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ERRATUM TO "LEFT CELLS AND CONSTRUCTIBLE REPRESENTATIONS"

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Abstract. Thomas Pietraho pointed out a problem in the proof of one of the main results in "Left cells and constructible representations", Represent. Theory 9 (2005), 385–416. The purpose of this note is to correct this problem.

The author is grateful to Thomas Pietraho for pointing out a problem in the proof of [2, Theorem 3.7(b)]. That problem is caused by a reference to [2, Remark 2.4], which in turn is taken from [3, Prop. 18.4]. When writing [2], I overlooked the fact that Lusztig only proved the latter result assuming that W, L is split.

Now, Example 3 below actually shows that [2, Remark 2.4] does not hold in the general case of unequal parameters. We will be able, however, to replace the reference to [2, Remark 2.4] by an alternative argument so that all of the main results of [2] remain valid.

Let (W, S) be a finite Coxeter system and L be a weight function on W such that $L(s) > 0$ for all $s \in S$. We assume that Lusztig's conjectures **P1**–**P15** in [3, 14.2] hold for W, L. We shall need the following characterisation of two-sided cells.

Lemma 1. *Let $x, y \in W$. Then $x \sim_{\mathcal{LR}} y$ if and only if there exists a sequence of left cells $\Gamma_0, \Gamma_1, \ldots, \Gamma_n$ of W such that $x \in \Gamma_0$, $y \in \Gamma_n$ and $\Gamma_{i-1}^{-1} \cap \Gamma_i \neq \varnothing$ for all i. All the left cells $\Gamma_0, \Gamma_1, \ldots, \Gamma_n$ are contained in the same two-sided cell.*

Indeed, since **P4**, **P9**, **P10** are assumed to hold, we have $x \sim_{\mathcal{LR}} y$ if and only if there exists a sequence $x = x_0, x_1, \ldots, x_k = y$ of elements of W such that, for each i, we have $x_{i-1} \sim_{\mathcal{L}} x_i$ or $x_{i-1} \sim_{\mathcal{R}} x_i$. Furthermore, note that $x_{i-1} \sim_{\mathcal{R}} x_i$ if and only if $x_{i-1}^{-1} \sim_{\mathcal{L}} x_i^{-1}$. This readily implies Lemma 1.

Now let Γ, Γ' be left cells. The condition that $\Gamma^{-1} \cap \Gamma' \neq \varnothing$ can be rephrased as follows. By **P13**, every left cell contains a unique element from $\mathcal{D}$. Let $\mathcal{D} \cap \Gamma = \{d\}$ and $\mathcal{D} \cap \Gamma' = \{d'\}$. Then we claim that $\Gamma^{-1} \cap \Gamma' \neq \varnothing$ if and only if $t_d J t_{d'} \neq 0$. Indeed, if $w \in \Gamma^{-1} \cap \Gamma'$, then $t_d t_w = \pm t_w$ and $t_w t_{d'} = \pm t_w$, using **P2**, **P5**, **P7**, **P13**. Hence $t_d t_w t_{d'} = \pm t_w \neq 0$. Conversely, if $t_d J t_{d'} \neq 0$, then $t_d t_w t_{d'} \neq 0$ for some $w \in W$. In particular, $t_d t_w \neq 0$ and so $d \sim_{\mathcal{L}} w^{-1}$, using **P8**. Similarly, $t_w t_{d'} \neq 0$ and so $w \sim_{\mathcal{L}} d'^{-1} = d'$, using **P6**, **P8**. Hence, we have $w \in \Gamma^{-1} \cap \Gamma'$. Thus, our claim is proved. Combining this with Lemma 1, we obtain:

Corollary 2. *Let $d, d' \in \mathcal{D}$. Then $d \sim_{\mathcal{LR}} d'$ if and only if there exists a sequence $d_0, d_1, \ldots, d_n$ of elements in $\mathcal{D}$ such that $d = d_0$, $d' = d_n$ and $t_{d_{i-1}} J t_{d_i} \neq 0$ for all i.*

Example 3. Assume that the statements in [2, Remark 2.4] hold. In particular, this would mean that if $d, d' \in \mathcal{D}$ are such that $d \sim_{\mathcal{LR}} d'$, then $t_d t_w d_{d'} \neq 0$ for some

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$w \in W$. (Thus, we would only need chains with $n = 1$ in Corollary 2.) Now let Γ be the left cell containing d and Γ' be the left cell containing d'. As is shown in the proof of [3, Prop. 23.3], the fact that $t_d t_w t_{d'} \neq 0$ implies that $\mathrm{Hom}_W([\Gamma], [\Gamma']) \neq 0$. However, this conclusion is not valid in type F_4 with parameters 1 and 2 on simple reflections corresponding to long roots and short roots, respectively. See Table 2 on p. 362 in [1]: there is a two-sided cell containing left cells $\Gamma, \Gamma', \Gamma''$ such that $[\Gamma] \cong 1_3 \oplus 8_3$, $[\Gamma'] \cong 2_1 \oplus 9_1$ and $[\Gamma''] \cong 9_1 \oplus 8_3$.

We are now ready to address the problems caused by the reference to [2, Remark 2.4].

Remark 4. In the proof of [2, Theorem 3.7(b)], we need to consider elements $d, d' \in \mathcal{D}$ such that $d \sim_{\mathcal{LR}} d'$. We used [2, Remark 2.4] to conclude that then $t_d t_w d_{d'} \neq 0$ for some $w \in W$. As Example 3 shows, that conclusion is not true in general. However, one easily sees that the argument in the proof of [2, Theorem 3.7(b)] actually goes through when we use a chain of elements from $\mathcal{D}$ as in Corollary 2.

Remark 5. Another reference to [2, Remark 2.4] is implicitly used in the proof of the equivalence of (a) and (b) in [2, Theorem 4.3]. This can be corrected in the following (not very elegant) way. By standard arguments, the proof of the equivalence of (a) and (b) in [2, Theorem 4.3] can be reduced to the case where (W, S) is irreducible. Now assume that (W, S) is irreducible. If W is a Weyl group and L is a positive multiple of the length function, then the required equivalence is proved in [3, Prop. 23.3]. To deal with the remaining cases, note that the desired equivalence follows once [2, Conjecture 2.1], i.e., the coincidence of constructible representations and left cell representations, is known to hold. If W is of type $I_2(m)$, H_3, H_4 or F_4 (with unequal parameters), that coincidence can be checked using explicit computations; see [2, 7.1–7.4] and the references there. Finally, if W is of type B_n, then that coincidence is proved in [2, Example 4.10] and [2, Prop. 6.3].

References

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