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# ERRATA TO "CLASSIFICATION OF THREE-DIMENSIONAL FLIPS"

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This paper makes two corrections to [KM92].

*Remark* 1. The statement [KM92, (2.2.3)] is false as it stands. (2.2.3) comes out of two sources: one from (2.13.4) and the other from (2.13.10). The latter (2.13.10) is correct and proves that $m \geq 5$ and the index-two point is of type $cA/2$. However, the former (2.13.4) proves only a weaker assertion that $m \geq 3$ (not $m \geq 5$), the index-two point is of type $cA/2$, $cAx/2$, or $cD/2$ (not just of type $cA/2$) and with axial multiplicity $k \geq 2$ (because the index-one cover is not smooth). Since $f : X \supset C \to Y \ni Q$ is divisorial in the case (2.13.4) [KM92, Prop. (9.3)], the following revision $(2.2.3)_{\mathrm{rev}}$ should replace (2.2.3).

> $(2.2.3)_{\mathrm{rev}}$ Case for exceptional $IA + IA$: The two $IA$ points are an ordinary point of odd index $m$ ($m \geq 5$ if $f$ is isolated (that is, a flipping contraction); $m \geq 3$ if divisorial) and an index-two point (of type $cA/2$ if $f$ is isolated; of type $cA/2$, $cAx/2$ or $cD/2$ if divisorial) and with axial multiplicity $k$ with $2k + m \geq 7$, and we have $(K_X \cdot C) = -1/(2m)$. $(E_Y, Q)$ is $D_{2k+m}$, $\mathrm{Sing} E_X$ is $A_{m-1} + D_{2k}$ ($A_{m-1} + A_1 + A_1$ if $k = 1$) and $\Delta(E_X \supset C)$ is

(*kAD*)

$$\begin{array}{l} \hphantom{\circ - \cdots - \circ - \bullet - \circ - \cdots - {}}\circ \\ \hphantom{\circ - \cdots - \circ - \bullet - \circ - \cdots - {}}| \\ \underbrace{\circ - \cdots - \circ}_{m-1} - \bullet - \circ - \cdots - \circ - \circ \end{array}$$

*Remark* 2. [KM92, Lemma (2.12.9)] holds under an extra assumption that $f|_{X \setminus C} : X \setminus C \to Y \setminus \{Q\}$ is an isomorphism since it is used in the second line of the proof. The following Lemma 3 can be used as a substitute for [KM92, Lemma (2.12.9)]. The arguments in [KM92, (2.12)] work after this modification.

**Lemma 3.** *Let $f : X \to (Y, Q)$ be a projective bimeromorphic morphism of irreducible normal 3-folds such that $R^1 f_* \mathcal{O}_X = 0$ and $C = f^{-1}(Q)_{\mathrm{red}}$ is 1-dimensional. Let $I \subset \mathcal{O}_X$ be a sheaf of ideals such that $\mathrm{Supp}\, \mathcal{O}_X/I = C$. For each $n > 0$, let $I^{(n)}$ be the sheaf of ideals such that $I^n \subset I^{(n)} \subset \mathcal{O}_X$ and $I^{(n)}/I^n$ is the largest subsheaf of $\mathcal{O}_X/I^n$ with 0-dimensional support. Then $\chi(\mathcal{O}_X/I^{(n)}) \geq O(n^3)$ as $n$ grows.*

*Proof.* Since $f$ is bimeromorphic, let $E \subset X$ be an effective Cartier divisor with very ample $\mathcal{O}(-E)$. We note that $E \supset C$ because $E \cdot C_i < 0$ for each irreducible

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component $C_i$ of $C$. Since $f^{-1}(Q)$ is 1-dimensional, we can choose a hyperplane section $\bar{D} \ni Q$ of $Y \ni Q$ such that $D \cap E$ is a curve, where $D = f^*(\bar{D})$.

For each subspace $Z \subset X$, let $I_Z \subset \mathcal{O}_X$ denote the sheaf of ideals such that $\mathcal{O}_X/I_Z = \mathcal{O}_Z$. Let $F \subset \mathcal{O}_X$ be any coherent subsheaf such that Supp $\mathcal{O}_X/F$ is a curve containing $C$. We consider a primary decomposition $F = \bigcap Q_i$ and set

$$F' = \bigcap \{Q_j | \sqrt{Q_j} = I_{C_k} \text{ for some } k\},$$

which does not depend on the choice of $Q_i$'s. We note $I^{(n)} = (I^n)'$.

We set $J = (I_D + I_E)'$ and first prove the lemma for the case $I = J$.

By the normality of $X$, $J$ is generated by a regular sequence outside a finite set. Hence the natural homomorphism

$$a_1 : (\mathcal{O}/J)(-D) \oplus (\mathcal{O}/J)(-E) \to J/J^{(2)}$$

is injective and Supp Coker$(a_1)$ is at most 0-dimensional, and the induced

$$a_n = S^n(a_1) : \bigoplus_{k=0}^{n} (\mathcal{O}/J)(-kD - (n-k)E) \to J^{(n)}/J^{(n+1)}$$

has similar properties: $\mathrm{Ker}(a_n) = 0$, dim Supp $\mathrm{Coker}(a_n) \leq 0$. By $(-E \cdot C_i) > 0$ and $(D \cdot C_i) = 0$, we have

$$\chi(J^{(n)}/J^{(n+1)}) \geq \sum_{k=0}^{n} (\chi(\mathcal{O}/J) + (n-k)) \geq n \cdot \chi(\mathcal{O}/J) + n(n+1)/2,$$

and the lemma holds for the case $I = J$.

Let $a$ be a natural number such that $I^{(a)} \subset J$. We note

$$\chi(\mathcal{O}_X/I^{(n)}) = \chi(\mathcal{O}_X/J^{([n/a])}) + \chi(J^{([n/a])}/I^{(n)}),$$

by $I^{(n)} \subset I^{(a[n/a])} \subset J^{([n/a])}$. Thus it remains to prove $\chi(J^{([n/a])}/I^{(n)}) \geq 0$. Since the cokernel of $S^{[n/a]}(\mathcal{O}(-D) \oplus \mathcal{O}(-E)) \to J^{([n/a])}/I^{(n)}$ is supported on a finite set and $\mathcal{O}(-D) \oplus \mathcal{O}(-E)$ is generated by global sections, we have $H^1(X, J^{([n/a])}/I^{(n)}) = 0$ by the following well-known lemma, and we are done. $\square$

**Lemma 4.** *Let $f : X \to (Y, Q)$ be a proper morphism to a germ such that $R^1 f_* \mathcal{O}_X = 0$ and $C = f^{-1}(Q)_{\mathrm{red}}$ is 1-dimensional. Let $F, G$ be coherent sheaves on $X$ with a homomorphism $a : F \to G$ such that $F$ is generated by global sections and* Supp Coker$(a)$ *is finite over $Y$. Then $R^1 f_* G = 0$.*

*Proof.* Since $C$ is 1-dimensional, $R^2 f_* \mathcal{H} = 0$ for each coherent sheaf $\mathcal{H}$ on $X$. Since $F$ is globally generated, there exist an integer $n > 0$ and a surjection $b : \mathcal{O}_X^{\oplus n} \to F$. By $R^2 f_* \operatorname{Ker} b = 0$, we have $R^1 f_* F = 0$. By $R^2 f_* \operatorname{Ker} a = 0$, we have $R^1 f_* \operatorname{Im} a = 0$. Since $R^1 f_* \operatorname{Coker} a = 0$ by the assumption, we have $R^1 f_* G = 0$. $\square$

## References

[KM92] Kollár, J. and Mori, S., Classification of three-dimensional flips, *J. Amer. Math. Soc.*, **5** (1992), 533–703. MR1149195 (93i:14015)

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