

## WONDERFUL VARIETIES OF TYPE E

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ABSTRACT. The classification of spherical varieties is already known for semi-simple groups of types A and D. Adding type E, we complete the classification for all semisimple groups with a simply laced Dynkin diagram.

### INTRODUCTION

Let  $G$  be a connected semisimple algebraic group over  $\mathbb{C}$ . In [Lu1] D. Luna associates to a wonderful  $G$ -variety a combinatorial invariant called spherical system. He proves that if  $G$  is adjoint of type A, then wonderful  $G$ -varieties are classified by means of their spherical systems, and he conjectures that this holds for all types. Moreover, he shows that spherical  $G$ -homogeneous spaces are classified by certain combinatorial invariants if its conjecture holds for the adjoint group  $G/Z(G)$  of  $G$ . Using the Luna-Vust theory of spherical embeddings, this leads to the classification of spherical  $G$ -varieties.

In [BP] Luna's conjecture is proved for  $G$  adjoint with simple components of type A or D. Here we extend this result also allowing simple components of type E, hence completing the cases of root systems with simply laced Dynkin diagrams.

Let  $H$  be a spherical subgroup in  $G$  equal to its normaliser. The normality of the Demazure embedding of  $G/H$ , conjectured by M. Brion, is then obtained as a corollary.

In the first section we recall all the definitions and state the main result.

In the second section we explain how to reduce the proof to a finite number of cases, called primitive. This is the same approach as that in [Lu1] and [BP], it makes use of a reduction step via the projective fibration of D. Luna, which is up to now proved only in the simply laced case (*see* Remark 2.1). The list of the primitive spherical systems for  $G$  with at least one simple component of type E is given.

In the third section we prove that for every primitive spherical system there exists exactly one wonderful variety. This is done case by case, but the argument is essentially the same for all cases. It is similar to that used in [loc. cit.] for types A and D, but with some differences since we cannot easily handle groups of type E as groups of matrices.

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*Note.* A recent preprint of V. Losev ([Lo1]) proves that a spherical  $G$ -homogeneous space is uniquely determined, up to  $G$ -isomorphism, by the above mentioned combinatorial invariants, for any  $G$ . This implies the uniqueness part of Luna’s conjecture, while the existence part remains open. In a second preprint ([Lo2]) the same author completes the proof of Brion’s conjecture, in all types.

1. DEFINITIONS AND MAIN STATEMENT

1.1. **Wonderful varieties.** Let  $G$  be a complex semisimple linear algebraic group,  $B$  and  $B^-$  two mutually opposite Borel subgroups, and  $T = B \cap B^-$  a maximal torus.

An algebraic  $G$ -variety  $X$  is *wonderful* of rank  $r$  if:

- (i)  $X$  is smooth and complete,
- (ii)  $G$  has a dense orbit in  $X$  whose complement is the union of  $r$  smooth prime divisors  $D_i$ ,  $i = 1, \dots, r$ , with normal crossings,
- (iii) the intersection of the divisors  $D_i$  is nonempty and, for all  $x, x' \in X$ ,  $G.x = G.x'$  if and only if  $\{i : x \in D_i\} = \{i : x' \in D_i\}$ .

A wonderful  $G$ -variety is spherical, namely it is normal and it contains an open dense  $B$ -orbit.

The lattice of  $T$ -weights of  $B$ -semi-invariant rational functions on  $X$  is denoted by  $\Xi_X$ . Every  $\mathbb{Q}$ -valued discrete valuation  $\nu$  of  $\mathbb{C}(X)$  over  $\mathbb{C}$  defines a functional  $\rho_X(\nu)$  on  $\Xi_X$ :  $\langle \rho_X(\nu), \chi \rangle = \nu(f_\chi)$  where  $\chi \in \Xi_X$  and  $f_\chi \in \mathbb{C}(X)$  is  $B$ -semi-invariant with weight  $\chi$ . The map  $\nu \mapsto \rho_X(\nu)$  restricted to the set  $\mathcal{V}_X$  of  $G$ -invariant valuations of  $X$  is injective, the set  $\mathcal{V}_X$  as a subset of  $\text{Hom}_{\mathbb{Z}}(\Xi_X, \mathbb{Q})$  is a polyhedral convex cone. This cone is equal to  $\{v \in \text{Hom}_{\mathbb{Z}}(\Xi_X, \mathbb{Q}) : v(\chi) \leq 0 \ \forall \chi \in \Sigma_X\}$  for a uniquely determined set  $\Sigma_X$  of indecomposable elements of  $\Xi_X$ , and are called *spherical roots* of  $X$ .

The  $B$ -stable but non- $G$ -stable prime divisors of  $X$  are called *colours*; the set of colours is denoted by  $\Delta_X$ . The discrete valuation  $\nu_D$  associated to a colour  $D$  defines a functional  $\rho_X(D)$  on  $\Xi_X$ .

The set of spherical roots of all wonderful  $G$ -varieties of rank 1 is denoted by  $\Sigma(G)$ , we have  $\Sigma_X \subset \Sigma(G)$  for any wonderful  $G$ -variety  $X$ .

Let  $S$  be the set of simple roots associated to  $B$ .

For every  $\alpha \in S$ , let  $P_{\{\alpha\}}$  be the standard parabolic subgroup associated to  $\alpha$ . The set of non- $P_{\{\alpha\}}$ -stable colours of a wonderful variety  $X$  is denoted by  $\Delta_X(\alpha)$ . For all  $\alpha \in S$ , the set  $\Delta_X(\alpha)$  contains at most two elements. Moreover, its cardinality is exactly two if and only if  $\alpha$  is a spherical root.

The subset of simple roots  $\alpha$  such that  $\Delta_X(\alpha)$  is empty is denoted by  $S_X^p$ . The union of  $\Delta_X(\alpha)$  for  $\alpha \in \Sigma_X$  is denoted by  $\mathbf{A}_X$ .

1.2. **Spherical systems.** Let  $S^p$  be a subset of  $S$ ,  $\Sigma$  a subset of  $\Sigma(G)$  and  $\mathbf{A}$  a finite set with a map  $\rho : \mathbf{A} \rightarrow \Xi^*$ , where  $\Xi = \langle \Sigma \rangle \subset \Xi(T)$ . For every  $\alpha \in \Sigma \cap S$ , let  $\mathbf{A}(\alpha)$  denote the set  $\{\delta \in \mathbf{A} : \langle \rho(\delta), \alpha \rangle = 1\}$ . The triple  $(S^p, \Sigma, \mathbf{A})$  is called a *spherical system* for  $G$  if:

- (A1) for every  $\delta \in \mathbf{A}$  and  $\gamma \in \Sigma$  we have  $\langle \rho(\delta), \gamma \rangle \leq 1$ , and if  $\langle \rho(\delta), \gamma \rangle = 1$ , then  $\gamma \in S \cap \Sigma$ ;
- (A2) for every  $\alpha \in \Sigma \cap S$ ,  $\mathbf{A}(\alpha)$  contains two elements and by denoting with  $\delta_\alpha^+$  and  $\delta_\alpha^-$  these elements, it holds that  $\langle \rho(\delta_\alpha^+), \gamma \rangle + \langle \rho(\delta_\alpha^-), \gamma \rangle = \langle \alpha^\vee, \gamma \rangle$ , for every  $\gamma \in \Sigma$ ;

- (A3) the set  $\mathbf{A}$  is the union of  $\mathbf{A}(\alpha)$  for all  $\alpha \in \Sigma \cap S$ ;
- ( $\Sigma$ 1) if  $2\alpha \in \Sigma \cap 2S$ , then  $\frac{1}{2}\langle \alpha^\vee, \gamma \rangle$  is a nonpositive integer for every  $\gamma \in \Sigma \setminus \{2\alpha\}$ ;
- ( $\Sigma$ 2) if  $\alpha, \beta \in S$  are orthogonal and  $\alpha + \beta \in \Sigma$  (or  $\frac{1}{2}(\alpha + \beta) \in \Sigma$ ), then  $\langle \alpha^\vee, \gamma \rangle = \langle \beta^\vee, \gamma \rangle$  for every  $\gamma \in \Sigma$ ;
- (S) for every  $\gamma \in \Sigma$ , there exists a wonderful  $G$ -variety  $X$  of rank 1 with  $\gamma$  as the spherical root and  $S^p = S_X^p$ .

The cardinality of  $\Sigma$  is the *rank* of the spherical system.

The set  $\Delta$  of colours of a spherical system  $(S^p, \Sigma, \mathbf{A})$  is abstractly defined to be  $\Delta^a \sqcup \Delta^{a'} \sqcup \Delta^b$  where

- (i)  $\Delta^a = \mathbf{A}$ ,
- (ii)  $\Delta^{a'} = \{\delta'_\alpha : \forall \alpha \in S \cap \frac{1}{2}\Sigma\} \cong S \cap \frac{1}{2}\Sigma$ ,
- (iii)  $\Delta^b = \{\delta_\alpha : \forall \alpha \in S^b\}$ , where  $S^b = (S \setminus (\Sigma \cup \frac{1}{2}\Sigma \cup S^p))$  and  $\delta_\alpha = \delta_\beta$  if and only if  $\alpha$  is orthogonal to  $\beta$  and  $\alpha + \beta \in \Sigma$  (or  $\frac{1}{2}(\alpha + \beta) \in \Sigma$ ),

with the map  $\rho: \Delta \rightarrow \Xi^*$  extended as follows:

- (i)  $\langle \rho(\delta'_\alpha), \gamma \rangle = \frac{1}{2}\langle \alpha^\vee, \gamma \rangle \quad \forall \alpha \in S \cap \frac{1}{2}\Sigma, \forall \gamma \in \Xi$ ;
- (ii)  $\langle \rho(\delta_\alpha), \gamma \rangle = \langle \alpha^\vee, \gamma \rangle \quad \forall \alpha \in S^b, \forall \gamma \in \Xi$ .

**1.3. Classification.** By definition, for every wonderful  $G$ -variety  $X$  the triple  $(S_X^p, \Sigma_X, \mathbf{A}_X)$  is a spherical system, and the map  $X \mapsto (S_X^p, \Sigma_X, \mathbf{A}_X)$  is a bijection between rank one (resp. rank two) wonderful varieties (up to  $G$ -isomorphism) and rank one (resp. rank two) spherical systems (see [W]).

**Theorem 1.1.** *Let  $G$  be a semisimple adjoint algebraic group of mixed types A, D and E. Then the map associating to a wonderful  $G$ -variety  $X$  the triple  $(S_X^p, \Sigma_X, \mathbf{A}_X)$  is a bijection between wonderful  $G$ -varieties (up to  $G$ -isomorphism) and spherical systems for  $G$ .*

**1.4. Normality of the Demazure embedding.** Let  $H \subset G$  be a wonderful subgroup equal to its normaliser  $N(H)$ . Then  $G$  acts on the Grassmannian  $\text{Gr}_{\dim \mathfrak{h}}(\mathfrak{g})$ , where  $\mathfrak{h}$  and  $\mathfrak{g}$  denote the Lie algebras of  $H$  and  $G$ , and the stabiliser of  $\mathfrak{h}$  is equal to  $H$ .

The embedding of  $G/H$  given by its immersion in  $\text{Gr}_{\dim \mathfrak{h}}(\mathfrak{g})$  is called the Demazure embedding. Its normalisation is proved to be wonderful in [B2] and it is conjectured to be normal itself (hence wonderful).

The proof of the normality of the Demazure embedding in [Lu2] for type A can be extended here as in [BP, Section 5]. Indeed, by Theorem 1.1 and the list of primitive cases given in Table 3, one can reduce to certain wonderful varieties having rank at most two, with at least one simple spherical root, but no distinct colours with the same functional. There are no new such varieties occurring in type E (see Table 1 and Table 2). Therefore, for  $G$  of mixed types A, D and E we have the following.

**Corollary 1.2.** *The Demazure embedding of  $G/H$  is wonderful.*

## 2. REDUCTION TO THE PRIMITIVE CASES

**2.1. Luna diagrams.** We recall here the construction of the *Luna diagram* of a spherical system as in [BP]. It is a convenient way of representing a spherical system, and we make extensive use of it in the tables below.

Let  $G$  be an adjoint group of mixed types A, D and E. The Luna diagram of a spherical system for  $G$  is drawn on the Dynkin diagram of the root system of  $G$ .

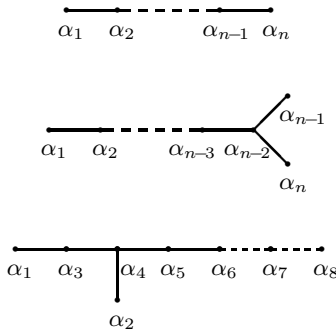
Let us recall for simplicity that, for every simple root  $\alpha$ ,  $\Delta(\alpha)$  denotes the subset of colours  $\mathbf{A}(\alpha) = \{\delta_\alpha^+, \delta_\alpha^-\}$  if  $\alpha \in \Sigma$ ,  $\{\delta'_\alpha\}$  if  $2\alpha \in \Sigma$ ,  $\{\delta_\alpha\}$  if  $\alpha \in S^b$ , or equal to the empty set if  $\alpha \in S^p$ .

The colours of the spherical system are drawn as circles in the Luna diagram; different circles corresponding to the same colour are joined by a line. In particular, for every simple root  $\alpha$ , we draw one or two circles near the corresponding vertex of the Dynkin diagram to represent the colours in  $\Delta(\alpha)$ . If  $\Delta(\alpha_i)$  has cardinality two, the circles are drawn immediately below and above the  $i$ -th vertex. If  $\Delta(\alpha_i)$  has cardinality one, the circle is drawn below or centered on the  $i$ -th vertex, when respectively  $\alpha_i$  is in  $\frac{1}{2}\Sigma$  or in  $S^b$ . In particular, simple roots in  $S^p$  correspond to vertices without circles.

Further notation is used to describe the set  $\Sigma$  of spherical roots (see Table 1). Two circles near the  $i$ -th vertex denote the spherical root  $\alpha_i$ . A zigzag line joining two vertices, say the  $i+1$ -th and the  $i+m$ -th, denotes the spherical root  $\alpha_{i+1} + \dots + \alpha_{i+m}$ . A unique circle below the  $i$ -th vertex denotes the spherical root  $2\alpha_i$ . Two circles joined by a line and centered on two nonadjacent vertices, say at positions  $i$  and  $j$ , denote the spherical root  $\alpha_i + \alpha_j$ . A shadowed circle on the  $i$ -th vertex denotes a spherical root of type  $d_m$  ( $m \geq 3$ ), whose support is the basis of a root subsystem of type  $D_m$ . The Dynkin subdiagram of this support is the connected component of  $\alpha_i$  in  $S^p \cup \{\alpha_i\}$ .

Finally, to describe the map  $\rho: \mathbf{A} \rightarrow \Xi^*$ , we use some arrows. By convention, the circle above a simple root  $\alpha$  (denoting  $\delta_\alpha^+$ ) always satisfies  $\langle \rho(\delta_\alpha^+), \gamma \rangle \geq -1$  for all  $\gamma \in \Sigma$ . Then we draw one arrow,  $>$  or  $<$ , near such a circle pointing towards the spherical root  $\gamma$  when  $\gamma$  is not orthogonal to  $\alpha$  and  $\langle \rho(\delta_\alpha^+), \gamma \rangle = -1$ . By axioms (A1) and (A2), this is enough to determine  $\rho$  without ambiguity. So, to avoid confusion in some special cases, we draw somemore arrows following the same rule for the circles below the Dynkin diagram.

**2.2. Labelling of simple roots.** For the labelling of simple roots  $\alpha_1, \dots, \alpha_n$  of an irreducible root system (of rank  $n$ ) we follow N. Bourbaki.




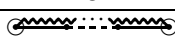




Simple roots belonging to different irreducible components (of rank  $n_1$  and  $n_2$ ) are labelled  $\alpha_1, \dots, \alpha_{n_1}$  and  $\alpha'_1, \dots, \alpha'_{n_2}$ .

**2.3. Low rank cases.** A wonderful variety  $X$  with open  $G$ -orbit  $X_G^\circ \cong G/H$  is said to be *prime* if: (i) for any parabolic subgroup  $P$  such that  $P^r \subseteq H \subseteq P$  we

have  $P = G$ ; (ii) if  $G = G_1 \times G_2$  and  $H = H_1 \times H_2$  with  $H_i \subseteq G_i$ , then  $G_1 = G$  or  $G_2 = G$ .

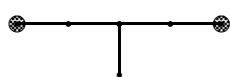
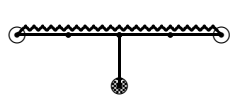
The list in Table 1 of rank 1 prime wonderful  $G$ -varieties for  $G$  an adjoint group of types A, D, and E, up to isomorphism, is equal to that of types A and D since no new rank 1 prime wonderful variety comes out in type E ([W]). For each variety we report the spherical root, the Luna diagram of the spherical system and the open orbit as homogeneous space  $G/H$ .

TABLE 1: Rank one prime wonderful  $G$ -varieties for  $G$  adjoint of types A, D, and E, up to isomorphism.

Spherical root	Diagram	$G / H$
$\alpha_1$		$SL(2) / GL(1)$
$\alpha_1 + \dots + \alpha_n$		$SL(n+1) / GL(n), n \geq 2$
$2\alpha_1$		$SL(2) / N_G(GL(1))$
$\alpha_1 + \alpha'_1$		$SL(2) \times SL(2) / N_G(SL(2))$
$\alpha_1 + 2\alpha_2 + \alpha_3$		$SL(4) / N_G(Sp(4))$
$2\alpha_1 + \dots + 2\alpha_{n-2} + \alpha_{n-1} + \alpha_n$		$SO(2n) / N_G(SO(2n-1)), n \geq 4$

In Table 2 we report the rank 2 prime wonderful varieties for adjoint groups of types A, D, and E, not of types A and D. There are only two cases, both for  $G$  simple of type  $E_6$ . The list for types A and D can be found in [BP, Table 2]. We use here the same labels which are found in [W] and report, respectively: the Luna diagram, the set of spherical roots, and the set of colours with the table of values taken by the associated functional, the open orbit.

TABLE 2: Rank two prime wonderful  $G$ -varieties for  $G$  adjoint of type E, up to isomorphism.

Diagram	$\Sigma, \delta_{\alpha_i}(\gamma_j)$	$G / H$									
	$\{2\alpha_1 + \alpha_2 + 2\alpha_3 + 2\alpha_4 + \alpha_5, \alpha_2 + \alpha_3 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6\},$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>\delta_{\alpha_1}</math></td> <td>2</td> <td>-1</td> </tr> <tr> <td><math>\delta_{\alpha_6}</math></td> <td>-1</td> <td>2</td> </tr> </table>	$\delta_{\alpha_1}$	2	-1	$\delta_{\alpha_6}$	-1	2	$E_6 / F_4$			
$\delta_{\alpha_1}$	2	-1									
$\delta_{\alpha_6}$	-1	2									
	$\{\alpha_1 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6, 2\alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5\},$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>\delta_{\alpha_1}</math></td> <td>1</td> <td>-1</td> </tr> <tr> <td><math>\delta_{\alpha_2}</math></td> <td>-1</td> <td>2</td> </tr> <tr> <td><math>\delta_{\alpha_6}</math></td> <td>1</td> <td>-1</td> </tr> </table>	$\delta_{\alpha_1}$	1	-1	$\delta_{\alpha_2}$	-1	2	$\delta_{\alpha_6}$	1	-1	$E_6 / GL(1) \cdot Spin(10)$
$\delta_{\alpha_1}$	1	-1									
$\delta_{\alpha_2}$	-1	2									
$\delta_{\alpha_6}$	1	-1									

**2.4. Morphisms.** We briefly recall here some technical definitions about spherical systems that have precise geometric counterparts in the class of wonderful varieties (see [Lu1] or [BP, 3.1]). These notions are used to reduce the proof of Theorem 1.1 to a finite number of cases (see 2.5).

A subset  $\Delta'$  of colours is called *distinguished* if there exists a linear combination with positive coefficients of the functionals  $\rho(\delta)$ , for  $\delta \in \Delta'$ , which takes nonnegative values on the spherical roots.

Let  $\Sigma(\Delta')$  be the maximal subset of spherical roots where such a linear combination can be positive. Let us denote with  $\{\gamma^* : \gamma \in \Sigma\}$  the basis of  $\Xi^*$  dual with respect to  $\Sigma$ . The intersection of the kernels of  $\rho(\delta)$ , for all  $\delta \in \Delta'$ , and  $\gamma^*$ , for all  $\gamma \in \Sigma(\Delta')$ , is denoted by  $\Xi/\Delta'$ . The set of indecomposable elements of the semigroup  $\mathbb{N}\Sigma \cap \Xi/\Delta'$  is denoted by  $\Sigma/\Delta'$ .

Let  $X$  be a wonderful  $G$ -variety. The dominant  $G$ -morphisms with connected fibers between wonderful varieties with source equal to  $X$  are in correspondence with the distinguished subsets  $\Delta'$  of  $\Delta_X$  such that  $\Sigma_X/\Delta'$  is a basis of  $\Xi_X/\Delta'$  ([Lu1, Proposition 3.3.2]).

A distinguished subset  $\Delta'$  of colours is called *smooth* if  $\Sigma/\Delta' = \Sigma \setminus \Sigma(\Delta')$ . It is called *parabolic* if  $\Sigma(\Delta') = \Sigma$ .

A distinguished subset is smooth if and only if the corresponding morphism is smooth. It is parabolic if and only if the target of the corresponding morphism is a flag variety ([Lu1, Proposition 3.3.3]).

**2.5. Reduction.** In [Lu1, Section 3]), certain geometric operations on wonderful varieties are well described in terms of the corresponding spherical systems. Such operations are

- *parabolic induction*: a wonderful  $G$ -variety  $X$  is obtained by parabolic induction if  $X \cong G \times_P Y$  for a parabolic subgroup  $P$  and a wonderful  $P$ -variety  $Y$  (the radical of  $P$  acting trivially on  $Y$ ),
- *fiber product*: a wonderful  $G$ -variety  $X$  is obtained as fiber product if there exist  $X_1, X_2, X_3$  wonderful  $G$ -varieties with  $\phi_1: X_1 \rightarrow X_3$  and  $\phi_2: X_2 \rightarrow X_3$   $G$ -morphisms with  $X \cong X_1 \times_{X_3} X_2$ ,
- *projective fibration*: a wonderful  $G$ -variety  $X$  is obtained as projective fibration if there exists a wonderful  $G$ -variety  $Y$  with  $\phi: X \rightarrow Y$  smooth  $G$ -morphism with fibers isomorphic to a projective space of fixed dimension.

Then it is proved that a wonderful variety  $X$  obtained as above from  $Y$  (or  $X_1, X_2, X_3$ ) is uniquely determined by its spherical system if and only if  $Y$  (or respectively  $X_1, X_2, X_3$ ) is (are) uniquely determined by its (their) spherical system(s).

A wonderful variety is called *primitive* if it cannot be obtained as nontrivial parabolic induction, fiber product or projective fibration. This property can be read off its spherical system.

A spherical system is called primitive if it is

- *cuspidal*: the union of the supports of the spherical roots covers the whole set of simple roots,
- *without projective elements*: an element of  $\mathbf{A}$  is called projective when its functional takes only nonnegative values on spherical roots,
- *indecomposable*: as fiber product, two disjoint distinguished subsets of colours  $\Delta_1$  and  $\Delta_2$  decompose a spherical system if:  $\Sigma/\Delta_1$ ,  $\Sigma/\Delta_2$  and  $\Sigma/(\Delta_1 \cup \Delta_2)$  respectively are bases of  $\Xi/\Delta_1$ ,  $\Xi/\Delta_2$  and  $\Xi/(\Delta_1 \cup \Delta_2)$ ,  $(\Sigma \setminus (\Sigma/\Delta_1)) \cap (\Sigma \setminus (\Sigma/\Delta_2)) = \emptyset$ , the simple roots  $\alpha \notin S^p$  with  $\Delta(\alpha) \subset \Delta_1$  are orthogonal to the simple roots  $\alpha \notin S^p$  with  $\Delta(\alpha) \subset \Delta_2$ , and  $\Delta_1$  or  $\Delta_2$  are smooth.

The spherical system of a primitive wonderful variety is primitive and to prove Theorem 1.1 is enough to show that for every primitive spherical system there exists a uniquely determined wonderful variety.

*Remark 2.1.* Notice that here we use the hypothesis on the type of the group  $G$  while supposing that there are no projective elements. Indeed, if the spherical system of a wonderful variety  $X$  admits a projective element  $\delta$ , then by [Lu1, Propositin 3.6])  $X$  is obtained by projective fibration from a wonderful variety with a set of colours equal to  $\Delta_X \setminus \{\delta\}$ . The proof of this result is based on a lemma ([loc. cit.] 3.6.2) that holds with groups with a simply laced Dynkin diagram. Such a lemma does not hold in general, but the above result about projective fibrations is conjectured by D. Luna and G. Pezzini to be valid in all types.

**2.6. The list of primitive cases.** To obtain all primitive cases we start constructing all cuspidal strongly  $\Delta$ -connected spherical systems and proceed by gluing some of them together. In the following we report the list of such spherical systems, but skip the technical details of the construction and refer to [BP, 3.2] for definitions.

The list of strongly  $\Delta$ -connected spherical systems of type **E** is the following. The complete list is obtained by adding the following list to that of types **A** and **D** in [loc. cit.].

- **ed(6)** for  $E_6$ : the rank 2 case labelled with  $E2$  in Table 2.
- **ef(n)** for  $E_n$ ,  $n = 6, 7$ : if  $n = 6$ , we have the rank 2 case labelled with  $E1$  in Table 2; if  $n = 7$ , we have  $S^p = \{\alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ ,  $\Sigma = \{\gamma_1 = 2\alpha_1 + 2\alpha_3 + 2\alpha_4 + \alpha_2 + \alpha_5, \gamma_2 = \alpha_2 + \alpha_3 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6, \gamma_3 = \alpha_7\}$ ,  $\mathbf{A} = \{\delta_{\alpha_7}^+, \delta_{\alpha_7}^-\}$  with  $\rho(\delta_{\alpha_7}^+) = \rho(\delta_{\alpha_7}^-)$  that is equal to 0, -1, 1 on  $\gamma_1, \gamma_2, \gamma_3$ , respectively.
- **ef'(n)** for  $E_n$ ,  $n = 7$  (or 8):  $S^p = \{\alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ ,  $\Sigma = \{2\alpha_1 + 2\alpha_3 + 2\alpha_4 + \alpha_2 + \alpha_5, \alpha_2 + \alpha_3 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6, 2\alpha_7, (2\alpha_8)\}$ ,  $\mathbf{A} = \emptyset$ .
- **ec(7)** for  $E_7$ :  $S^p = \{\alpha_2, \alpha_5, \alpha_7\}$ ,  $\Sigma = \{2\alpha_1, 2\alpha_3, \alpha_2 + 2\alpha_4 + \alpha_5, \alpha_5 + 2\alpha_6 + \alpha_7\}$ ,  $\mathbf{A} = \emptyset$ .
- **ea(6)** for  $E_6$ :  $S^p = \emptyset$ ,  $\Sigma = \{\alpha_1 + \alpha_6, \alpha_3 + \alpha_5, 2\alpha_2, 2\alpha_4\}$ ,  $\mathbf{A} = \emptyset$ .
- **eo(n)** for  $E_n$ ,  $n = 6, 7, 8$ :  $S^p = \emptyset$ ,  $\Sigma = \{2\alpha_1, \dots, 2\alpha_n\}$ ,  $\mathbf{A} = \emptyset$ .
- **ee(p, p)** for  $E_p \times E_p$ ,  $p = 6, 7, 8$ :  $S^p = \emptyset$ ,  $\Sigma = \{\alpha_1 + \alpha'_1, \dots, \alpha_p + \alpha'_p\}$ ,  $\mathbf{A} = \emptyset$ .
- **ec\*(n)** for  $E_n$ ,  $n = 6, 7, 8$ :  $S^p = \emptyset$ ,  $\Sigma = \{\alpha_1 + \alpha_3, \alpha_2 + \alpha_4, \alpha_3 + \alpha_4, \dots, \alpha_i + \alpha_{i+1}, \dots, \alpha_{n-1} + \alpha_n\}$ ,  $\mathbf{A} = \emptyset$ .
- **ey(8)** for  $E_8$ :  $S^p = \emptyset$ ,  $\Sigma = S$ ,  $\mathbf{A}$  as in Table 3.

The list of primitive spherical systems, of rank greater than two, of types **A**, **D**, and **E** not of types **A** and **D** is reported in Table 3. For every primitive case we report the label, the spherical system, and the generic stabiliser  $H$  of the corresponding wonderful variety (see Section 3).

The spherical system is often described via its Luna diagram. Sometimes it is just given as a localisation of an already defined case. Let us recall the definition of localisation: let  $S'$  be a subset of simple roots, the *localisation* of  $(S^p, \Sigma, \mathbf{A})$  in  $S'$  is the spherical system  $(S'^p, \Sigma', \mathbf{A}')$  where  $S'^p = S^p \cap S'$ ,  $\Sigma'$  is the set of spherical roots with support in  $S'$  and  $\mathbf{A}'$  is the union of  $\mathbf{A}(\alpha)$  for  $\alpha \in S'$  with  $\rho'$  equal to the composition of the restriction of  $\rho$  and the projection of  $\Xi^*$  onto  $\langle \Sigma' \rangle^*$ .

To describe the subgroup  $H$  we provide a Levi factor  $L$  up to isogeny and, if  $H$  is not reductive, the subset  $S'$  of simple roots associated to the standard parabolic subgroup  $Q$  (with Levi decomposition  $Q = L(Q)Q^u$  such that  $L^\circ \subseteq L(Q)$  and  $H^u \subseteq Q^u$ ) and the complementary  $\mathfrak{m}$  of the Lie algebra of  $H^u$  in the Lie algebra  $\mathfrak{n}_1$

of  $Q^u$  (as representation of the semisimple part  $(L^\circ)'$  of the connected component of  $L$ ). If necessary, we specify the submodule of  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1]$  in which  $\mathfrak{m}$  is embedded diagonally.

TABLE 3: Primitive wonderful  $G$ -varieties with rank at least three for  $G$  adjoint of types A, D, and E, not of type A and D, up to isomorphism.

Spherical system and generic stabiliser	
1. $\mathbf{ef}(7)$ .	
$GL(1) \cdot E_6$	
2. $\mathbf{ef}'(n)$ , $n = 7, 8$ .	
$N(GL(1) \cdot E_6)$ if $n = 7$ , $SL(2) \cdot E_7$ if $n = 8$	
3. $\mathbf{ec}(7)$ .	
$SL(2) \cdot Spin(12)$	
4. $\mathbf{ea}(6)$ .	
$SL(2) \cdot SL(6)$	
5. $\mathbf{eo}(n)$ , $n = 6, 7, 8$ .	
$Sp(8)$ if $n = 6$ , $SL(8)$ if $n = 7$ , $Spin(16)$ if $n = 8$	
6. $\mathbf{ee}(p, p)$ , $p = 6, 7, 8$ .	
$N(E_p)$	



TABLE 3: Primitive wonderful  $G$ -varieties (continuation).

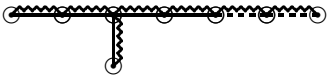
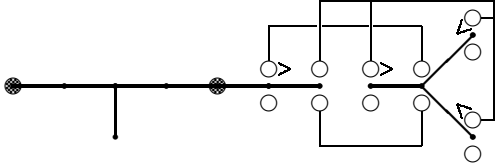
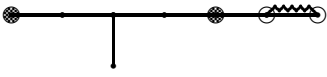
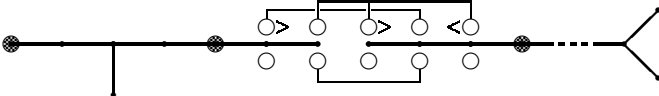
Spherical system and generic stabiliser																																																																																		
<p>7. <math>\mathbf{ec}^*(\mathbf{n})</math>, <math>n = 6, 7, 8</math>.</p>	 <p><math>n = 6</math>: <math>S' = S \setminus \{\alpha_2\}</math>, <math>L = Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1)</math>;  <math>n = 7</math>: <math>S' = S \setminus \{\alpha_2, \alpha_7\}</math>, <math>L = Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2}</math>;  <math>n = 8</math>: <math>S' = S \setminus \{\alpha_2\}</math>, <math>L = Sp(8) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1)</math></p>																																																																																	
<p>8. <math>\mathbf{ey}(\mathbf{8})</math>. <math>G</math> of type <math>E_8</math>. <math>\Sigma = S</math>,</p>	<table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_1}^+ = \delta_{\alpha_6}^+</math></td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_1}^-</math></td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_2}^+</math></td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_2}^- = \delta_{\alpha_8}^-</math></td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_3}^+ = \delta_{\alpha_5}^+ = \delta_{\alpha_7}^+</math></td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_3}^- = \delta_{\alpha_6}^-</math></td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_4}^+ = \delta_{\alpha_8}^+</math></td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_4}^- = \delta_{\alpha_7}^-</math></td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">-1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"><math>\delta_{\alpha_5}^-</math></td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">-1</td> <td style="padding: 2px 10px;">0</td> </tr> </table> <p><math>S' = S \setminus \{\alpha_1, \alpha_5\}</math>, <math>L = SL(4) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2}</math></p>	$\delta_{\alpha_1}^+ = \delta_{\alpha_6}^+$	1	0	-1	0	0	1	0	0	$\delta_{\alpha_1}^-$	1	0	0	0	0	-1	0	0	$\delta_{\alpha_2}^+$	0	1	0	0	0	0	0	-1	$\delta_{\alpha_2}^- = \delta_{\alpha_8}^-$	0	1	0	-1	0	0	0	1	$\delta_{\alpha_3}^+ = \delta_{\alpha_5}^+ = \delta_{\alpha_7}^+$	0	0	1	-1	1	-1	1	0	$\delta_{\alpha_3}^- = \delta_{\alpha_6}^-$	-1	0	1	0	-1	1	-1	0	$\delta_{\alpha_4}^+ = \delta_{\alpha_8}^+$	0	-1	0	1	0	0	-1	1	$\delta_{\alpha_4}^- = \delta_{\alpha_7}^-$	0	0	-1	1	-1	0	1	-1	$\delta_{\alpha_5}^-$	0	0	-1	0	1	0	-1	0
$\delta_{\alpha_1}^+ = \delta_{\alpha_6}^+$	1	0	-1	0	0	1	0	0																																																																										
$\delta_{\alpha_1}^-$	1	0	0	0	0	-1	0	0																																																																										
$\delta_{\alpha_2}^+$	0	1	0	0	0	0	0	-1																																																																										
$\delta_{\alpha_2}^- = \delta_{\alpha_8}^-$	0	1	0	-1	0	0	0	1																																																																										
$\delta_{\alpha_3}^+ = \delta_{\alpha_5}^+ = \delta_{\alpha_7}^+$	0	0	1	-1	1	-1	1	0																																																																										
$\delta_{\alpha_3}^- = \delta_{\alpha_6}^-$	-1	0	1	0	-1	1	-1	0																																																																										
$\delta_{\alpha_4}^+ = \delta_{\alpha_8}^+$	0	-1	0	1	0	0	-1	1																																																																										
$\delta_{\alpha_4}^- = \delta_{\alpha_7}^-$	0	0	-1	1	-1	0	1	-1																																																																										
$\delta_{\alpha_5}^-$	0	0	-1	0	1	0	-1	0																																																																										
<p>9. <math>\mathbf{dz}(\mathbf{4}, \mathbf{2}) + \mathbf{ef}(\mathbf{6})</math>.</p>	 <p><math>S' = S \setminus \{\alpha_7, \alpha'_1, \alpha'_3, \alpha'_4\}</math>, <math>L = F_4 \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 4}</math></p>																																																																																	
<p>10. <math>\mathbf{az}(\mathbf{3}, \mathbf{2}) + \mathbf{ef}(\mathbf{6})</math>. <math>G</math> of type <math>A_3 \times E_8</math>, localisation of <math>\mathbf{dz}(\mathbf{4}, \mathbf{2}) + \mathbf{ef}(\mathbf{6})</math> on <math>\{\alpha_1, \dots, \alpha_8; \alpha'_1, \alpha'_2, \alpha'_3\}</math>.</p>	<p><math>S' = \{\alpha_1, \dots, \alpha_6, \alpha_8, \alpha'_2\}</math>, <math>L = F_4 \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 3}</math></p>																																																																																	
<p>11. <math>\mathbf{ay}(\mathbf{2}, \mathbf{2}) + \mathbf{ef}(\mathbf{6})</math>. <math>G</math> of type <math>A_2 \times E_8</math>, localisation of <math>\mathbf{dz}(\mathbf{4}, \mathbf{2}) + \mathbf{ef}(\mathbf{6})</math> on <math>\{\alpha_1, \dots, \alpha_8; \alpha'_1, \alpha'_2\}</math>.</p>	<p><math>S' = \{\alpha_1, \dots, \alpha_6, \alpha_8, \alpha'_2\}</math>, <math>L = F_4 \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 2}</math></p>																																																																																	
<p>12. <math>\mathbf{ay}(\mathbf{2}, \mathbf{1}) + \mathbf{ef}(\mathbf{6})</math>. <math>G</math> of type <math>A_1 \times E_8</math>, localisation of <math>\mathbf{dz}(\mathbf{4}, \mathbf{2}) + \mathbf{ef}(\mathbf{6})</math> on <math>\{\alpha_1, \dots, \alpha_8; \alpha'_2\}</math>.</p>	<p><math>S' = \{\alpha_1, \dots, \alpha_6, \alpha_8, \alpha'_2\}</math>, <math>L = F_4 \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1)</math></p>																																																																																	
<p>13. <math>\mathbf{ef}(\mathbf{6}) + \mathbf{aa}(\mathbf{2})</math>.</p>	 <p><math>S' = S \setminus \{\alpha_7\}</math>, <math>L = F_4 \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1)</math></p>																																																																																	
<p>14. <math>\mathbf{d}(\mathbf{n}) + \mathbf{az}(\mathbf{3}, \mathbf{2}) + \mathbf{ef}(\mathbf{6})</math>, <math>n \geq 2</math>.</p>	 <p><math>S' = S \setminus \{\alpha_7, \alpha'_1, \alpha'_3\}</math>, <math>L = F_4 \cdot SL(2) \cdot SO(2n-1) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 3}</math></p>																																																																																	

TABLE 3: Primitive wonderful  $G$ -varieties (continuation).

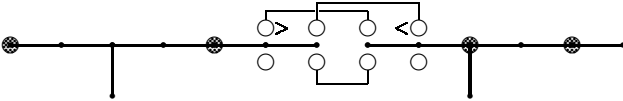
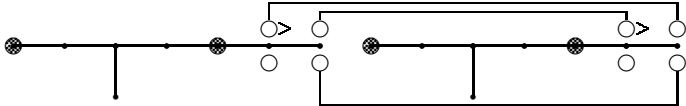
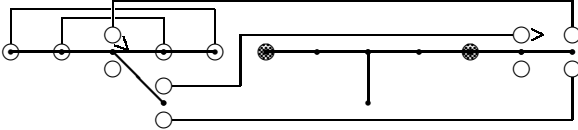
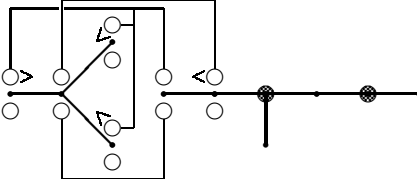
Spherical system and generic stabiliser
<p>15. <math>\mathbf{d}(n) + \mathbf{ay}(2, 2) + \mathbf{ef}(6)</math>, <math>n \geq 2</math>. <math>G</math> of type <math>D_{n+2} \times E_8</math>, localisation of <math>d(n) + az(3, 2) + ef(6)</math> on <math>\{\alpha_1, \dots, \alpha_8; \alpha'_2, \dots, \alpha'_{n+3}\}</math>.  <math>S' = \{\alpha_1, \dots, \alpha_6, \alpha_8, \alpha'_2, \alpha'_4, \dots, \alpha'_{n+3}\}</math>, <math>L = F_4 \cdot SL(2) \cdot SO(2n - 1) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 2}</math></p>
<p>16. <math>\mathbf{ac}(5) + \mathbf{ay}(2, 2) + \mathbf{ef}(6)</math>.</p>  <p><math>S' = S \setminus \{\alpha_7, \alpha'_3\}</math>, <math>L = F_4 \cdot SL(2) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 2}</math></p>
<p>17. <math>\mathbf{ef}(6) + \mathbf{ay}(2, 2) + \mathbf{ef}(6)</math>.</p>  <p><math>S' = S \setminus \{\alpha_7, \alpha'_7\}</math>, <math>L = F_4 \cdot SL(2) \cdot F_4 \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 2}</math></p>
<p>18. <math>\mathbf{aa}(2, 2) + \mathbf{ay}(2, 2) + \mathbf{ef}(6)</math>.</p>  <p><math>S' = S \setminus \{\alpha_4, \alpha'_7\}</math>, <math>L = SL(3) \cdot SL(2) \cdot F_4 \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 2}</math></p>
<p>19. <math>\mathbf{dz}(4, 2) + \mathbf{ac}(5)</math>.</p>  <p><math>S' = S \setminus \{\alpha_1, \alpha_3, \alpha_4, \alpha'_3\}</math>, <math>L = SL(2) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 4}</math></p>
<p>20. <math>\mathbf{az}(3, 2) + \mathbf{ac}(5)</math>. <math>G</math> of type <math>A_3 \times E_7</math>, localisation of <math>dz(4, 2) + ac(5)</math> on <math>\{\alpha_1, \alpha_2, \alpha_3; \alpha'_1, \dots, \alpha'_7\}</math>.  <math>S' = \{\alpha_2, \alpha'_1, \alpha'_2, \alpha'_4, \dots, \alpha'_7\}</math>, <math>L = SL(2) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 3}</math></p>
<p>21. <math>\mathbf{ay}(2, 2) + \mathbf{ac}(5)</math>. <math>G</math> of type <math>A_2 \times E_7</math>, localisation of <math>dz(4, 2) + ac(5)</math> on <math>\{\alpha_1, \alpha_2; \alpha'_1, \dots, \alpha'_7\}</math>.  <math>S' = \{\alpha_2, \alpha'_1, \alpha'_2, \alpha'_4, \dots, \alpha'_7\}</math>, <math>L = SL(2) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2}</math></p>
<p>22. <math>\mathbf{ay}(2, 1) + \mathbf{ac}(5)</math>. <math>G</math> of type <math>A_1 \times E_7</math>, localisation of <math>dz(4, 2) + ac(5)</math> on <math>\{\alpha_2; \alpha'_1, \dots, \alpha'_7\}</math>.  <math>S' = \{\alpha_2, \alpha'_1, \alpha'_2, \alpha'_4, \dots, \alpha'_7\}</math>, <math>L = SL(2) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1)</math></p>

TABLE 3: Primitive wonderful  $G$ -varieties (continuation).

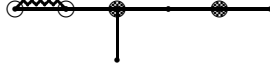
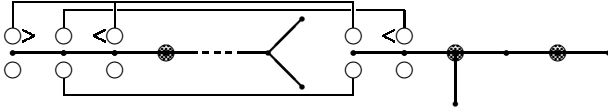
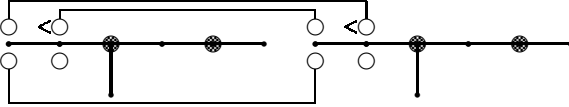
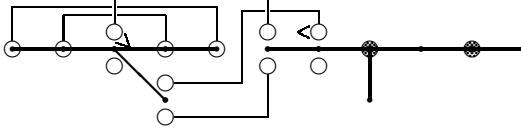
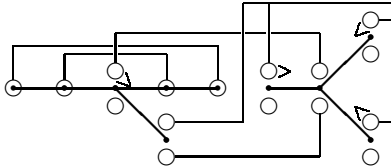
Spherical system and generic stabiliser
<p>23. <math>\mathbf{ac}(5) + \mathbf{aa}(2)</math>.</p>  <p><math>S' = S \setminus \{\alpha_3\}</math>, <math>L = SL(2) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1)</math></p>
<p>24. <math>\mathbf{d}(n) + \mathbf{az}(3, 2) + \mathbf{ac}(5)</math>, <math>n \geq 2</math>.</p>  <p><math>S' = S \setminus \{\alpha_1, \alpha_3, \alpha'_3\}</math>, <math>L = SL(2) \cdot SO(2n-1) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 3}</math></p>
<p>25. <math>\mathbf{d}(n) + \mathbf{ay}(2, 2) + \mathbf{ac}(5)</math>, <math>n \geq 2</math>. <math>G</math> of type <math>D_{n+2} \times E_7</math>, localisation of <math>d(n) + az(3, 2) + ac(5)</math> on <math>\{\alpha_2, \dots, \alpha_{n+3}; \alpha'_1, \dots, \alpha'_7\}</math>.  <math>S' = \{\alpha_2, \alpha_4, \dots, \alpha_{n+3}; \alpha'_1, \alpha'_2, \alpha'_4, \dots, \alpha'_7\}</math>, <math>L = SL(2) \cdot SO(2n-1) \cdot Sp(6) \cdot GL(1)</math>,  <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2}</math></p>
<p>26. <math>\mathbf{ac}(5) + \mathbf{ay}(2, 2) + \mathbf{ac}(5)</math>.</p>  <p><math>S' = S \setminus \{\alpha_3, \alpha'_3\}</math>, <math>L = SL(2) \cdot Sp(6) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2}</math></p>
<p>27. <math>\mathbf{aa}(2, 2) + \mathbf{ay}(2, 2) + \mathbf{ac}(5)</math>.</p>  <p><math>S' = S \setminus \{\alpha_4, \alpha'_3\}</math>, <math>L = SL(3) \cdot SL(2) \cdot Sp(6) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 2}</math></p>
<p>28. <math>\mathbf{dz}(4, 2) + \mathbf{aa}(2, 2)</math>.</p>  <p><math>S' = S \setminus \{\alpha_4, \alpha'_1, \alpha'_3, \alpha'_4\}</math>, <math>L = SL(3) \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 4}</math></p>
<p>29. <math>\mathbf{az}(3, 2) + \mathbf{aa}(2, 2)</math>. <math>G</math> of type <math>A_3 \times E_6</math>, localisation of <math>dz(4, 2) + aa(2, 2)</math> on <math>\{\alpha_1, \dots, \alpha_6; \alpha'_1, \alpha'_2, \alpha'_3\}</math>.  <math>S' = \{\alpha_1, \alpha_2, \alpha_3, \alpha_5, \alpha_6, \alpha'_2\}</math>, <math>L = SL(3) \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 3}</math></p>
<p>30. <math>\mathbf{ay}(2, 2) + \mathbf{aa}(2, 2)</math>. <math>G</math> of type <math>A_2 \times E_6</math>, localisation of <math>dz(4, 2) + aa(2, 2)</math> on <math>\{\alpha_1, \dots, \alpha_6; \alpha'_1, \alpha'_2\}</math>.  <math>S' = \{\alpha_1, \alpha_2, \alpha_3, \alpha_5, \alpha_6, \alpha'_2\}</math>, <math>L = SL(3) \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1) \subset V(\omega'_1)^{\oplus 2}</math></p>
<p>31. <math>\mathbf{ay}(2, 1) + \mathbf{aa}(2, 2)</math>. <math>G</math> of type <math>A_1 \times E_6</math>, localisation of <math>dz(4, 2) + aa(2, 2)</math> on <math>\{\alpha_1, \dots, \alpha_6; \alpha'_2\}</math>.  <math>S' = \{\alpha_1, \alpha_2, \alpha_3, \alpha_5, \alpha_6, \alpha'_2\}</math>, <math>L = SL(3) \cdot SL(2) \cdot GL(1)</math>, <math>\mathfrak{m} = V(\omega'_1)</math></p>

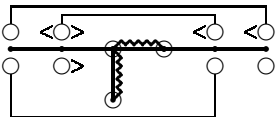
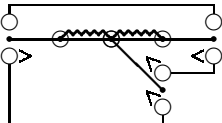
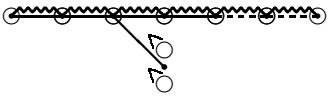
TABLE 3: Primitive wonderful  $G$ -varieties (continuation).

Spherical system and generic stabiliser																																																																																											
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<p>33. <b>d(n) + az(3, 2) + aa(2, 2), <math>n \geq 2</math>.</b></p>																																																																																											
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<p>34. <b>d(n) + ay(2, 2) + aa(2, 2), <math>n \geq 2</math>.</b> <math>G</math> of type <math>D_{n+2} \times E_6</math>, localisation of <math>d(n) + az(3, 2) + aa(2, 2)</math> on <math>\{\alpha_1, \dots, \alpha_6; \alpha'_2, \dots, \alpha'_{n+3}\}</math>.</p>																																																																																											
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<p>35. <b>aa(2, 2) + ay(2, 2) + aa(2, 2).</b></p>																																																																																											
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<p>36. <b>az(3, 3) + 2-comb.</b> <math>G</math> of type <math>E_8, \Sigma = S</math>,</p>	<table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_1}^+ = \delta_{\alpha_4}^+ = \delta_{\alpha_7}^+</math></td> <td>1</td><td>0</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_1}^-</math></td> <td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>-1</td><td>0</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_2}^+ = \delta_{\alpha_5}^+</math></td> <td>0</td><td>1</td><td>0</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>0</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_2}^-</math></td> <td>0</td><td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>0</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_3}^+ = \delta_{\alpha_6}^+ = \delta_{\alpha_8}^+</math></td> <td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>-1</td><td>1</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_3}^- = \delta_{\alpha_7}^-</math></td> <td>-1</td><td>0</td><td>1</td><td>-1</td><td>0</td><td>-1</td><td>1</td><td>-1</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_4}^-</math></td> <td>-1</td><td>-1</td><td>0</td><td>1</td><td>-1</td><td>0</td><td>-1</td><td>0</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_5}^-</math></td> <td>0</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>-1</td><td>0</td><td>0</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_6}^-</math></td> <td>0</td><td>0</td><td>-1</td><td>0</td><td>-1</td><td>1</td><td>0</td><td>-1</td> </tr> <tr> <td style="padding: 2px;"><math>\delta_{\alpha_8}^-</math></td> <td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td> </tr> </table>	$\delta_{\alpha_1}^+ = \delta_{\alpha_4}^+ = \delta_{\alpha_7}^+$	1	0	-1	1	0	0	1	0	$\delta_{\alpha_1}^-$	1	0	0	-1	0	0	-1	0	$\delta_{\alpha_2}^+ = \delta_{\alpha_5}^+$	0	1	0	-1	1	0	0	0	$\delta_{\alpha_2}^-$	0	1	0	0	-1	0	0	0	$\delta_{\alpha_3}^+ = \delta_{\alpha_6}^+ = \delta_{\alpha_8}^+$	0	0	1	0	0	1	-1	1	$\delta_{\alpha_3}^- = \delta_{\alpha_7}^-$	-1	0	1	-1	0	-1	1	-1	$\delta_{\alpha_4}^-$	-1	-1	0	1	-1	0	-1	0	$\delta_{\alpha_5}^-$	0	-1	0	0	1	-1	0	0	$\delta_{\alpha_6}^-$	0	0	-1	0	-1	1	0	-1	$\delta_{\alpha_8}^-$	0	0	-1	0	0	-1	0	1
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TABLE 3: Primitive wonderful  $G$ -varieties (continuation).

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TABLE 3: Primitive wonderful  $G$ -varieties (continuation).

Spherical system and generic stabiliser
<p>44. <math>\mathbf{ay}(2, 2) + \mathbf{ac}^*(3)</math>.</p>  <p><math>S' = S \setminus \{\alpha_2, \alpha_7\}, L = SL(4) \cdot SL(2) \cdot GL(1)^2 \subset SL(6) \cdot GL(1)^2, \mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2}</math></p>
<p>45. <math>\mathbf{ay}(2, 1) + \mathbf{ac}^*(3)</math>. <math>G</math> of type <math>E_6</math>, localisation of <math>\mathbf{ay}(2, 2) + \mathbf{ac}^*(3)</math> on <math>\{\alpha_1, \dots, \alpha_6\}</math>.  <math>S' = \{\alpha_1, \alpha_3, \dots, \alpha_6\}, L = SL(4) \cdot SL(2) \cdot GL(1)^2 \subset SL(6) \cdot GL(1)^2, \mathfrak{m} = V(\omega_1)</math></p>
<p>46. <math>\mathbf{ax}(1, 1, 1) + \mathbf{ac}^*(3)</math>.</p>  <p><math>S' = S \setminus \{\alpha_3, \alpha_5\}, L = SL(2) \cdot SL(3) \cdot GL(1), \mathfrak{m} = V(\omega_1 + \omega'_1) \subset V(\omega_1 + \omega'_1)^{\oplus 2}</math></p>
<p>47. <math>\mathbf{ac}^*(n) + \mathbf{aa}(1), n = 5, 6, 7</math>.</p>  <p><math>n = 5: S' = S \setminus \{\alpha_1, \alpha_6\}, L = SL(4) \cdot GL(1)^2 \subset SO(8) \cdot GL(1)^2, \mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2};</math>  <math>n = 6: S' = S \setminus \{\alpha_1\}, L = SL(6) \cdot GL(1)^2 \subset SO(12) \cdot GL(1)^2, \mathfrak{m} = V(\omega_1);</math>  <math>n = 7: S' = S \setminus \{\alpha_1, \alpha_8\}, L = SL(6) \cdot GL(1)^2 \subset SO(12) \cdot GL(1)^2, \mathfrak{m} = V(\omega_1) \subset V(\omega_1)^{\oplus 2}</math></p>

*Remark 2.2.* In some cases of Table 3 the generic stabiliser  $H$  can actually be described more explicitly. For example, looking at dominant  $G$ -morphisms with connected fibers between wonderful  $G$ -varieties, D. Luna observed that:

- in case  $\mathbf{ec}^*(6)$ ,  $H$  is the parabolic subgroup of semisimple type  $C_3$  of the symmetric subgroup  $F_4$  (the case  $E1$  in Table 2) of  $G$  (see [Lu3]),
- in case  $\mathbf{ds}^*(4) + \mathbf{aa}(1) + \mathbf{aa}(1)$ ,  $H$  is the parabolic subgroup of semisimple type  $D_4$  of the symmetric subgroup  $GL(1) \cdot Spin(10)$  (the case  $E2$  in Table 2).

### 3. THE PROOF OF THE PRIMITIVE CASES

We now proceed as in [Lu1] and [BP, Section 4] by checking that every primitive spherical system, for  $G$  of types A, D, and E not of types A and D, corresponds to a wonderful variety, unique up to  $G$ -isomorphism, by determining its generic stabiliser  $H$ .

The proof of the cases where the subgroup  $H$  is reductive (the first six cases in Table 3) follows from the classification of reductive spherical subgroups ([Kr], [B1], [BP, 4.1]).

**3.1. Uniqueness.** For each non-reductive spherical system we find a distinguished subset  $\Delta'$  of  $\Delta$  such that  $(S^p, \Sigma, \mathbf{A})/\Delta'$  is a parabolic induction of a spherical system corresponding to a reductive case and there exists a minimal parabolic distinguished subset  $\Delta''$  such that it contains  $\Delta'$ .

As in [Lu1] and [BP, 4.2], these subsets of colours provide us with a wonderful subgroup  $H_1$  (i.e. such that  $G/H_1$  admits a wonderful embedding) and two parabolic subgroups  $P \subseteq Q$ :  $H \subset H_1 \subset P$ . Being  $H = L H^u$ ,  $H_1 = L_1 H_1^u$ ,  $P = L(P) P^u$ ,  $Q = L(Q) Q^u$  the respective Levi decompositions, we prove that  $(L^\circ)'$  is spherical in  $L(P)$  (not contained in any proper parabolic subgroup of  $L(P)$ ) and  $H^u \subset H_1^u = Q^u \subseteq P^u$ . Moreover, the codimension of the connected center  $C$  of  $H$  in the connected center  $C_1$  of  $H_1$  and the codimension of the  $(L^\circ)'$ -submodule  $\mathfrak{n}$  (Lie algebra of  $H^u$ ) in  $\mathfrak{n}_1$  (Lie algebra of  $H_1^u$ ) are known.

The Lie subalgebra  $\mathfrak{n} \subset \mathfrak{n}_1$  contains for dimensional reasons  $[\mathfrak{n}_1, \mathfrak{n}_1]$ ; then we decompose into simple  $(L^\circ)'$ -modules the  $L(Q)$ -module

$$\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = \bigoplus_{\alpha \in S \setminus S'} V(\lambda_{-\alpha}),$$

where  $S'$  is the subset of simple roots associated to  $Q$  and  $\lambda_{-\alpha}$  is the dominant weight of  $L(Q)$  such that  $(\beta^\vee, \lambda_{-\alpha}) = -(\beta^\vee, \alpha)$ , for any  $\beta \in S'$ .

Case by case this leads to a subgroup  $H$ , uniquely determined up to conjugation.

**3.2. Existence.** After having proved that the subgroup  $H$  is spherical we check that it is equal to its normaliser.

In general the normaliser  $N(H)$  of  $H$  is included in the normaliser  $N(H^u)$  of its unipotent radical. The latter is easier to compute: looking at the normaliser of the Lie algebra  $\mathfrak{n}$  of  $H^u$ , we have that  $Q^u$  is a normal subgroup in  $N(H^u)$  (and the latter is contained in  $Q$ ), therefore the reductive group  $N(H^u)/Q^u$  can be computed in  $L(Q)$ . Case by case it is equal to  $N(H)/H^u$ .

Finally, it is possible to compute the spherical system of the wonderful variety  $G/\overline{H}$  with ad hoc arguments, as in [Lu1] and [BP, 4.2].

**3.3. Non-reductive primitive cases.** For each case we report here the distinguished subsets of colours  $\Delta'$  and  $\Delta''$ , the semisimple part of the connected component of  $L$  and the corresponding decomposition into simple modules of  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1]$ . We denote with  $c_1$ ,  $c$  and  $d$  respectively the dimensions of  $C_1$ ,  $C$  and  $\mathfrak{m} = \mathfrak{n}_1/\mathfrak{n}$ . These data lead to the description of the subgroup  $H$  reported in Table 3.

7. **ec\*(n).**  $n = 6$ :  $\Delta' = \{\delta_{\alpha_1}, \delta_{\alpha_4}, \delta_{\alpha_6}\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_2}\}$ ,  $(L^\circ)' = Sp(6)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_1) \oplus V(\omega_3)$ ,  $c_1 = c = 1$ ,  $d = 6$ .  $n = 7$ :  $\Delta' = \{\delta_{\alpha_1}, \delta_{\alpha_4}, \delta_{\alpha_6}\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_2}, \delta_{\alpha_7}\}$ ,  $(L^\circ)' = Sp(6)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_1)^{\oplus 2} \oplus V(\omega_3)$ ,  $c_1 = 2$ ,  $c = 1$ ,  $d = 6$ .  $n = 8$ :  $\Delta' = \{\delta_{\alpha_1}, \delta_{\alpha_4}, \delta_{\alpha_6}, \delta_{\alpha_8}\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_2}\}$ ,  $(L^\circ)' = Sp(8)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_1) \oplus V(\omega_3)$ ,  $c_1 = c = 1$ ,  $d = 8$ .

8. **ey(8).**  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+, \delta_{\alpha_4}^+\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_1}^-, \delta_{\alpha_5}^-\}$ ,  $(L^\circ)' = SL(4)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_1)^{\oplus 2} \oplus V(\omega_2 + \omega_3)$ ,  $c_1 = 2$ ,  $c = 1$ ,  $d = 4$ .

9. **dz(4, 2) + ef(6).**  $\Delta' = \{\delta_{\alpha_7}^+, \delta_{\alpha_8}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}, \delta_{\alpha_6}, \delta_{\alpha_7}^+, \delta_{\alpha_8}^+, \delta_{\alpha_8}^-\}$ ,  $(L^\circ)' = F_4 \cdot SL(2)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_4 + \omega'_1) \oplus V(\omega'_1)^{\oplus 4}$ ,  $c_1 = 4$ ,  $c = 1$ ,  $d = 2$ .

10. **az(3, 2) + ef(6).**  $\Delta' = \{\delta_{\alpha_7}^+, \delta_{\alpha_8}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}, \delta_{\alpha_6}, \delta_{\alpha_7}^+, \delta_{\alpha_8}^+, \delta_{\alpha_8}^-\}$ ,  $(L^\circ)' = F_4 \cdot SL(2)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_4 + \omega'_1) \oplus V(\omega'_1)^{\oplus 3}$ ,  $c_1 = 3$ ,  $c = 1$ ,  $d = 2$ .

11. **ay(2, 2) + ef(6).**  $\Delta' = \{\delta_{\alpha_7}^+, \delta_{\alpha_8}^+\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_7}^-, \delta_{\alpha_1}^-\}$ ,  $(L^\circ)' = F_4 \cdot SL(2)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_4 + \omega'_1) \oplus V(\omega'_1)^{\oplus 2}$ ,  $c_1 = 2$ ,  $c = 1$ ,  $d = 2$ .

12. **ay(2, 1) + ef(6).**  $\Delta' = \{\delta_{\alpha_7}^+, \delta_{\alpha_8}^+\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_7}^-\}$ ,  $(L^\circ)' = F_4 \cdot SL(2)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_4 + \omega'_1) \oplus V(\omega'_1)$ ,  $c_1 = c = 1$ ,  $d = 2$ .

13. **ef(6) + aa(2).**  $\Delta' = \{\delta_{\alpha_8}\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_7}\}$ ,  $(L^\circ)' = F_4 \cdot SL(2)$ ,  $\mathfrak{n}_1/[\mathfrak{n}_1, \mathfrak{n}_1] = V(\omega_4 + \omega'_1) \oplus V(\omega'_1)$ ,  $c_1 = c = 1$ ,  $d = 2$ .





33.  $\mathbf{d}(\mathbf{n}) + \mathbf{az}(\mathbf{3}, \mathbf{2}) + \mathbf{aa}(\mathbf{2}, \mathbf{2})$ ,  $n \geq 2$ .  $\Delta' = \{\delta_{\alpha_2}^+, \delta_{\alpha_4}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}, \delta_{\alpha_3}, \delta_{\alpha_2}^+, \delta_{\alpha_2}^-, \delta_{\alpha_4}^+, \delta_{\alpha_4}^-\}$ ,  $(L^\circ)' = SL(3) \cdot SL(2) \cdot SO(2n-1)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_1 + \omega_2 + \omega'_1) \oplus V(\omega'_1)^{\oplus 3} \oplus V(\omega'_1 + \omega''_1)$ ,  $c_1 = 3$ ,  $c = 1$ ,  $d = 2$ .

34.  $\mathbf{d}(\mathbf{n}) + \mathbf{ay}(\mathbf{2}, \mathbf{2}) + \mathbf{aa}(\mathbf{2}, \mathbf{2})$ ,  $n \geq 2$ .  $\Delta' = \{\delta_{\alpha_2}^+, \delta_{\alpha_4}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}, \delta_{\alpha_3}, \delta_{\alpha_2}^+, \delta_{\alpha_2}^-, \delta_{\alpha_4}^+, \delta_{\alpha_4}^-\}$ ,  $(L^\circ)' = SL(3) \cdot SL(2) \cdot SO(2n-1)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_1 + \omega_2 + \omega'_1) \oplus V(\omega'_1)^{\oplus 2} \oplus V(\omega'_1 + \omega''_1)$ ,  $c_1 = 2$ ,  $c = 1$ ,  $d = 2$ .

35.  $\mathbf{aa}(\mathbf{2}, \mathbf{2}) + \mathbf{ay}(\mathbf{2}, \mathbf{2}) + \mathbf{aa}(\mathbf{2}, \mathbf{2})$ .  $\Delta' = \{\delta_{\alpha_2}^+, \delta_{\alpha_4}^+\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_4}^-, \delta_{\alpha_4}^-\}$ ,  $(L^\circ)' = SL(3) \cdot SL(2) \cdot SL(3)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_1 + \omega_2 + \omega'_1) \oplus V(\omega'_1)^{\oplus 2} \oplus V(\omega'_1 + \omega''_1 + \omega'''_1)$ ,  $c_1 = 2$ ,  $c = 1$ ,  $d = 2$ .

36.  $\mathbf{az}(\mathbf{3}, \mathbf{3}) + \mathbf{2-comb}$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+, \delta_{\alpha_3}^-\}$ ,  $(L^\circ)' = SL(2)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(0)^{\oplus 2} \oplus V(\omega_1)^{\oplus 4}$ ,  $c_1 = 6$ ,  $c = 2$ ,  $d = 3$ .

37.  $\mathbf{az}(\mathbf{3}, \mathbf{2}) + \mathbf{2-comb}$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+, \delta_{\alpha_3}^-\}$ ,  $(L^\circ)' = SL(2)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(0)^{\oplus 2} \oplus V(\omega_1)^{\oplus 3}$ ,  $c_1 = 5$ ,  $c = 2$ ,  $d = 3$ .

38.  $\mathbf{ay}(\mathbf{3}, \mathbf{3}) + \mathbf{1-comb}$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+, \delta_{\alpha_4}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_1}^-, \delta_{\alpha_2}^+, \delta_{\alpha_2}^-, \delta_{\alpha_3}^+, \delta_{\alpha_3}^-, \delta_{\alpha_4}^+, \delta_{\alpha_4}^-\}$ ,  $(L^\circ)' = SL(3)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(0)^{\oplus 2} \oplus V(\omega_1)^{\oplus 2}$ ,  $c_1 = 4$ ,  $c = 2$ ,  $d = 4$ .

39.  $\mathbf{ay}(\mathbf{3}, \mathbf{3}) + \mathbf{2-comb}$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+, \delta_{\alpha_4}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_1}^-, \delta_{\alpha_2}^+, \delta_{\alpha_2}^-, \delta_{\alpha_3}^+, \delta_{\alpha_3}^-, \delta_{\alpha_4}^+, \delta_{\alpha_4}^-\}$ ,  $(L^\circ)' = SL(4) \cdot SL(3)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(0) \oplus V(\omega_2 + \omega'_2) \oplus V(\omega_3 + \omega'_3) \oplus V(\omega_1)^{\oplus 2} \oplus V(\omega'_1)$ ,  $c_1 = 3$ ,  $c = 2$ ,  $d = 4$ .

40.  $\mathbf{ay}(\mathbf{3}, \mathbf{2}) + \mathbf{2-comb}$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_2}^+, \delta_{\alpha_3}^+, \delta_{\alpha_4}^+\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_1}^-, \delta_{\alpha_2}^+, \delta_{\alpha_2}^-, \delta_{\alpha_3}^+, \delta_{\alpha_3}^-, \delta_{\alpha_4}^+, \delta_{\alpha_4}^-\}$ ,  $(L^\circ)' = SL(4) \cdot SL(3)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(0) \oplus V(\omega_2 + \omega'_2) \oplus V(\omega_3 + \omega'_3) \oplus V(\omega_1)$ ,  $c_1 = c = 2$ ,  $d = 4$ .

41.  $\mathbf{ds}(\mathbf{n}) + \mathbf{2-comb}$ ,  $n = 4, 5, 6$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_{n-1}}\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_3}, \delta_{\alpha_{n-1}}\}$ ,  $(L^\circ)' = SL(n-2)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(0) \oplus V(\omega_1)^{\oplus 2} \oplus V(\omega_2)$ ,  $c_1 = 3$ ,  $c = 2$ ,  $d = n-2$ .

42.  $\mathbf{ds}^*(\mathbf{4}) + \mathbf{aa}(\mathbf{1}) + \mathbf{aa}(\mathbf{1})$ .  $\Delta' = \{\delta_{\alpha_2}, \delta_{\alpha_5}, \delta_{\alpha_6}^-\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_1}^+, \delta_{\alpha_6}^+\}$ ,  $(L^\circ)' = Spin(8)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_3) \oplus V(\omega_4)$ ,  $c_1 = c = 2$ ,  $d = 8$ . There are two possible choices for  $H$ , they are conjugated since they correspond to two conjugated copies of  $Spin(8)$  in  $Spin(10)$ .

43.  $\mathbf{ds}^*(\mathbf{4}) + \mathbf{2-comb}$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_3}, \delta_{\alpha_5}\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_1}^-, \delta_{\alpha_6}^-\}$ ,  $(L^\circ)' = Spin(7)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_3)^{\oplus 2}$ ,  $c_1 = 2$ ,  $c = 1$ ,  $d = 8$ .

44.  $\mathbf{ay}(\mathbf{2}, \mathbf{2}) + \mathbf{ac}^*(\mathbf{3})$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_3}^+, \delta_{\alpha_4}\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_1}^-, \delta_{\alpha_3}^+, \delta_{\alpha_3}^-, \delta_{\alpha_4}\}$ ,  $(L^\circ)' = SL(4) \cdot SL(2)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_3) \oplus V(\omega_2 + \omega'_1) \oplus V(\omega_1)^{\oplus 2} \oplus V(\omega'_1)$ ,  $c_1 = 3$ ,  $c = 2$ ,  $d = 4$ .

45.  $\mathbf{ay}(\mathbf{2}, \mathbf{1}) + \mathbf{ac}^*(\mathbf{3})$ .  $\Delta' = \{\delta_{\alpha_1}^+, \delta_{\alpha_3}^+, \delta_{\alpha_4}\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_1}^-, \delta_{\alpha_3}^+, \delta_{\alpha_3}^-, \delta_{\alpha_4}\}$ ,  $(L^\circ)' = SL(4) \cdot SL(2)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_3) \oplus V(\omega_2 + \omega'_1) \oplus V(\omega_1)$ ,  $c_1 = c = 2$ ,  $d = 4$ . There are two possible choices for  $H$ , they are conjugated via an inner automorphism of  $SL(4)$ .

46.  $\mathbf{ax}(\mathbf{1}, \mathbf{1}, \mathbf{1}) + \mathbf{ac}^*(\mathbf{3})$ .  $\Delta' = \{\delta_{\alpha_1}^-, \delta_{\alpha_2}^+, \delta_{\alpha_4}\}$ ,  $\Delta'' = \{\delta_{\alpha_1}^+, \delta_{\alpha_1}^-, \delta_{\alpha_2}^+, \delta_{\alpha_4}\}$ ,  $(L^\circ)' = SL(2) \cdot SL(3)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_1 + \omega'_1)^{\oplus 2}$ ,  $c_1 = 2$ ,  $c = 1$ ,  $d = 6$ .

47.  $\mathbf{ac}^*(\mathbf{n}) + \mathbf{aa}(\mathbf{1})$ ,  $n = 5, 6, 7$ .  $n = 5$ :  $\Delta' = \{\delta_{\alpha_3}, \delta_{\alpha_5}\}$ ,  $\Delta'' = \{\delta_{\alpha_2}^+, \delta_{\alpha_3}, \delta_{\alpha_4}, \delta_{\alpha_5}\}$ ,  $(L^\circ)' = SL(4)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_1)^{\oplus 2} \oplus V(\omega_3)^{\oplus 2}$ ,  $c_1 = 3$ ,  $c = 2$ ,  $d = 4$ . There are two conjugated choices for  $H$ , via an inner automorphism of  $SL(4)$ .  $n = 6$ :  $\Delta' = \{\delta_{\alpha_3}, \delta_{\alpha_5}, \delta_{\alpha_7}\}$ ,  $\Delta'' = \Delta \setminus \{\delta_{\alpha_1}, \delta_{\alpha_2}^-\}$ ,  $(L^\circ)' = SL(6)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_1) \oplus V(\omega_3) \oplus V(\omega_5)$ ,  $c_1 = c = 2$ ,  $d = 6$ . The two choices for  $H$  are conjugated since they correspond to two conjugated copies of  $SL(6)$  in  $SO(12)$ .  $n = 7$ :  $\Delta' = \{\delta_{\alpha_3}, \delta_{\alpha_5}, \delta_{\alpha_7}\}$ ,  $\Delta'' = \{\delta_{\alpha_2}^+, \delta_{\alpha_3}, \dots, \delta_{\alpha_7}\}$ ,  $(L^\circ)' = SL(6)$ ,  $\mathbf{n}_1/[\mathbf{n}_1, \mathbf{n}_1] = V(\omega_1)^{\oplus 2} \oplus V(\omega_3) \oplus V(\omega_5)^{\oplus 2}$ ,  $c_1 = 3$ ,  $c = 2$ ,  $d = 6$ . The two choices for  $H$  are analogously conjugated.

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