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THE FISCHER-CLIFFORD MATRICES OF A MAXIMAL SUBGROUP OF Fi'_{24}

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ABSTRACT. The Fischer group Fi'_{24} is the largest sporadic simple Fischer group of order

$$1255205709190661721292800 = 2^{21}.3^{16}.5^2.7^3.11.13.17.23.29\ .$$

The group Fi'_{24} is the derived subgroup of the Fischer 3-transposition group Fi_{24} discovered by Bernd Fischer. There are five classes of elements of order 3 in Fi'_{24} as represented in ATLAS by $3A$, $3B$, $3C$, $3D$ and $3E$. A subgroup of Fi'_{24} of order 3 is called of type $3X$, where $X \in \{A, B, C, D, E\}$, if it is generated by an element in the class $3X$. There are six classes of maximal 3-local subgroups of Fi'_{24} as determined by Wilson. In this paper we determine the Fischer-Clifford matrices and conjugacy classes of one of these maximal 3-local subgroups $\bar{G} := N_{Fi'_{24}}(\langle N\rangle) \cong 3^7{\cdot}O_7(3)$, where $N \cong 3^7$ is the natural orthogonal module for $\bar{G}/N \cong O_7(3)$ with 364 subgroups of type $3B$ corresponding to the totally isotropic points. The group $\bar{G}$ is a nonsplit extension of N by $G \cong O_7(3)$.

1. INTRODUCTION

Let $\bar{G} = N{\cdot}G$ be an extension of N by G, where N is a normal subgroup of $\bar{G}$ and $\bar{G}/N \cong G$. Let $\bar{g} \in \bar{G}$ be a lifting of $g \in G$ under the natural homomorphism $\bar{G} \longrightarrow G$ and let $[g]$ be a conjugacy class of elements of G with representative g. Let $X(g) = \{x_1, x_2, \ldots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\bar{G}$ from the coset $N\bar{g}$ whose images under the natural homomorphism $\bar{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. Let $\{\theta_1, \theta_2, \ldots, \theta_t\}$ be a set of representatives of the orbits of $\bar{G}$ on $\mathrm{Irr}(N)$ such that for $1 \le i \le t$, we have $\bar{H}_i = I_{\bar{G}}(\theta_i)$ with the corresponding inertia factors H_i and let ψ_i be a projective character of $\bar{H}_i$ with the factor set $\bar{\alpha}_i$ such that $(\psi_i)_N = \theta_i$. By Gallagher [18] we have

$$\mathrm{Irr}(\bar{G}) = \bigcup_{i=1}^{t} \{(\psi_i\bar{\beta})^{\bar{G}} \mid \beta \in \mathrm{IrrProj}(H_i),\ \text{ with the factor set } \alpha_i^{-1}\},$$

where α_i is obtained from $\bar{\alpha}_i$ and $\bar{\beta}$ from β. Without loss of generality, suppose that $\theta_1 = 1_N$ is the identity character of N. Then $\bar{H}_1 = \bar{G}$ and $H_1 = G$. Now choose $y_1, y_2, \ldots, y_r$ to be the representatives of the α_i^{-1}-conjugacy classes of elements of H_i that fuse to $[g]$ in G. Since $y_k \in H_i$ for $1 \le k \le r$, then we define $y_{l_k} \in \bar{H}_i$ such that y_{l_k} ranges over all representatives of the conjugacy classes of elements of $\bar{H}_i$

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which map to y_k under the homomorphism $\bar{H}_i \longrightarrow H_i$ whose kernel is N. Now by using the formula for induced characters, we have

$$\begin{aligned}(\psi_i\bar{\beta})^{\bar{G}}(x_j) &= \sum_{1\le k\le r}\sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|}\psi_i\bar{\beta}(y_{\ell_k}) \\ &= \sum_{1\le k\le r}\sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|}\psi_i(y_{\ell_k})\bar{\beta}(y_{\ell_k}) \\ &= \sum_{1\le k\le r}(\sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|}\psi_i(y_{\ell_k}))\beta(y_k)\end{aligned}$$

where $\sum_{\ell}{}'$ is the summation over all ℓ for which $y_{\ell_k} \sim x_j$ in $\bar{G}$. Now we define a matrix $M_i(g)$ by $M_i(g) = (a_{uv})$, where $1 \le u \le r$ and $1 \le v \le c(g)$, and

$$a_{uv} = \sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|}\psi_i(y_{\ell_k}) \ .$$

Then we get

$$(\psi_i\beta)^{\bar{G}}(x_j) = \sum_{1\le k\le r} a_{uv}\hat{\beta}(y_k) \ .$$

By doing so for all $1 \le i \le t$ such that H_i contains an element in $[g]$ we obtain the matrix $M(g)$ given by

$$M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix} ,$$

where $M_i(g)$ is the submatrix corresponding to the inertia group $\bar{H}_i$ and its inertia factor H_i. If $H_i \cap [g] = \emptyset$, then $M_i(g)$ will not exist and $M(g)$ does not contain $M_i(g)$. The size of the matrix $M(g)$ is $l \times c(g)$ where l is the number of α_i^{-1}-regular conjugacy classes of elements of the inertia factors H_i for $1 \le i \le t$ which fuse into $[g]$ in G, and $c(g)$ is the number of conjugacy classes of elements of $\bar{G}$ which correspond to the coset $\bar{g}N$. Then $M(g)$ is the *Fischer-Clifford matrix* of $\bar{G}$ corresponding to the coset $\bar{g}N$. We will see later that $M(g)$ is a $c(g) \times c(g)$ nonsingular matrix. Let

$$R(g) = \{(i, y_k) \mid 1 \le i \le t \ , \ H_i \cap [g] \ne \emptyset \ , \ 1 \le k \le r\} \ ,$$

and we note that y_k runs over representatives of the α_i^{-1}-conjugacy classes of elements of H_i which fuse into $[g]$ in G. Following the notation used by Fischer [9], Mpono [25] and Whitely [30] we denote $M(g)$ by writing $M(g) = Cl(N\bar{g}) = (a_j^{(i,y_k)})$, where

$$a_j^{(i,y_k)} = \sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|}\psi_i(y_{\ell_k}) \ ,$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$. Then the partial character table of $\bar{G}$ on the classes $\{x_1, x_2, \ldots, x_{c(g)}\}$ is given by

$$\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix}$$

where the Fischer-Clifford matrix $M(g)$ is divided into blocks with each block corresponding to an inertia group $\bar{H}_i$ and $C_i(g)$ is the partial projective character table of H_i with the factor set α_i^{-1} consisting of the columns corresponding to the α_i^{-1}-regular classes that fuse into $[g]$ in G. We obtain the characters of $\bar{G}$ by multiplying the relevant columns of the projective characters of H_i with the factor set α_i^{-1} by the rows of $M(g)$. We can also observe that the number of irreducible characters of $\bar{G}$ is the sum of numbers of projective characters of the inertia factors H_i with the factor set α_i^{-1}, for all i, $1 \leq i \leq t$.

In the present paper we determine Fischer-Clifford matrices and conjugacy classes of $3^7 \cdot O_7(3)$. Then the character table of $3^7 \cdot O_7(3)$ can be constructed by using these matrices and the partial character tables of the inertia factor groups.

Let K be a group and $A \leq \mathrm{Aut}(K)$. Then by Brauer's theorem (see [14]) A acts on the conjugacy classes of elements of K and on the irreducible characters of K resulting in the same number of orbits.

Lemma 1.1. *Suppose we have the following matrix describing the above actions:*

$$\begin{array}{c} \\ s_1 \\ s_2 \\ \vdots \\ s_i \\ \vdots \\ s_t \end{array}
\begin{array}{c} \begin{array}{cccccc} 1=l_1 & l_2 & \cdots & l_j & \cdots & l_t \end{array} \\
\begin{pmatrix} 1 & 1 & \cdots & 1 & \cdots & 1 \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2t} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{it} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{tj} & \cdots & a_{tt} \end{pmatrix} \end{array}$$

where $a_{1j} = 1$ for $j \in \{1, 2, \ldots, t\}$, l_j's are lengths of orbits of A on the conjugacy classes of K, s_i's are lengths of orbits of A on $\mathrm{Irr}(K)$ and a_{ij} is the sum of s_i irreducible characters of K on the element x_j, where x_j is an element of the orbit of length l_j. Then the following relation holds for $i, i' \in \{1, 2, \ldots, t\}$:
$\sum_{j=1}^{t} a_{ij}\overline{a_{i'j}}l_j = |K|s_i\delta_{ii'}$.

Proof. This result has been proved as Lemma 2.2.2 in [27] and as Lemma 4.2.2 in [30]. □

For arithmetical properties the weights are important and hence we attach to $M(g)$ the corresponding *weights*. Let $x_j \in X(g)$. For a fixed coset $X = \bar{g}N \in \bar{G}/N$, we define $m_j = [N_{\bar{G}}(X) : C_{\bar{G}}(x_j)]$.

The Fischer-Clifford matrix $M(g)$ is partitioned row-wise into blocks, where each block corresponds to an inertia group. The columns of $M(g)$ are indexed by $X(g)$ and for each $x_j \in X(g)$, at the top of the columns of $M(g)$, we write $|C_{\bar{G}}(x_j)|$ and at the bottom we write m_j. The rows of $M(g)$ are indexed by $R(g)$ and on the left of each row we write $|C_{H_i}(y_k)|$, where y_k fuses into $[g]$ in G. Then, in general, we can write $M(g)$ with corresponding weights for rows and columns as follows, where blocks corresponding to the inertia groups are separated by horizontal lines:

$$
\begin{array}{c}
\\
|C_G(g)| \\
\\
|C_{H_2}(y_1)| \\
|C_{H_2}(y_2)| \\
\vdots \\
\\
|C_{H_i}(y_1)| \\
|C_{H_i}(y_2)| \\
\vdots \\
\\
|C_{H_t}(y_1)| \\
|C_{H_t}(y_2)| \\
\vdots \\
\\
\\
\end{array}
\begin{array}{c}
\begin{array}{cccc}
|C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})|
\end{array} \\
\left(
\begin{array}{cccc}
a_1^{(1,g)} & a_2^{(1,g)} & \dots & a_{c(g)}^{(1,g)} \\
\hline
a_1^{(2,y_1)} & a_2^{(2,y_1)} & \dots & a_{c(g)}^{(2,y_1)} \\
a_1^{(2,y_2)} & a_2^{(2,y_2)} & \dots & a_{c(g)}^{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
a_1^{(i,y_1)} & a_2^{(i,y_1)} & \dots & a_{c(g)}^{(i,y_1)} \\
a_1^{(i,y_2)} & a_2^{(i,y_2)} & \dots & a_{c(g)}^{(i,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
a_1^{(t,y_1)} & a_2^{(t,y_1)} & \dots & a_{c(g)}^{(t,y_1)} \\
a_1^{(t,y_2)} & a_2^{(t,y_2)} & \dots & a_{c(g)}^{(t,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
\end{array}
\right) \\
\\
\begin{array}{cccc}
m_1 & m_2 & \dots & m_{c(g)}
\end{array}
\end{array}
$$

The following result gives the orthogonality relation for $M(g)$. Its proof was obtained from Whitley [30], Proposition 4.2.3.

Proposition 1.2 ([30], [25] (*Column orthogonality*))**.** *Let $\bar{G} = N{\cdot}G$. Then*

$$\sum_{(i,y_k)\in R(g)} |C_{H_i}(y_k)| a_j^{(i,y_k)} \overline{a_{j'}^{(i,y_k)}} = \delta_{jj'} |C_{\bar{G}}(x_j)| \quad .$$

Proof. The partial character table of $\bar{G}$ at classes $x_1, \dots, x_{c(g)}$ is given by

$$\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix}.$$

By column orthogonality of the character table of $\bar{G}$, we have

$$\begin{aligned}
|C_{\bar{G}}(x_j)|\delta_{jj'} &= \sum_{i=1}^{t} \sum_{\beta_i\in \mathrm{IrrProj}(H_i)} \Big(\sum_{y_k:(i,y_k)\in R(g)} a_j^{(i,y_k)}\beta_i(y_k)\Big)\overline{\Big(\sum_{y'_k:(i,y'_k)\in R(g)} a_{j'}^{(i,y'_k)}\beta_i(y'_k)\Big)} \\
&= \sum_{i=1}^{t} \sum_{\beta_i\in \mathrm{IrrProj}(H_i)} \Big(\sum_{y_k} a_j^{(i,y_k)}\overline{a_{j'}^{(i,y'_k)}}\beta_i(y_k)\overline{\beta_i(y_k)} \\
&\quad + \sum_{y_k}\sum_{y'_k\neq y_k} a_j^{(i,y_k)}\overline{a_{j'}^{(i,y'_k)}}\beta_i(y_k)\overline{\beta_i(y'_k)}\Big) \\
&= \sum_{i=1}^{t} \Big(\sum_{y_k} a_j^{(i,y_k)}\overline{a_{j'}^{(i,y_k)}} \sum_{\beta_i\in \mathrm{IrrProj}(H_i)} \beta_i(y_k)\overline{\beta_i(y_k)} \\
&\quad + \sum_{y_k}\sum_{y'_k\neq y_k} a_j^{(i,y_k)}\overline{a_{j'}^{(i,y'_k)}} \sum_{\beta_i\in \mathrm{IrrProj}(H_i)} \beta_i(y_k)\overline{\beta_i(y'_k)}\Big) \\
&= \sum_{i=1}^{t} \Big(\sum_{y_k} a_j^{(i,y_k)}\overline{a_{j'}^{(i,y_k)}}|C_{H_i}(y_k)| \quad + \quad 0\Big) \\
&= \sum_{(i,y_k)\in R(g)} a_j^{(i,y_k)}\overline{a_{j'}^{(i,y_k)}}|C_{H_i}(y_k)|.
\end{aligned}$$

□

Theorem 1.3. $a_j^{(1,g)} = 1$ *for all* $j \in \{1, 2, \ldots, c(g)\}$.

Proof. For $y_{\ell_k} \sim x_j$ in $\bar{G}$, we have $|C_{\bar{G}}(x_j)| = |C_{\overline{H}_1}(y_{\ell_k})|$. Thus we obtain that

$$a_j^{(1,g)} = {\sum_{\ell}}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\overline{H}_1}(y_{\ell_k})|}\psi_1(y_{\ell_k}) = {\sum_{\ell}}' 1 = 1 \quad .$$

Hence the result. □

Proposition 1.4 ([19], [30])**.** *The matrix* $M(1_G)$ *is the matrix with rows equal to the orbit sums of the action of* $\bar{G}$ *on* $\mathrm{Irr}(N)$ *with duplicate columns discarded. For this matrix we have* $a_j^{(i,1_G)} = [G : H_i]$*, and an orthogonality relation for rows:*

$$\sum_{j=1}^{t} \frac{1}{|C_{\bar{G}}(x_j)|} a_j^{(i,1_G)} a_j^{(i',1_G)} = \frac{1}{|C_{H_i}(1_G)|}\delta_{ii'} = \frac{1}{|H_i|}\delta_{ii'} \ .$$

Proof. See [25]. □

As a consequence of Lemma 1.1, Proposition 1.3 and from Fischer [10], we have the following properties:

(a) $|X(g)| = |R(g)|$,
(b) $\sum_{j=1}^{c(g)} m_j a_j^{(i,y_k)}\overline{a_j^{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \frac{|C_G(g)|}{|C_{H_i}(y_k)|}|N|$,
(c) $\sum_{(i,y_k)\in R(g)} a_j^{(i,y_k)}\overline{a_{j'}^{(i,y_k)}}|C_{H_i}(y_k)| = \delta_{jj'}|C_{\bar{G}}(x_j)|$,
(d) $M(g)$ is square and nonsingular.

2. Split cosets

From now on suppose that N is an elementary abelian normal p-subgroup of $\bar{G}$ and $\bar{g}N = X$ is a fixed coset of $\bar{G}/N \cong G$. Let $M = C_{\bar{g}} = N_{\bar{G}}(X)$. We define

$$N_{\bar{g}} := \langle [\bar{g}, n],\ n \in N \rangle.$$

With this notation we have the following lemma.

Lemma 2.1. (i) *$N_x = N_{\bar{g}}$ for all $x \in X$ and $[\bar{g}, u_1].[\bar{g}, u_2] = [\bar{g}, u_1u_2]$ for all $u_1, u_2 \in N$.*
(ii) *$N_{\bar{g}} \lhd M$ and $N_{\bar{g}} \leq N$.*
(iii) *If $\varphi \in \mathrm{Irr}(N)$, then $N_{\bar{g}} \leq \ker(\varphi)$ or $I_{\bar{G}}(\varphi) \cap \bar{g}N = \emptyset$.*

Proof. (i) Let $x = \bar{g}n \in \bar{g}N$ and $u \in N$, then

$$\begin{aligned} [x, u] &= [\bar{g}n, u] = n^{-1}(u^{-1})^{\bar{g}}nu \\ &= (u^{-1})^{\bar{g}}u \qquad \text{since } N \text{ is abelian} \\ &= \bar{g}^{-1}u^{-1}\bar{g}u = [\bar{g}, u] \end{aligned}$$

which implies that

$$N_x = N_{\bar{g}} \quad \text{for all} \quad x \in \bar{g}N.$$

Also, since N is abelian, we obtain for all $u_1, u_2 \in N$,

$$\begin{aligned} [\bar{g}, u_1].[\bar{g}, u_2] &= (u_1^{-1})^{\bar{g}}u_1(u_2^{-1})^{\bar{g}}u_2 \\ &= (u_1^{-1}u_2^{-1})^{\bar{g}}u_1u_2 \\ &= [\bar{g}, u_1u_2]\ . \end{aligned}$$

Hence

$$[\bar{g}, u_1].[\bar{g}, u_2] = [\bar{g}, u_1u_2] \quad \text{for} \quad u_1, u_2 \in N.$$

(ii) Since $[\bar{g}, u] = (u^{-1})^{\bar{g}}u \in N$, we obtain $N_{\bar{g}} \leq N \leq M$. Conversely, let $m \in M$. Then

$$\begin{aligned} [\bar{g}, u]^m &= m^{-1}[\bar{g}, u]m \\ &= (\bar{g}^{-1})^m(u^{-1})^m\bar{g}^m u^m \\ &= (\bar{g}^m)^{-1}(u^m)^{-1}\bar{g}^m u^m \\ &= [\bar{g}^m, u^m] \in N_{\bar{g}}\ . \end{aligned}$$

Hence $N_{\bar{g}} \lhd M$.

(iii) Let $\varphi \in \mathrm{Irr}(N)$ be fixed. Then

$$\begin{aligned} N_{\bar{g}} \leq \mathrm{Ker}(\varphi) &\Leftrightarrow \varphi([\bar{g}, u]) = \varphi(1) = 1 \quad \text{for all} \quad u \in N \\ &\Leftrightarrow \varphi(\bar{g}^{-1}u^{-1}\bar{g}u) = \varphi((u^{-1})^{\bar{g}}u) = 1 \\ &\Leftrightarrow \varphi((u^{-1})^{\bar{g}}) = (\varphi(u))^{-1} = \varphi(u^{-1}) \\ &\Leftrightarrow \varphi^{\bar{g}}(u^{-1}) = \varphi(u^{-1}) \\ &\Leftrightarrow \varphi^{\bar{g}} = \varphi \\ &\Leftrightarrow \bar{g}N \subseteq I_{\bar{G}}(\varphi) \\ &\Leftrightarrow \bar{g}N \cup I_{\bar{G}}(\varphi) \neq \emptyset\ . \end{aligned}$$

□

Remark 2.2. We can easily show that $\langle X \rangle / N_{\bar{g}}$ is abelian and $X/N_{\bar{g}}$ is a coset of $\langle X \rangle / N_{\bar{g}}$.

Lemma 2.3 ([10]). *The rows of the Fischer-Clifford matrix $Cl(X)$ can be identified with restrictions of M-invariant characters of $\langle X\rangle/N_{\bar{g}}$ to $X/N_{\bar{g}}$.*

Proof. This is Lemma 5.2 in [10]. □

Remark 2.4. In the above lemma, the rows of $Cl(X)$ will be an independent set of orbit sums, under the action of M on $\langle X\rangle/N_{\bar{g}}$. This observation was first given in Fischer [10].

Definition 2.5. A coset X is said to be a *split coset* if it contains an element x such that $M = N.C_{\bar{G}}(x)$.

Note that we do not require $\langle x\rangle \cap N = \langle 1\rangle$ in the above definition.

Lemma 2.6 ([28]). *If the extension splits, then every coset is a split coset.*

Proof. Let $X = \bar{g}N$ and $h \in C_{\bar{G}}(\bar{g})$. Then $h(\bar{g}n)h^{-1} = (h\bar{g}h^{-1})(hnh^{-1}) = \bar{g}hnh^{-1} = \bar{g}n^h \in \bar{g}N$. Now, since $N \leq M$ and $C_{\bar{G}}(\bar{g}) \leq M$, then $M \geq N.C_{\bar{G}}(\bar{g})$. Let C be the complement of N in $\bar{G}$ such that $\bar{g} \in C$. Let $m \in M$. Then $m = n.k$, for some $k \in C$. Since $M = N_{\bar{G}}(\bar{g}N)$, $(\bar{g}N)^m = \bar{g}N$. Hence

$$\begin{aligned}\bar{g}N &= (\bar{g}N)^m = m(\bar{g}N)m^{-1}\\ &= n(k\bar{g}Nk^{-1})n^{-1} = n(k\bar{g}Nk^{-1})n^{-1} = n(\bar{g}N)^k n^{-1},\end{aligned}$$

so that $n^{-1}(\bar{g}N)n = (\bar{g}N)^k$ and $n^{-1}\bar{g}N = (\bar{g}N)^k$. Hence $\bar{g}N = (\bar{g}N)^k$. It follows that $\bar{g}N = (\bar{g}N)^k = \bar{g}^kN$, which implies that $\bar{g}^k \in \bar{g}N$. Hence $\bar{g}^k \in C \cap \bar{g}N = \{\bar{g}\}$ and so $k \in C_{\bar{G}}(\bar{g})$, which implies that $m = n.k \in N.C_{\bar{G}}(\bar{g})$ and so $M \leq N.C_{\bar{G}}(\bar{g})$. Thus $M = N.C_{\bar{G}}(\bar{g})$. □

The following result is of fundamental importance and very helpful to fill the entries of Fischer-Clifford matrices.

Lemma 2.7 ([9]). *Let X be a split coset. Then the rows of $Cl(X)$ can be identified with M-invariant characters of $N/N_{\bar{g}}$ multiplied by a p-th root of unity.*

Proof. See [9] and [28]. □

Lemma 2.8. *Let $X = \bar{g}N$ be a split coset and $N_{\bar{G}}(X) = NC_{\bar{G}}(x)$ for $x \in X(g)$. Then we have the following:*

(i) $a_1^{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$,

(ii) $|a_j^{(i,y_k)}| \leq |a_1^{(i,y_k)}|$ *for all* $1 \leq j \leq r$,

(iii) *If* $|N| = p^w$, *then* $a_j^{(i,y_k)} \equiv a_1^{(i,y_k)} (\mathrm{mod}\ p)$.

Proof. See [28]. □

3. Nonsplit extensions

Let $\bar{G} = N{\cdot}G$ be a nonsplit extension, where N is an elementary abelian normal p-subgroup of $\bar{G}$. Let $\bar{g}N$ be a conjugacy class representative of $\bar{G}/N$ and let φ be a representative of $\bar{G}$-orbit irreducible characters of N with the projective extension $\bar{\varphi}$ to $\bar{G}$. We consider the groups $\langle\bar{g}\rangle N \leq \bar{G}$ and $\langle\bar{g}N\rangle \leq \bar{G}/N$.

Lemma 3.1.

$$\langle\bar{g}\rangle N/N = \langle\bar{g}N\rangle\ .$$

Proof. Let $x \in \langle \bar{g} \rangle N/N$. Then $x = \bar{g}^m nN = \bar{g}^m N$ for some $m \in \mathbb{Z}$. So that $x = (\bar{g}N)^m \in \langle \bar{g}N \rangle$. Hence $\langle \bar{g} \rangle N/N \leq \langle \bar{g}N \rangle$. Conversely, let $x \in \langle \bar{g}N \rangle$. Then $x = (\bar{g}N)^m = \bar{g}^m N$ for some $m \in \mathbb{Z}$. Hence $x = (\bar{g}^m N) \in \langle \bar{g} \rangle N/N$. Thus $\langle \bar{g}N \rangle \leq \langle \bar{g} \rangle N/N$. Therefore, $\langle \bar{g} \rangle N/N = \langle \bar{g}N \rangle$. □

Lemma 3.2. *With the above notation, we have the following:*

(a) $\langle \bar{g} \rangle N \leq M$.
(b) $(\langle \bar{g} \rangle N)' = N_{\bar{g}}$ *where* $(\langle \bar{g} \rangle N)'$ *denotes the commutator subgroup of* $\langle \bar{g} \rangle N$.
(c) $\langle \bar{g} \rangle N \leq I_M(\varphi)$ *where* $\varphi \in \mathrm{Irr}(N)$.
(d) *Given* $\varphi \in \mathrm{Irr}(N)$ *there exists an extension* $\eta\beta$ *to* $\langle \bar{g} \rangle N$ *where* $\eta = (\bar{\varphi})_{\langle \bar{g} \rangle N}$ *and* β *is a projective character of* $\langle \bar{g}N \rangle$.

Proof. (a) Let $x \in \langle \bar{g} \rangle N$. Then $x = \bar{g}^m N$ for some $m \in \mathbb{Z}$. Now

$$\begin{aligned} x(\bar{g}N) &= \bar{g}^m n(\bar{g}N) = \bar{g}^m n\bar{g}N = \bar{g}^m nN\bar{g} \quad (\text{since} \quad N \trianglelefteq \bar{G}) \\ &= \bar{g}^m N\bar{g} = N\bar{g}^{m+1}. \end{aligned}$$

Similarly, $(\bar{g}N)x = N\bar{g}^{m+1}$. Hence $x \in M = N_{\bar{G}}(\bar{g}N)$ and so $\langle \bar{g} \rangle N \leq M$.

(b) First suppose that $[\bar{g}, n] \in N_{\bar{g}}$. Then $[\bar{g}, n] \in (\langle \bar{g} \rangle N)'$ and thus $N_{\bar{g}} \leq (\langle \bar{g} \rangle N)'$. Also, for $n \in N$, by the definition of $N_{\bar{g}}$, we have

$$(\bar{g}N_{\bar{g}})(nN_{\bar{g}}) = (nN_{\bar{g}})(\bar{g}N_{\bar{g}}).$$

Therefore, $(\langle \bar{g} \rangle N/N_{\bar{g}})$ is abelian, and hence $(\langle \bar{g} \rangle N)' \leq N_{\bar{g}}$ and we deduce that $(\langle \bar{g} \rangle N)' = N_{\bar{g}}$.

(c) Let $\varphi \in \mathrm{Irr}(N)$. Then $N_{\bar{g}} \leq \mathrm{Ker}(\varphi)$. Now by Lemma 2.1, we have $\bar{g}N \cap I_M(\varphi) \neq \emptyset$. Therefore, $\bar{g}$ lies in $I_M(\varphi)$ and so $\langle \bar{g} \rangle \leq I_M(\varphi)$. Hence $\langle \bar{g} \rangle N \leq I_M(\varphi)$.

(d) Notice that by part (c), $W = \langle \bar{g} \rangle N$ is a subgroup of $I_M(\varphi)$. Hence φ is invariant under W. So we can apply the Theorem 5.8 in [26] to φ and W. Let $\chi \in \mathrm{Irr}(\langle \bar{g} \rangle N, \varphi)$. Then by the Clifford theorem (see [26]) we obtain $\chi = ((\bar{\varphi})_{\langle \bar{g} \rangle N})\beta = \eta\beta$ where β is an $\bar{\alpha}^{-1}$-projective character of $\langle \bar{g} \rangle N/N = \langle \bar{g}N \rangle$ and $\bar{\alpha}$ is the factor set of $\langle \bar{g} \rangle N \times \langle \bar{g} \rangle N$ obtained from α. If N is abelian, then χ is linear since $\chi_N = \varphi$ is linear (because $\deg(\chi) = \deg(\varphi) = 1$). □

Theorem 3.3 ([11])**.** *Let* $\bar{g} \in \bar{G}$, *so that* $\langle \bar{g} \rangle N$ *is abelian. Then* $\langle \bar{g}N \rangle \leq Z(\bar{G}/N)$ *and the rows of the Fischer-Clifford matrix of* $\bar{g}N$ *for regular classes of the inertia group of* φ *in* $\bar{G}$ *can be regarded as restrictions to* $\bar{g}N$ *of the* $\bar{G}$*-orbit sums of the (projective) extension* $\eta\beta$ *to* $\langle \bar{g} \rangle N$ *of* φ.

Proof. See [11] and [28]. □

4. The action of $O_7(3)$ on 3^7

We know that $O_7(3)$ acts naturally on 3^7. The action of $O_7(3)$ on 3^7 gives rise to four orbits of lengths 1, 702, 728 and 756 and hence four point stabilizes $O_7(3)$, $2U_4(3)$, $3^5{:}U_4(2)$ and $L_4(3)$ respectively. Let $\chi(O_7(3)|2U_4(3))$, $\chi(O_7(3)|3^5{:}U_4(2))$ and $\chi(O_7(3)|L_4(3))$ be the permutation characters of $O_7(3)$ on $2U_4(3)$, $3^5{:}U_4(2)$ and $L_4(3)$ respectively. Then using GAP [29] and by considering the fact that

$\chi(O_7(3)|3^7)(g) = 3^n$ for all $g \in O_7(3)$ and for some $n \in \{0, 1, \ldots, 7\}$, we obtain that

$$\begin{aligned}\chi(O_7(3)|2U_4(3)) &= 1a + 78a + 168a + 182a + 273a,\\ \chi(O_7(3)|3^5{:}U_4(2)) &= 1a + 91a + 168a + 195a + 273a,\\ \chi(O_7(3)|L_4(3)) &= 1a + 105a + 182a + 195a + 273a,\end{aligned}$$

where $1a$, $78a$, $91a$, $105a$, $168a$, $182a$, $195a$ and $273a$ are irreducible characters of $O_7(3)$ of degrees 1, 91, 105, 168, 182, 195 and 273 respectively. Then we have

$$\begin{aligned}\chi(O_7(3)|3^7) &= 1 + I_{2U_4(3)}^{O_7(3)} + I_{3^5:U_4(2)}^{O_7(3)} + I_{L_4(3)}^{O_7(3)}\\ &= 1a + 1a + 91a + 168a + 195a + 273a + 1a + 105a + 182a + 195a\\ &\quad + 273a + 1a + 105a + 182a + 195a + 273a\\ &= 4 \times 1a + 91a + 2 \times 105a + 168a + 2 \times 182a + 3 \times 195a + 3 \times 273a\end{aligned}$$

where $I_{2U_4(3)}^{O_7(3)}$, $I_{3^5:U_4(2)}^{O_7(3)}$ and $I_{L_4(3)}^{O_7(3)}$ are the identity characters of $2U_4(3)$, $3^5{:}U_4(2)$ and $L_4(3)$ respectively induced to $O_7(3)$. For each class representative $g \in O_7(3)$, $\chi(O_7(3)|3^7)(g)$ will give us the number of fixed points k which we provide in Table 1.

5. The inertia groups of $3^7{\cdot}O_7(3)$

When $O_7(3)$ acts on the conjugacy classes of 3^7 it produces four orbits of lengths 1, 702, 728 and 756. Hence by Brauer's theorem (see [14]) $O_7(3)$ acting on $\mathrm{Irr}(3^7)$ will also produce four orbits of lengths $1, s, t$ and u such that $s + t + u = 2186$. Now by checking the indices of the maximal subgroups of $O_7(3)$ given in the ATLAS, we can see that the only possibility is that $s = 702$, $t = 728$ and $u = 756$. We deduce that the four inertia groups are $\bar{H}_i = 3^7{:}H_i$ of indices 1, 702, 728 and 756 in $3^7{\cdot}O_7(3)$ respectively, where $i \in \{1, 2, 3, 4\}$ and $H_i \leq O_7(3)$ are the inertia factors. We also observed that the inertia factors are $H_1 = O_7(3)$, $H_2 = 2U_4(3)$, $H_3 = 3^5{:}U_4(2)$ and $H_4 = L_4(3)$ of indices 1, 702, 728 and 756 in $O_7(3)$ respectively. The (ordinary) character tables of H_1, H_2, H_3 and H_4 are given in the GAP [29]. We will see in Section 7 that we need to use the projective characters of $H_2 = 2U_4(3)$. The first step to find projective characters of a group is to find its Schur multiplier. In [15], D. Holt has developed a package called *Cohomolo* for calculating Schur multipliers of finite groups which is now available in GAP. Using the Cohomolo package we obtained that the Schur multiplier of H_2 is the group $C_3 \times C_6$. The projective characters of H_2 with the factor set α^{-1} where $\alpha^3 \sim 1$, can be obtained from ordinary characters of the threefold proper covering group $3.H_2$ of H_2. We will see later in Section 7 that the projective characters of H_2 with the factor set β^{-1} where $\beta^2 \sim 1$ are not required for our computations. The projective characters of H_2 are not listed here, but these are just a subset of the projective characters of $U_4(3)$ which are available in the ATLAS. In Section 7 we will use these projective characters to compute the irreducible characters of $\bar{G} = 3^7{\cdot}O_7(3)$. Note that H_2 has 31 α^{-1}-regular classes and 31 irreducible projective characters with the factor set α^{-1}.

TABLE 1

$[g]_{O_7(3)}$	$1A$	$2A$	$2B$	$2C$	$3A$	$3B$	$3C$	$3D$	$3E$	$3F$	$3G$	$4A$
$\chi(O_7(3)\|2U_4(3))$	702	2	90	6	54	90	72	0	0	18	0	0
$\chi(O_7(3)\|3^5{:}U_4(2))$	728	0	80	8	80	62	98	26	26	8	8	0
$\chi(O_7(3)\|L_4(3))$	756	0	72	12	108	90	72	0	0	0	18	2
k	2187	3	243	27	243	243	243	27	27	9	27	3

$[g]_{O_7(3)}$	$4B$	$4C$	$4D$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$6H$
$\chi(O_7(3)\|2U_4(3))$	6	2	12	12	2	2	2	18	0	12	6	6
$\chi(O_7(3)\|3^5{:}U_4(2))$	8	0	8	8	0	0	0	8	2	2	8	14
$\chi(O_7(3)\|L_4(3))$	12	0	6	6	0	0	0	0	0	12	12	6
k	27	3	27	27	3	1	3	27	3	27	27	27

$[g]_{O_7(3)}$	$6I$	$6J$	$6K$	$6L$	$6M$	$6N$	$6O$	$6P$	$7A$	$8A$	$8B$	$9A$
$\chi(O_7(3)\|2U_4(3))$	6	6	2	0	0	0	0	0	2	0	2	12
$\chi(O_7(3)\|3^5{:}U_4(2))$	8	8	0	2	2	2	2	2	0	0	0	2
$\chi(O_7(3)\|L_4(3))$	12	12	0	0	0	0	0	0	0	2	0	12
k	27	27	1	3	3	3	3	3	3	3	3	27

$[g]_{O_7(3)}$	$9B$	$9C$	$9D$	$10A$	$10B$	$12A$	$12B$	$12C$	$12D$	$12E$	$12F$	$12G$
$\chi(O_7(3)\|2U_4(3))$	6	0	0	0	2	0	6	0	0	2	0	0
$\chi(O_7(3)\|3^5{:}U_4(2))$	14	2	2	0	0	2	8	0	0	0	2	2
$\chi(O_7(3)\|L_4(3))$	6	0	0	2	0	0	12	2	2	0	0	0
k	27	3	3	3	3	3	27	3	3	3	3	3

$[g]_{O_7(3)}$	$12H$	$13A$	$13B$	$14A$	$15A$	$18A$	$18B$	$18C$	$18D$	$20A$
$\chi(O_7(3)\|2U_4(3))$	0	0	0	2	0	2	2	2	0	0
$\chi(O_7(3)\|3^5{:}U_4(2))$	2	0	0	0	2	0	0	0	2	0
$\chi(O_7(3)\|L_4(3))$	0	2	2	0	0	0	0	0	0	2
k	3	3	3	3	3	3	3	3	3	3

6. THE FUSION OF INERTIA FACTOR GROUPS INTO $O_7(3)$

Using the permutation characters of $O_7(3)$ on $H_2 = 2U_4(3)$, $H_3 = 3^5{:}U_4(2)$ and $H_4 = L_4(3)$ of degrees 702, 728 and 756 respectively we are able to obtain partial fusions of $H_2 = 2U_4(3)$, $H_3 = 3^5{:}U_4(2)$ and $H_4 = L_4(3)$ into $O_7(3)$. We completed the fusions by using direct matrix conjugation in $O_7(3)$. The complete fusions of $2U_4(3)$, $3^5{:}U_4(2)$ and $L_4(3)$ into $O_7(3)$ are given in Tables 2, 3 and 4 respectively.

TABLE 2. The fusion of $2U_4(3)$ into $O_7(3)$

$[h]_{2U_4(3)}$	$\longrightarrow$ $[g]_{O_7(3)}$	$[h]_{2U_4(3)}$	$\longrightarrow$ $[g]_{O_7(3)}$	$[h]_{2U_4(3)}$	$\longrightarrow$ $[g]_{O_7(3)}$
$1A$	$1A$	$2A$	$2A$	$2B$	$2C$
$2C$	$2B$	$3A$	$3A$	$3B$	$3C$
$3C$	$3B$	$3D$	$3F$	$4A$	$4B$
$4B$	$4C$	$4C$	$4D$	$5A$	$5A$
$6A$	$6A$	$6B$	$6C$	$6C$	$6B$
$6D$	$6K$	$6E$	$6G$	$6F$	$6D$
$6G$	$6J$	$6H$	$6F$	$6I$	$6I$
$6J$	$6H$	$7A$	$7A$	$7B$	$7A$
$8A$	$8B$	$8B$	$8B$	$9A$	$9B$
$9B$	$9B$	$9C$	$9A$	$9D$	$9A$
$10A$	$10B$	$12A$	$12B$	$12B$	$12E$
$14A$	$14A$	$14B$	$14A$	$18A$	$18C$
$18B$	$18C$	$18C$	$18B$	$18D$	$18A$

TABLE 3. The fusion of $3^5{:}U_4(2)$ into $O_7(3)$

$[h]_{3^5:U_4(2)}$	$\longrightarrow [g]_{O_7(3)}$	$[h]_{3^5:U_4(2)}$	$\longrightarrow [g]_{O_7(3)}$	$[h]_{3^5:U_4(2)}$	$\longrightarrow [g]_{O_7(3)}$
$1A$	$1A$	$2A$	$2C$	$2B$	$2B$
$3A$	$3B$	$3B$	$3A$	$3C$	$3C$
$3D$	$3A$	$3E$	$3D$	$3F$	$3E$
$3G$	$3G$	$3H$	$3A$	$3I$	$3D$
$3J$	$3E$	$3K$	$3G$	$3L$	$3B$
$3M$	$3G$	$3N$	$3G$	$3O$	$3F$
$3P$	$3C$	$3Q$	$3F$	$3R$	$3F$
$3S$	$3G$	$3T$	$3D$	$3U$	$3E$
$4A$	$4B$	$4B$	$4D$	$5A$	$5A$
$6A$	$6E$	$6B$	$6F$	$6C$	$6D$
$6D$	$6H$	$6E$	$6G$	$6F$	$6M$
$6G$	$6L$	$6H$	$6G$	$6I$	$6M$
$6J$	$6L$	$6K$	$6I$	$6L$	$6P$
$6M$	$6I$	$6N$	$6P$	$6O$	$6J$
$6P$	$6O$	$6Q$	$6N$	$6R$	$6H$
$9A$	$9A$	$9B$	$9B$	$9C$	$9B$
$9D$	$9D$	$9E$	$9C$	$9F$	$9B$
$9G$	$9D$	$9H$	$9C$	$12A$	$12A$
$12B$	$12A$	$12C$	$12H$	$12D$	$12B$
$12E$	$12G$	$12F$	$12F$	$12G$	$12B$
$12H$	$12G$	$12I$	$12F$	$15A$	$15A$
$15B$	$15A$	$18A$	$18D$		

TABLE 4. The fusion of $L_4(3)$ into $O_7(3)$

$[h]_{L_4(3)}$	$\longrightarrow [g]_{O_7(3)}$	$[h]_{L_4(3)}$	$\longrightarrow [g]_{O_7(3)}$	$[h]_{L_4(3)}$	$\longrightarrow [g]_{O_7(3)}$
$1A$	$1A$	$2A$	$2B$	$2B$	$2C$
$3A$	$3A$	$3B$	$3B$	$3C$	$3C$
$3D$	$3G$	$4A$	$4A$	$4B$	$4B$
$4C$	$4D$	$5A$	$5A$	$6A$	$6H$
$6B$	$6F$	$6C$	$6G$	$6D$	$6I$
$6E$	$6J$	$8A$	$8A$	$9A$	$9B$
$9B$	$9A$	$10A$	$10A$	$12A$	$12D$
$12B$	$12C$	$12C$	$12B$	$13A$	$13A$
$13B$	$13A$	$13C$	$13B$	$13D$	$13B$
$20A$	$20A$	$20B$	$20A$		

7. THE FISCHER-CLIFFORD MATRICES OF $3^7{\cdot}O_7(3)$

Having obtained the fusions of the inertia factors $2U_4(3)$, $3^5{:}U_4(2)$ and $L_4(3)$ into $O_7(3)$ (Tables 2, 3 and 4) together with properties of the Fischer-Clifford matrices discussed in Sections 1, 2 and 3, we are able to compute the Fischer-Clifford matrices of the nonsplit extension $3^7{\cdot}O_7(3)$.

Consider the coset corresponding to the identity of $O_7(3)$. Clearly this is a split coset, and since the action of $O_7(3)$ on $\mathrm{Irr}(N)$ is self-dual, we can easily determine the column weights and hence the orders of the centralizers for the classes of elements of $3^7{\cdot}O_7(3)$ corresponding to this coset. We used the results given in Sections 1, 2 and 3 to complete the entries of the rows of $M(1A)$. We obtain four conjugacy classes of elements of $\bar{G} = 3^7{\cdot}O_7(3)$ of orders 1, 3, 3 and 3 respectively corresponding to the identity coset. We have the following Fischer-Clifford matrix for the identity of $O_7(3)$:

$$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 702 & 0 & -27 & 27 \\ 728 & 26 & -1 & -28 \\ 756 & -27 & 27 & 0 \end{pmatrix}.$$

Let $\mathrm{Irr}(Fi'_{24}) = \{\psi_i : 1 \leq i \leq 108\}$ be the set of irreducible characters of Fi'_{24} as listed in the ATLAS. Then we have

$[x]_{Fi'_{24}}$	$1A$	$3A$	$3B$	$3C$
ψ_2	8671	247	-77	85
ψ_3	57477	534	615	210
ψ_4	249458	370	2705	869

Let γ_1, γ_2, γ_3 and γ_4 be the rows of the Fischer-Clifford matrix $M(1A)$. Since $\langle(\psi_2)_N, 1_N\rangle = 91$, $\langle(\psi_3)_N, 1_N\rangle = 483$ and $\langle(\psi_4)_N, 1_N\rangle = 1392$, we have the decompositions $(\psi_2)_N = 91\gamma_1 + 6\gamma_2 + 6\gamma_3$, $(\psi_3)_N = 483\gamma_1 + 21\gamma_2 + 30\gamma_3 + 27\gamma_4$ and $(\psi_4)_N = 1392\gamma_1 + 105\gamma_2 + 145\gamma_3 + 91\gamma_4$. Now by considering the coefficients of γ_2, we deduce that there are characters $\chi_1, \chi_2, \chi_3 \in \mathrm{Irr}(\bar{G})$, with $\deg(\chi_1) = 6 \times 702 = 4212$, $\deg(\chi_2) = 21 \times 702 = 14742$ and $\deg(\chi_3) = 105 \times 702 = 73710$. Let $[\ x_1\ x_2\ \ldots\ x_s\ ]$ be the transpose of the partial entries for the projective characters of $H_2 = 2U_4(3)$ on $1A \in O_7(3)$. Then $C_2(1A)M_2(1A)$ is a $s \times 4$ matrix with the entries of the first column $702x_1 = 4212$, $702x_2 = 14742$ and $702x_3 = 73710$. Hence $x_1 = 6$, $x_2 = 21$ and $x_3 = 105$. This shows that the partial projective character table of H_2 that we need to use should contain characters $\hat{\beta}_{21}$, $\hat{\beta}_{22}$ and $\hat{\beta}_{23}$ of degrees 6, 21 and 105 respectively. Now by checking the ordinary character table of $H_2 = 2U_4(3)$ which is available in GAP and ATLAS we can see that $\hat{\beta}_{21}$, $\hat{\beta}_{22}$ and $\hat{\beta}_{23}$ do not come from the ordinary characters of H_2 as there is no ordinary character of degree 6 in H_2. Since the Schur multiplier of H_2 is the group $C_3 \times C_6$, then three distinct projective character tables occur corresponding to the factor sets β^{-1}, α^{-1} and δ^{-1} with $\beta^2 \sim 1$, $\alpha^3 \sim 1$ and $\delta^6 \sim 1$. Now by Lemma 3.2.6 in [1] we know that each irreducible projective character with a factor set ω has its degree divisible by $o([\omega])$. Since $o([\beta]) = 2$ and 2 does not divide 21 and since $o([\delta]) = 6$ and 6 does not divide 105, we deduce that $\hat{\beta}_{21}$, $\hat{\beta}_{22}$ and $\hat{\beta}_{23}$ belong to the projective characters of H_2 with the factor set α^{-1} such that $\alpha^3 \sim 1$. Hence we need to use the projective characters of H_2 with the factor set α^{-1} to obtain the irreducible characters of $\bar{G} = 3^7{\cdot}O_7(3)$ corresponding to this inertia factor.

Similarly, by considering the coefficients of γ_3 in the decompositions of $(\psi_2)_N$, $(\psi_3)_N$ and $(\psi_4)_N$, we obtain that there are irreducible characters of $\bar{G}$ of degrees 4368, 21840, 7280, 10920 and 65520. Let $[\ y_1\ y_2\ \ldots\ y_t]$ be the transpose of the partial entries for the projective character table of $H_3 = 3^5{:}U_4(2)$ on $1A$. Then $C_3(1A)M_3(1A)$ is a $t \times 4$ matrix and we obtain that $y_1 = 6$, $y_2 = 30$, $y_3 = 10$, $y_4 = 15$ and $y_5 = 90$. This shows that the partial projective character table of H_3 needed, should contain characters $\hat{\beta}_{31}$, $\hat{\beta}_{32}$, $\hat{\beta}_{33}$, $\hat{\beta}_{34}$ and $\hat{\beta}_{35}$ of degrees 6, 30, 10, 15 and 90 respectively. Since the Schur multiplier of $H_3 = 3^5{:}U_4(2)$ is also the cyclic group of order 6, again three distinct projective character tables occur. Now by similar arguments as for H_2 we deduce that $\hat{\beta}_{31}$, $\hat{\beta}_{32}$, $\hat{\beta}_{33}$, $\hat{\beta}_{34}$, and $\hat{\beta}_{35}$ come from the ordinary characters of H_3. Hence we need to use the ordinary characters of H_3. The character table of $H_3 = 3^5{:}U_4(2)$ is available in GAP [29].

Finally, by considering the coefficients of γ_4 in the decompositions of $(\psi_3)_N$ and $(\psi_4)_N$, it can be shown that the partial projective character table of $H_4 = L_4(3)$ needed, contains a character of degree 1. Hence this partial projective character table comes from the ordinary characters of $H_4 = L_4(3)$.

To summarize, for our computations of Fischer-Clifford matrices we need to use the projective characters of $H_2 = 2U_4(3)$ corresponding to the factor set α^{-1} with

$\alpha^3 \sim 1$, the ordinary characters of $H_1 = O_7(3)$, $H_3 = 3^5{:}U_4(2)$ and $H_4 = L_4(3)$. This forces the shape of all Fischer-Clifford matrices by the results of Section 2. We produce altogether 183 conjugacy classes of elements of $\bar{G}$.

We use the results of Sections 1 and 2 to compute the entries of the Fischer-Clifford matrices of $\bar{G}$ for split cosets. The fusion of $\bar{G}$ into Fi'_{24} together with the restriction of characters of Fi'_{24} to $\bar{G}$ forces the signs of the Fischer-Clifford matrices and the orders of the elements of $\bar{G}$.

For example, consider the conjugacy class $3A$ of $O_7(3)$. Let $g \in 3A$. Since there are fusions from the α-regular class $3A$ of H_2 and from classes of H_3 and H_4 into $3A$ of $O_7(3)$, we deduce that the coset $\bar{g}N$ is a split coset. We obtain the following form of the Fischer-Clifford matrix $M(3A)$ with the possibility of change of signs for rows:

	306110016	*25509168*	*1417176*	*51018336*	*34012224*	*34012224*
1259712	1	1	1	1	1	1
23328	54	0	0	27	−27	−27
157464	8	8	−1	8	8	8
34992	36	9	0	−18	$18\alpha - 9\bar{\alpha}$	$-9\alpha + 18\bar{\alpha}$
34992	36	9	0	−18	$-9\alpha + 18\bar{\alpha}$	$18\alpha - 9\bar{\alpha}$
11664	108	−27	0	0	27	27
	1	*12*	*216*	*6*	*4*	*4*

where $\alpha = (-1+\sqrt{-3})/2$ and $\bar{\alpha} = (-1-\sqrt{-3})/2$.

Let $3D$, $3E$, $3F$, $3G$, $3H$ and $3I$ be conjugacy classes of $\bar{G}$, obtained by corresponding to this coset. First suppose that the above matrix is the Fischer-Clifford matrix $M(3A)$ as it is. Then by considering the restriction of $\psi_2 \in \mathrm{Irr}(Fi'_{24})$ to $\bar{G}$, we notice that there will be no fusion from the $3D$ class of $\bar{G}$ into Fi'_{24}. Hence this is not the required Fischer-Clifford matrix and the sign of the rows has to be changed. Now if we multiply rows 4 and 5 by $\alpha = (-1+\sqrt{-3})/2$ and $\bar{\alpha} = (-1-\sqrt{-3})/2$ respectively, then the resulting character table of $\bar{G}$ does not satisfy the congruence relations (Lemma 2.8). By checking various other possibilities we deduce that rows 4 and 5 have to be multiplied by $\bar{\alpha}$ and α respectively in order to get the proper character table of $\bar{G}$. Hence

$$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 54 & 0 & 0 & 27 & -27 & -27 \\ 8 & 8 & -1 & 8 & 8 & 8 \\ 36\bar{\alpha} & 9\bar{\alpha} & 0 & -18\bar{\alpha} & 18-9\alpha & -9+18\alpha \\ 36\alpha & 9\alpha & 0 & -18\alpha & -9\bar{\alpha}+18 & 18\bar{\alpha}-9 \\ 108 & -27 & 0 & 0 & 27 & 27 \end{pmatrix}.$$

Now consider the coset corresponding to the class $3B$ of $O_7(3)$. This is a nonsplit coset since there is a fusion from α-irregular class $3C$ of H_2. Using Programme C, given in [1], we find the appropriate column weights. We obtained the centralizer orders of the relevant classes by considering these column weights and by using the restriction of irreducible characters of Fi'_{24} to $\bar{G}$. We have the matrix corresponding to this coset in the following shape:

$$\begin{array}{c} \\ 174960 \\ 87480 \\ 2916 \\ 1944 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 7085880 & 2834352 & 708588 & 262440 \end{array} \\ \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 2 & a & f & m \\ -30 & b & g & n \\ 45 & c & h & p \end{array}\right) \\ \begin{array}{cccc} 2 & 5 & 20 & 54 \end{array} \end{array}$$

Notice that the restrictions of irreducible characters of Fi'_{24} to $\bar{G}$ determine the entries for the first column. We find the remaining entries of $M(3B)$ using properties of the Fischer-Clifford matrices discussed in Sections 1 and 3. We have

$$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & -1 \\ -30 & 24 & -3 & 0 \\ 45 & 18 & -9 & 0 \end{pmatrix}.$$

We use similar types of arguments to compute all other Fischer-Clifford matrices and conjugacy classes of $\bar{G}$. The complete list of Fischer-Clifford matrices and conjugacy classes of $\bar{G}$ is given in Tables 5 and 6 respectively. Please note that the fusion of elements of $\bar{G}$ into Fi'_{24} are given in the last column of Table 6.

TABLE 5. The Fischer-Clifford matrices of $3^7{\cdot}O_7(3)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 702 & 0 & -27 & 27 \\ 728 & 26 & -1 & -28 \\ 756 & -27 & 27 & 0 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 90 & -9 & 0 & 9 \\ 80 & 8 & -10 & -1 \\ 72 & 0 & 9 & -9 \end{pmatrix}$	$M(2C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 6 & 3 & -3 & 0 \\ 8 & -4 & -1 & 2 \\ 12 & 0 & 3 & -3 \end{pmatrix}$
$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 54 & 0 & 0 & 27 & -27 & -27 \\ 8 & 8 & -1 & 8 & 8 & 8 \\ 36\bar{\alpha} & 9\bar{\alpha} & 0 & -18\bar{\alpha} & 18-9\alpha & -9+18\alpha \\ 36\alpha & 9\alpha & 0 & -18\alpha & -9\bar{\alpha}+18 & 18\bar{\alpha}-9 \\ 108 & -27 & 0 & 0 & 27 & 27 \end{pmatrix}$	$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & -1 \\ -30 & 24 & -3 & 0 \\ 45 & 18 & -9 & 0 \end{pmatrix}$
$M(3C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 72 & -9 & 0 & 18 & -9 \\ 2 & 2 & -1 & 2 & 2 \\ 96 & 15 & 0 & -12 & -12 \\ 72 & -9 & 0 & -9 & 18 \end{pmatrix}$	$M(3D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & \alpha & \bar{\alpha} \\ 1 & 1 & \bar{\alpha} & \alpha \\ 24 & -3 & 0 & 0 \end{pmatrix}$
$M(3E) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & \alpha & \bar{\alpha} \\ 1 & 1 & \bar{\alpha} & \alpha \\ 24 & -3 & 0 & 0 \end{pmatrix}$	$M(3F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & -2 & -2 & 1 \\ 2 & -1 & 2 & -1 \\ 2 & 2 & -1 & -1 \end{pmatrix}$
$M(3G) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2\bar{\alpha} & -1 & -\alpha & -\bar{\alpha} & 2\alpha & 2 & 2\bar{\alpha} \\ 2\alpha & -1 & -\bar{\alpha} & -\alpha & 2\bar{\alpha} & 2 & 2\alpha \\ \alpha & 1 & \bar{\alpha} & \alpha & \bar{\alpha} & 1 & \alpha \\ \bar{\alpha} & 1 & \alpha & \bar{\alpha} & \alpha & 1 & \bar{\alpha} \\ 2 & -1 & -1 & -1 & 2 & 2 & 2 \\ 18 & 0 & 0 & 0 & 0 & 0 & -9 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 6 & 3 & -3 & 0 \\ 8 & -4 & -1 & 2 \\ 12 & 0 & 3 & -3 \end{pmatrix}$	$M(4C) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

TABLE 5. The Fischer-Clifford matrices of $3^7{\cdot}O_7(3)$ (continued)

$M(g)$	$M(g)$
$M(4D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 12 & 0 & 3 & -3 \\ 8 & -4 & -1 & 2 \\ 6 & 3 & -3 & 0 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 12 & 0 & 3 & -3 \\ 8 & -4 & -1 & 2 \\ 6 & 3 & -3 & 0 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6B) = \begin{pmatrix} 1 \end{pmatrix}$
$M(6C) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6D) = \begin{pmatrix} 1 & 1 & 1 \\ 18 & -9 & 0 \\ 8 & 8 & -1 \end{pmatrix}$
$M(6E) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 12 & -6 & 3 & 0 \\ 2 & 2 & 2 & -1 \\ 12 & 3 & -6 & 0 \end{pmatrix}$
$M(6G) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -3 & -3 & 3 & 0 \\ 4\bar{\alpha} & -1+2\alpha & 2-\alpha & -2\bar{\alpha} & \bar{\alpha} \\ 4\alpha & 2\bar{\alpha}-1 & -\bar{\alpha}+2 & -2\alpha & \alpha \\ 12 & 3 & 3 & 0 & -3 \end{pmatrix}$	$M(6H) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & -3 & -3 & 0 \\ 2 & 2 & 2 & 2 & -1 \\ 12 & -6 & -6 & 3 & 0 \\ 6 & -3 & 6 & -3 & 0 \end{pmatrix}$
$M(6I) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -3 & -3 & 3 & 0 \\ 4\bar{\alpha} & -1+2\alpha & 2-\alpha & -2\bar{\alpha} & \bar{\alpha} \\ 4\alpha & 2\bar{\alpha}-1 & -\bar{\alpha}+2 & -2\alpha & \alpha \\ 12 & 3 & 3 & 0 & -3 \end{pmatrix}$	$M(6J) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 6 & 3 & -3 & 0 \\ 8 & -4 & -1 & 2 \\ 12 & 0 & 3 & -3 \end{pmatrix}$
$M(6K) = \begin{pmatrix} 1 \end{pmatrix}$	$M(6L) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(6M) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(6N) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(6O) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(6P) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(7A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(8A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(8B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(9A) = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 6 & -3 & 0 \end{pmatrix}$
$M(9B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3\alpha & 3\bar{\alpha} & 3 & 0 & 3\bar{\alpha} & 3 & 3\alpha \\ 3\bar{\alpha} & 3\alpha & 3 & 0 & 3\alpha & 3 & 3\bar{\alpha} \\ 2 & 2 & 2 & -1 & 2 & 2 & 2 \\ 6\alpha & -3 & -3\bar{\alpha} & 0 & 6 & 6\bar{\alpha} & -3\alpha \\ 6\bar{\alpha} & -3 & -3\alpha & 0 & 6 & 6\alpha & -3\bar{\alpha} \\ 6 & -3 & -3 & 0 & 6 & 6 & -3 \end{pmatrix}$	$M(9C) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(9D) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(10A) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(10B) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(12A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$

TABLE 5. The Fischer-Clifford matrices of $3^7{\cdot}O_7(3)$ (continued)

$M(g)$	$M(g)$
$M(12B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 6 & -3 & -3 & 3 & 0 \\ 4\bar{\alpha} & -1+2\alpha & 2-\alpha & -2\bar{\alpha} & \bar{\alpha} \\ 4\alpha & 2\bar{\alpha}-1 & -\bar{\alpha}+2 & -2\alpha & \alpha \\ 12 & 3 & 3 & 0 & -3 \end{pmatrix}$	$M(12C) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(12D) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(12E) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$
$M(12F) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(12G) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(12H) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(13A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(13B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(14A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$
$M(15A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$	$M(18A) = \begin{pmatrix} 1 \end{pmatrix}$
$M(18B) = \begin{pmatrix} 1 \end{pmatrix}$	$M(18C) = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 6 & -3 & 0 \end{pmatrix}$
$M(18D) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$	$M(20A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}$

$\alpha = (-1+\sqrt{-3}\,)/2, \quad \bar{\alpha} = (-1-\sqrt{-3}\,)/2$

TABLE 6. The conjugacy classes of $3^7{\cdot}O_7(3)$

$[g]_{O_7(3)}$	$[x]_{3^7\cdot O_7(3)}$	$\lvert C_{3^7\cdot O_7(3)}(x)\rvert$	$\longrightarrow$ Fi'_{24}
$1A$	$1A$	10028164124160	$1A$
	$3A$	13264767360	$3A$
	$3B$	13774950720	$3B$
	$3C$	14285134080	$3C$
$2A$	$2A$	39191040	$2B$
	$6A$	19595520	$6B$
$2B$	$2B$	50388480	$2A$
	$6B$	559872	$6F$
	$6C$	699840	$6A$
	$6D$	629856	$6D$
$2C$	$2C$	373248	$2B$
	$6E$	62208	$6I$
	$6F$	46656	$6E$
	$6G$	31104	$6D$

TABLE 6. The conjugacy classes of $3^7{\cdot}O_7(3)$ (continued)

$[g]_{O_7(3)}$	$[x]_{3^7\cdot O_7(3)}$	$\lvert C_{3^7\cdot O_7(3)}(x)\rvert$	$\longrightarrow$ Fi'_{24}
$3A$	$3D$	153055008	$3D$
	$3E$	12754584	$3C$
	$3F$	708588	$3D$
	$3G$	25509168	$3B$
	$3H$	38263752	$3D$
	$3I$	38263752	$3A$
$3B$	$3J$	7085880	$3B$
	$3K$	2834352	$3C$
	$3L$	708588	$3D$
	$9A$	262440	$9B$
$3C$	$3M$	34012224	$3A$
	$3N$	1062882	$3D$
	$9B$	209952	$9C$
	$3O$	1417176	$3C$
	$3P$	1417176	$3D$
$3D$	$9C$	472392	$9A$
	$9D$	59049	$9D$
	$9E$	52488	$9C$
	$3Q$	52488	$3E$
$3E$	$9F$	472392	$9A$
	$9G$	59049	$9D$
	$9H$	52488	$9C$
	$3R$	52488	$3E$
$3F$	$3S$	26244	$3E$
	$9I$	13122	$9E$
	$9J$	13122	$9E$
	$9K$	6561	$9D$
$3G$	$9L$	39366	$9A$
	$9M$	6561	$9C$
	$9N$	6561	$9E$
	$9O$	6561	$9B$
	$3T$	13122	$3D$
	$9P$	13122	$9E$
	$9Q$	19683	$9D$
$4A$	$4A$	8640	$4B$
	$12A$	4320	$12D$
$4B$	$4B$	31104	$4A$
	$12B$	5184	$12E$
	$12C$	3888	$12B$
	$12D$	2592	$12A$
$4C$	$4C$	1152	$4C$
	$12E$	576	$12K$

TABLE 6. The conjugacy classes of $3^7{\cdot}O_7(3)$ (continued)

$[g]_{O_7(3)}$	$[x]_{3^7\cdot O_7(3)}$	$\lvert C_{3^7\cdot O_7(3)}(x)\rvert$	$\longrightarrow$ Fi'_{24}
$4D$	$4D$	5184	$4B$
	$12F$	648	$12D$
	$12G$	864	$12G$
	$12H$	432	$12L$
$5A$	$5A$	3240	$5A$
	$15A$	540	$15A$
	$15B$	405	$15B$
	$15C$	270	$15C$
$6A$	$6H$	69984	$6H$
	$6I$	34992	$6E$
$6B$	$6J$	3888	$6I$
$6C$	$6K$	11664	$6D$
	$6L$	5832	$6J$
$6D$	$6M$	69984	$6G$
	$6N$	34992	$6C$
	$6O$	2916	$6G$
$6E$	$6P$	5184	$6D$
	$18A$	2592	$18A$
$6F$	$6Q$	23328	$6A$
	$6R$	5832	$6F$
	$6S$	5832	$6G$
	$18B$	1296	$18B$
$6G$	$6T$	23328	$6J$
	$6U$	5832	$6D$
	$6V$	5832	$6J$
	$6W$	3888	$6E$
	$6X$	1944	$6I$
$6H$	$6Y$	11664	$6F$
	$6Z$	5832	$6C$
	$6AA$	5832	$6F$
	$6AB$	2916	$6G$
	$18C$	648	$18D$
$6I$	$6AC$	11664	$6B$
	$6AD$	2916	$6E$
	$6AE$	2916	$6H$
	$6AF$	1944	$6I$
	$6AG$	972	$6J$
$6J$	$6AH$	11664	$6D$
	$6AI$	1944	$6H$
	$6AJ$	1458	$6J$
	$6AK$	972	$6I$
$6K$	$6AL$	324	$6K$

TABLE 6. The conjugacy classes of $3^7{\cdot}O_7(3)$ (continued)

$[g]_{O_7(3)}$	$[x]_{3^7\cdot O_7(3)}$	$\lvert C_{3^7\cdot O_7(3)}(x)\rvert$	$\longrightarrow$ Fi'_{24}
$6L$	$18D$	648	$18C$
	$18E$	648	$18A$
	$6AM$	648	$6K$
$6M$	$18F$	648	$18C$
	$18G$	648	$18A$
	$6AN$	648	$6K$
$6N$	$6AO$	324	$6K$
	$18H$	162	$18E$
$6O$	$6AP$	324	$6K$
	$18I$	162	$18E$
$6P$	$18J$	162	$18E$
	$18K$	162	$18C$
	$6AQ$	162	$6J$
$7A$	$7A$	42	$7B$
	$21A$	42	$21C$
	$21B$	42	$21D$
$8A$	$8A$	48	$8A$
	$24A$	24	$24A$
$8B$	$8B$	48	$8C$
	$24B$	48	$24F$
	$24C$	48	$24G$
$9A$	$9R$	1458	$9B$
	$9S$	729	$9E$
	$9T$	972	$9F$
$9B$	$9U$	4374	$9E$
	$9V$	2187	$9C$
	$9W$	2187	$9D$
	$9X$	243	$9F$
	$9Y$	4374	$9C$
	$9Z$	4374	$9A$
	$9AA$	2187	$9E$
$9C$	$27A$	81	$27B$
	$27B$	81	$27C$
	$27C$	81	$27A$
$9D$	$27D$	81	$27C$
	$27E$	81	$27B$
	$27F$	81	$27A$
$10A$	$10A$	120	$10A$
	$30A$	60	$30A$
$10B$	$10B$	60	$10B$
	$30B$	30	$30B$

TABLE 6. The conjugacy classes of $3^7{\cdot}O_7(3)$ (continued)

$[g]_{O_7(3)}$	$[x]_{3^7\cdot O_7(3)}$	$\lvert C_{3^7\cdot O_7(3)}(x)\rvert$	$\longrightarrow$ Fi'_{24}
$12A$	$12I$	432	$12C$
	$36A$	432	$36A$
	$36B$	432	$36B$
$12B$	$12J$	3888	$12F$
	$12K$	972	$12A$
	$12L$	972	$12F$
	$12M$	648	$12B$
	$12N$	324	$12E$
$12C$	$12O$	216	$12D$
	$12P$	108	$12L$
$12D$	$12Q$	216	$12L$
	$12R$	108	$12G$
$12E$	$12S$	144	$12M$
	$12T$	72	$12H$
$12F$	$36C$	108	$36C$
	$36D$	108	$36B$
	$12U$	108	$12I$
$12G$	$36E$	108	$36C$
	$36F$	108	$36A$
	$12V$	108	$12J$
$12H$	$12W$	72	$12L$
	$36G$	36	$36D$
$13A$	$13A$	39	$13A$
	$39A$	39	$39A$
	$39B$	39	$39A$
$13B$	$13B$	39	$13A$
	$39C$	39	$39B$
	$39D$	39	$39B$
$14A$	$14A$	42	$14B$
	$42A$	42	$42B$
	$42B$	42	$42C$
$15A$	$15D$	45	$15B$
	$45A$	45	$45A$
	$45B$	45	$45B$
$18A$	$18L$	108	$18G$
$18B$	$18M$	108	$18H$
$18C$	$18N$	162	$18C$
	$18O$	162	$18E$
	$18P$	162	$18A$
$18D$	$18Q$	108	$18F$
	$18R$	54	$18D$
$20A$	$20A$	60	$20A$
	$60A$	60	$60A$
	$60B$	60	$60A$

The character table of $\bar{G} = 3^7{\cdot}O_7(3)$ can be obtained by using the Fischer-Clifford matrices (Table 5), the projective characters of the inertia factors $H_2 = 2U_4(3)$ with the factor set α^{-1} and the ordinary character tables of $H_3 = 3^5{:}U_4(2)$ and $H_4 = L_4(3)$ together with the fusions of H_2, H_3 and H_4 into $O_7(3)$ (Tables 3, 4 and 5). The full character table of $3^7{\cdot}O_7(3)$ which was also calculated independently by Alexander Hulpke, is available in GAP [29].

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