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ERRATUM TO "REAL BOUNDS, ERGODICITY AND NEGATIVE SCHWARZIAN FOR MULTIMODAL MAPS"

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In Part 1 of Theorem C of the paper *Real bounds, ergodicity and negative Schwarzian for multimodal maps*, see [1], the assumption that V is nice was, by mistake, omitted. We would like to thank Weixiao Shen for pointing this out. The correct version of Theorem C(1) is as follows:

Theorem C(1) (Improved Macroscopic Koebe Principle)**.** *Assume that $f\colon M \to M$ is contained in $\mathcal{A}^{1+\mathrm{Zygmund}}$. Then for each $\xi > 0$, there exists $\xi' > 0$ such that if I is a nice interval, V is nice and ξ-well-inside I and $x \in I$, $f^k(x) \in V$ (with $k \geq 1$ not necessarily minimal), then the pullback of V along $\{x, \dots, f^k(x)\}$ is ξ'-well-inside the return domain to I containing x.*

Here, as before, we define an open interval K to be nice if no iterate of ∂K enters K. This implies that if K_1 and K_2 are pullbacks of K, then they are either disjoint or nested.

In Lemma 9 (page 762) it was implicitly assumed that V is disjoint from J_n. It is for this reason that the proof of Theorem C(1) does not work unless we assume V is nice (or something similar). The proof of Theorem C(1) as stated above is essentially the same as before, using Lemma 6′ below instead of Lemma 6; then in Lemma 9 (page 762) we do not need to require that k_{n+1} is a jump time provided we assume that V is nice. Making the additional assumption that V is nice, Proposition 1 (and its proof) and the rest of the paper go through unchanged.

Lemma 6′. *For each $\rho > 0$ sufficiently small, there exists $\delta_3 > 0$ such that if I is a ρ-scaled neighbourhood of a nice interval $V \subset I$, then J is a δ_3-scaled neighbourhood of any component A of $\phi_{|J}^{-k}(V)$ (where $k \geq 1$ is arbitrary).*

Proof. Let V_i, $i = 0, \dots, k$ be the component of $\phi_{|J}^{-(k-i)}(V)$ containing $\phi^i(A)$. Of course, we may assume that k is large and that $V_0, \dots, V_k$ are disjoint.

Claim. There exists $\alpha > 0$ such that if $0 \leq j < k$ and V_{j+1} is contained in a neighbourhood of V_j of size $(1+\alpha)|V_j|$, then V_j is α-well-inside I. Similarly, if $0 \leq j < k-1$ and V_j lies between V_{j+1} and V_{j+2}, then V_j is α-well-inside I.

Proof of Claim. If V_{j+1} is contained in a neighbourhood of V_j of size $(1+\alpha)|V_j|$ and α is small enough, then ϕ' is close to zero on a definite neighbourhood of V_j. So V_j is contained in the basin of an attracting fixed point with multiplier close to zero. Since V_k is nice and δ_3-well-inside I, we easily get that V_j is δ_3'-well-inside

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I, proving the first part of the claim. The second part of the claim follows in the same way by first applying part 1 to V_j, V_{j+1} and then applying it to V_j, V_{j+2} while considering ϕ^2 instead of ϕ, completing the proof of the claim.

If both sides of J are small, then $|\phi'|$ is bounded on J. There are three possibilities.

(a) V_1 lies between V_2 and V_3, in which case, by the second part of the Claim, V_1 is well-inside I.

(b) V_2 lies between V_1 and V_3; in this case since $|V_1|/|V_2|$ is not small and by the first part of the Claim, V_2 is well-inside I.

(c) V_3 lies between V_1 and V_2; then, because $|V_1|/|V_2|$ and $|V_2|/|V_3|$ are not small, V_3 is well-inside I. In all cases, we get that V_0 is well-inside J.

So assume one of the sides, say the right side, of J is not small. Let $1 \leq j \leq k$ be the largest integer so that $V_1, \dots, V_j$ are all not α-well-inside I. By taking $\alpha > 0$ small, we may assume $j \leq k-2$. Since V_{j+1} is α-well-inside I, we may assume that $j \geq 1$ and that there exists $\alpha' > 0$ so that V_j, V_{j-1} are α'-well-inside $J \subset I$. By the claim, for each $i = 1, \dots, j$, V_i has an α-small and an α-big side, and V_{i+1} is contained in the α-big side. Since the right side of J is not small, $V_1, \dots, V_{j+1}$ lie therefore ordered from left to right. If for each $i = 1, \dots, j-1$, V_{i-1} is contained in a β-scaled neighbourhood of V_i, then V_1 is in a $(\beta+\beta^2+\cdots+\beta^j)$-scaled neighbourhood of V_{j-1}. So taking $\beta \in (0,1)$ so small that $\beta/(1-\beta) < \alpha'/2$, then, because V_{j-1} is α'-well-inside I, the left component of $I \setminus V_1$ has at least size $\frac{\alpha'}{2}|V_{j-1}| > \frac{\alpha'}{2}|V_1|$, i.e., V_1 is well-inside I, and V_0 is well-inside J. Hence we may assume there exists $i \in \{1, \dots, j-1\}$ so that V_{i-1} is not contained in a β-scaled neighbourhood of V_i. This and the first part of the Claim imply that V_i is well-inside the convex hull $H_i := [V_{i-1}, V_{i+1}]$ of V_{i-1} and V_{i+1}. Because the intervals $V_1, \dots, V_{i+2}$ lie ordered, it follows that the pullback of H_i along $V_1, \dots, V_i$ has intersection multiplicity at most 4 and therefore that V_1 is well-inside I. This again gives that V_0 is well-inside J. (This method of proof can also be used to provide a slightly shorter proof of Lemma 5.) $\square$

Reference

[1] S. van Strien, E. Vargas, *Real bounds, ergodicity and negative Schwarzian for multimodal maps,* Journal of the American Mathematical Society, **17**, 2004, 749–782. MR2083467 (2005i:37043)

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