2}+11H_{\alpha_3}+16H_{\alpha_4}+13H_{\alpha_5}+10H_{\alpha_6}+7H_{\alpha_7}+H_{\alpha_8}$
$e=\sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+2X_{-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}+X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_5}\oplus\mathbb{C}(H_{\alpha_3}+H_{\alpha_4})\oplus\mathbb{C}(2H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_6}+H_{\alpha_7}+H_{\alpha_8})$
$\oplus\mathbb{C}(2H_{\alpha_1}+H_{\alpha_3}+H_{\alpha_7}+H_{\alpha_8})$
$d\delta_e(z)=-z_3-z_4$ **sc-admissible** $\chi=-\lambda_8$ Not special

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23. 0110001 2 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq B_2+T_1$
$x=8H_{\alpha_1}+12H_{\alpha_2}+16H_{\alpha_3}+23H_{\alpha_4}+18H_{\alpha_5}+13H_{\alpha_6}+8H_{\alpha_7}+2H_{\alpha_8}$
$e=\sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8})$
$+2X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}+\sqrt{2}(X_{-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_5}\oplus\mathbb{C}(H_{\alpha_4}+H_{\alpha_6})\oplus\mathbb{C}(2H_{\alpha_1}+H_{\alpha_2}+2H_{\alpha_3}+H_{\alpha_4}+H_{\alpha_7}+H_{\alpha_8})$
$d\delta_e(z)=-8z_3$ **admissible** special

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26. 1010011 1 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq A_2+T_1$
$x=9H_{\alpha_1}+12H_{\alpha_2}+17H_{\alpha_3}+24H_{\alpha_4}+19H_{\alpha_5}+14H_{\alpha_6}+8H_{\alpha_7}+H_{\alpha_8}$
$e=2(X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}+X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8})$
$+\sqrt{6}(X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}+X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_2}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_3}+H_{\alpha_5}+2H_{\alpha_6}+2H_{\alpha_7}+H_{\alpha_8})$
$d\delta_e(z)=-5z_3$ **sc-admissible** $\chi=-5\lambda_8$ special

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27. 0110003 4 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq A_3$
$x=12H_{\alpha_1}+18H_{\alpha_2}+24H_{\alpha_3}+35H_{\alpha_4}+28H_{\alpha_5}+21H_{\alpha_6}+14H_{\alpha_7}+4H_{\alpha_8}$
$e=\sqrt{1+\sqrt{6}}X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{6}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}+\sqrt{1-\sqrt{6}}X_{-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{6-\sqrt{6}}X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{6+\sqrt{6}}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$
$d\delta_e(z) = 0$ **admissible** special

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29. 1000031 3 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq G_2 + T_1$
$x = 12H_{\alpha_1} + 17H_{\alpha_2} + 23H_{\alpha_3} + 34H_{\alpha_4} + 28H_{\alpha_5} + 22H_{\alpha_6} + 13H_{\alpha_7} + 3H_{\alpha_8}$
$e = \sqrt{5}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$
$+\sqrt{8}(X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8} + X_{-\alpha_1-2\alpha_2-2\alpha_3-3\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+3X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_3} + 2H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$
$d\delta_e(z) = -7z_3$ **sc-admissible** $\chi = -7\lambda_8$ Not special

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References

[A-K] L. Auslander and B. Kostant, *Polarization and unitary representations of solvable Lie groups*, Invent. Math. **14** (1971), 255-354. MR **45:**2092

[Bo] N. Bourbaki, *Groupes et Algèbre de Lie, Chapitres 4,5,6*, Elements de mathématique, Masson (1981).

[C] C. W. Curtis, *Corrections and additions to: 'On the degrees and rationality of certain characters of finite Chevalley groups'*, Trans. Amer. Math. Soc. **202** (1975), 405-406. MR **51:**737

[D] M. Duflo, *Construction de représentations unitaires d'un groupe de Lie*, Harmonic Analysis and Group Representations. **C.I.M.E.** (1982). MR **87b:**22028

[D1] D. Djoković, *Classification of nilpotent elements in simple exceptional real Lie algebras of inner type and description of their centralizers*, J. Alg. **112** (1987), 503-524. MR **89b:**170

[D2] D. Djoković, *Proof of a conjecture of Kostant*, Trans. Amer. Math. Soc. **302(2)** (1988), 577-585. MR **88j:**17007

[Dy] E. Dynkin, *Semisimple subalgebras of simple Lie algebras*, Selected papers of E.B. Dynkin with commentary edited by Yushkevich, Seitz and Onishchik AMS, (2000), 175-309.

[Dy1] E. Dynkin, *Semisimple subalgebras of simple Lie algebras*, Amer. Soc. Transl. Ser. 2 **6**, (1957), 111-245.

[K] A. A. Kirillov, *Unitary representations of nilpotent Lie groups*, Russian Math. Surveys **17** (1962), 57-110. MR **25:**5396

[K-R] B. Kostant and S. Rallis, *Orbits and representations associated with symmetric spaces*, Amer. J. Math. **93** (1971), 753–809. MR **47:**399

[Kn] A. W. Knapp, *Lie Groups Beyond an Introduction*, vol. **140**, Birkhäuser, Progress in Mathematics Boston, 1996. MR **98b:**22002

[Ne] M. Nevins, *Admissible nilpotent coadjoint orbits in the p-adic reductive groups*, Ph.D. Thesis M.I.T. Cambridge, MA (June 1998).

[L] G. Lusztig, *Some problems in the representation theory of finite Chevalley groups*, Proc. Symp. Pure Math., **37**, Amer. Math. Soc., (1980), 313-317. MR **82i:**20014

[No] A. G. Noël, *Nilpotent orbits and theta-stable parabolic Subalgebras*, Amer. Math. Soc., J. Representation Theory **2**, (1998), 1-32. MR **99g:**17023

[No2] A. G. Noël, *Classification of admissible nilpotent orbits in simple real Lie algebras $E_{6(6)}$ and $E_{6(-26)}$*, Amer. Math. Soc., J. Represent. Theory **5** (2001), 494-502.

[O] T. Ohta, *Classification of admissible nilpotent orbits in the classical real Lie algebras*, J. of Algebra **136**, No. 1 (1991), 290-333. MR **92j:**22032

[Sch] J. Schwartz, *The determination of the admissible nilpotent orbits in real classical groups*, Ph. D. Thesis M.I.T. Cambridge, MA (1987). MR **88g:**53053

[Se] J. Sekiguchi, *Remarks on real nilpotent orbits of a symmetric pair*, J. Math. Soc. Japan **39**, No. 1 (1987), 127-138.

[S-S] T. A. Springer and R. Steinberg, Conjugacy classes, *Seminar on Algebraic Groups and Related Finite Groups*, Lecture Notes in Mathematics **131** (1970). MR **42:**3091

[V] D. Vogan, Jr., *Unitary representations of reductive groups*, Annals of Mathematical Studies, Princeton University Press Study **118** (1987). MR **89g:**22024

[V1] D. Vogan, Jr., *Associated varieties and unipotent representations*, Harmonic Analysis on Reductive Groups (1991), 315-388. MR **93k:**22012

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