ERRATUM TO

"THE STRUCTURE OF RATIONAL AND RULED SYMPLECTIC 4-MANIFOLDS"

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Francois Lalonde pointed out that Theorem 1.3 in [2] about the structure of symplectic $S^2$-bundles needs an extra hypothesis. The argument which proves uniqueness works only for a restricted range of cohomology classes, and the condition $a^2(V) > (a(F))^2$ need not hold when $M$ has genus $> 0$. The mistake occurred in Lemma 4.15 where I failed to realise that the integral of the form $\rho$ over the section $\Gamma$ is not zero in general, but depends on the homology class of $\Gamma$. Thus this Lemma holds only under certain restrictions on $[\omega]$.

The methods developed in [2] suffice to prove the following theorems which replace Theorem 1.3, Corollary 1.5(iii), and Proposition 4.17: for complete details, see [3]. We will say that two symplectic forms $\omega_0$ and $\omega_1$ are pseudo-isotopic if they may be joined by a family of not necessarily cohomologous symplectic forms.

**Theorem 1.** Let $\omega$ be a symplectic form on $M \times S^2$ which is compatible with the projection onto the Riemann surface $M$.

(i) If $M$ is $S^2$ or $T^2$, $\omega$ is isotopic to a split form.

(ii) If $M$ has genus $g > 1$, the statement in (i) holds provided that $\mu_1 > q\mu_2$, where $\mu_1 = \omega(M \times \text{pt}), \mu_2 = \omega(\text{pt} \times S^2)$ and $q = \lceil \frac{q}{2} \rceil$.

(iii) $\omega$ is always pseudo-isotopic to a split form. Moreover, it is isotopic to a split form iff it has a symplectic section in class $[M \times \text{pt}]$.

In the case of the nontrivial $S^2$-bundle $V_M$ over $M$, we write $\{b_-, b_+\}$ for the basis of $H^2(M)$ dual to the homology basis $\{[M_-], [M_+]\}$ where $M_-$ (resp. $M_+$) is a section with self-intersection $-1$ (resp. $+1$).

**Theorem 2.** (i) When $M = S^2$, the class $a = \mu_+ b_+ + \mu_- b_-$ may be represented by a compatible symplectic form on $V_M$ only if $\mu_+ > \mu_- > 0$. Moreover, up to isotopy there is a unique such form in each class.

(ii) If $M$ has genus $g > 0$, every class with $\mu_+ > |\mu_-| > 0$ has a compatible symplectic representative. There is a unique form up to isotopy in each class such that $q\mu_- > (q - 1)\mu_+$, where $q = \lceil \frac{q+1}{2} \rceil$.

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(iii) All these forms are pseudo-isotopic.

Other inaccuracies:
1. In Lemma 4.9 one must assume that \( \omega_0 = \omega_1 \) at the point \( S_L \cap F \). Similarly in Proposition 4.18. A corrected version of Proposition 4.18 is given in [3].

2. The proof of Lemma 4.16 is not quite right because the complex structure \( J'_1 \) has too many cusp-curves. However, this is easy to rectify: one just has to replace \( J'_1 \) by a generic integrable \( J \). This is discussed in detail in [3].

3. The proof of (5.5) is inadequate because when I was considering blowing down I did not allow for the possibility that the intersection number \( C \cdot \Sigma \) might be \( \geq 2 \), so that the blow down of \( C \) would no longer be embedded. The given arguments prove that the category under consideration is closed under pseudo-isotopy, under blowing up, and under blowing down when \( C \cdot \Sigma \leq 1 \). When \( C \cdot \Sigma \geq 2 \), we reduce to the corresponding result in the integrable case as follows. Let \( (\bar{V}, \bar{\omega}) \) be a minimal reduction of \( (V, \omega) \) obtained by blowing down a family of exceptional spheres \( \Sigma_i \) which includes \( \Sigma \). Then \( C \) descends to an immersed \( J \)-holomorphic sphere in \( \bar{V} \) which is not embedded. If \( \bar{V} \) were an \( S^2 \)-bundle over a Riemann surface \( M \) of genus \( \geq 0 \) the projection of \( C \) onto \( M \) would be a map of positive degree, which is impossible. Therefore, \( (\bar{V}, \bar{\omega}) \) is rational, and hence Kähler. Furthermore, there is a Kähler form \( \omega' \) on \( V \) which is pseudo-isotopic to \( \omega \). (Take \( \omega' \) so that its integral over the \( \Sigma_i \) is small.) By deforming \( \omega' \) further, we may assume that the Kähler manifold \( (V, \omega') \) is a blow-up of \( \mathbb{C}P^2 \) rather than \( S^2 \times S^2 \). By Lemma 3.1, the homology class \([\Sigma]\) is represented by a \( J \)-holomorphic curve or cusp-curve, for each \( \omega \)-tame \( J \). If we choose \( J \) to be integrable and good and generic in the sense of [1, Ch. III], then \([\Sigma]\) must be represented by a curve \( \Sigma_f \). Then \( \Sigma_f \) is isotopic to \( \Sigma \), and, because our category is closed under pseudo-isotopy, it suffices to show that the complex surface obtained from \( V \) by blowing down \( \Sigma_f \) contains a suitable embedded rational curve. But this follows from the Castelnuovo criterion.

References


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