The authors would like to thank Professor Gross for pointing out an error in Lemma 3.3 of [4]. Conjecture 1.4 of [4] does not hold for \( d(\pi, \mu_{G/A, \psi}) \) in the non-archimedean case. We need to modify the choice of the Haar measure \( \mu_{G/A, \psi} \).

Following Gross and Gan [3], we take another Haar measure \( \mu'_{G/A, \psi} \) on \( G/A \) defined as follows.

Let \( F \) be a non-archimedean local field of characteristic zero and let \( \psi \) be a non-trivial additive character of \( F \). Let \( G \) be a connected reductive algebraic group over \( F \) and let \( A \) be the split component of the center of \( G \). We may assume that \( A = \{1\} \). Let \( G_0 \) be the split form of \( G \) and choose an isomorphism \( \eta_0 : G \to G_0 \) over \( \bar{F} \), which may not be an inner twist. Let \( G_0 \) be a Chevalley model of \( G_0 \) over \( \mathbb{O}_F \) and choose a differential form \( \omega_0 \) of top degree on \( G_0 \) over \( \mathbb{O}_F \) with non-zero reduction. Put \( \omega = \eta_0^*(\omega_0) \). Let \( \mu'_{G, \psi} \) denote the Haar measure on \( G \) determined by \( \omega \) and the self-dual measure on \( F \) with respect to \( \psi \). The Haar measure \( \mu'_{G, \psi} \) does not depend on the choice of \( \eta_0 \) and \( \omega_0 \) (cf. [3, §5]).

Conjecture 1.4 of [4] should be modified as follows.

**Conjecture 1.4’.** Let \( \phi : L_F \to L_G \) be an elliptic tempered Langlands parameter. Then

\[
\langle 1, \pi \rangle = \left| \frac{\gamma(0, \pi, \Ad, \psi)}{\gamma(1, \pi)} \right| d(\pi, \mu'_{G/A, \psi})
\]

for \( \pi \in \Pi_\phi(G) \).

The relation between \( \mu_{G/A, \psi} \) and \( \mu'_{G/A, \psi} \) has been studied by Gross and Gan [3]. For simplicity, we assume that the connected center of \( G \) is anisotropic. Let \( M \) be the motive of \( G \) defined in [1]. Recall that \( M = \bigoplus_{d \geq 1} V_d(1 - d) \), where \( V_d \) is the Artin motive given in [1, §1]. Following [3, §4], we define the Artin conductor \( a(M) \) of \( M \) by

\[
a(M) = \sum_{d \geq 1} (2d - 1)a(V_d),
\]

where \( a(V_d) \) is the Artin conductor of \( V_d \). We have

\[
\mu'_{G, \psi} = q^{-a(M)/2} \cdot \mu_{G, \psi}
\]

(cf. [3, §5]). In particular, \( \mu'_{G, \psi} = \mu_{G, \psi} \) if the quasi-split inner form \( G^* \) of \( G \) is unramified.

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Let \( \pi_0 \) be the Steinberg representation of \( G \). By [4] §3.3, we have
\[
d(\pi_0, \mu'_{G, \psi}) = |H^1(F, G)|^{-1} \cdot q^{a(M)/2} \cdot \frac{|L(M')(1)|}{|L(M)|}.
\]
Here \( \psi \) is a non-trivial additive character of \( F \) of order zero. We need to modify Lemma 3.3 of [4] as follows.

**Lemma 3.3'.**
\[
|\gamma(0, \pi_0, \text{Ad}, \psi)| = q^{a(M)/2} \cdot \frac{|L(M')(1)|}{|L(M)|}.
\]

**Proof.** Set \( \hat{\mathfrak{g}} = \text{Lie}(\hat{G}) \). Let \((\rho, N)\) be the representation of \( W'_F \) on \( \hat{\mathfrak{g}} \) associated to \( \text{Ad} \circ \phi_0 \), where \( \phi_0 : L_F \to \mathbf{L}G \) is the Langlands parameter associated to \( \pi_0 \). It remains to show that
\[
|\epsilon(0, \pi_0, \text{Ad}, \psi)| = q^{a(M)/2} \cdot \prod_{d \geq 1} q^{(d-1)\dim V^{lF}_d}.
\]
By definition, we have
\[
|\epsilon(s, \pi_0, \text{Ad}, \psi)| = q^{-a(\hat{\mathfrak{g}})(s-1/2)}.
\]
We remark that the equation \( a(\hat{\mathfrak{g}}) = \dim \hat{\mathfrak{g}}^{lF} - \dim \hat{\mathfrak{g}}^N_{lF} \) in the proof of Lemma 3.3 of [4] is incorrect and should be corrected as
\[
a(\hat{\mathfrak{g}}) = a(\rho) + \dim \hat{\mathfrak{g}}^{lF} - \dim \hat{\mathfrak{g}}^N_{lF},
\]
where \( a(\rho) \) is the Artin conductor of \( \rho \). By Proposition 5.2 of [2], we have \( \hat{\mathfrak{g}} = \bigoplus_{d \geq 1} V_d \otimes \rho_{2d-2} \) as a representation of \( \Gamma \times \text{SL}(2, \mathbb{C}) \). Hence we have
\[
a(\hat{\mathfrak{g}}) = \sum_{d \geq 1} (2d-1) a(V_d) + \sum_{d \geq 1} (2d-1) \dim V^{lF}_d - \sum_{d \geq 1} \dim V^N_{lF}_d
\]
\[
= a(M) + 2 \sum_{d \geq 1} (d-1) \dim V^{lF}_d.
\]
This completes the proof. \( \square \)

Thus we obtain
\[
d(\pi_0, \mu'_{G, \psi}) = \frac{1}{|S^e_{\phi_0}|} \cdot |\gamma(0, \pi_0, \text{Ad}, \psi)|.
\]

In particular, Conjecture 1.4' holds for \( \pi_0 \).

Theorem 8.6 of [4] does not hold for \( d(\pi_H) = d(\pi_H, \mu_H, \psi) \) unless \( H \) is unramified. Using Lemma 3.3', one can show that Proposition 8.5 of [4] holds for \( d(\pi_H, \mu'_{H, \psi}) \). Thus we obtain the following theorem.

**Theorem 8.6'.** Let \( H = \text{U}(3) \) be the quasi-split unitary group in three variables. Let \( \pi_H \) be a stable discrete series representation of \( H \). Then
\[
d(\pi_H, \mu'_{H, \psi}) = \frac{1}{2} \cdot |\gamma(0, \pi_H, \text{Ad}, \psi)|.
\]
In particular, Conjecture 1.4' holds for \( \pi_H \).
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