

ERRATUM TO
“MASS EQUIDISTRIBUTION FOR AUTOMORPHIC FORMS
OF COHOMOLOGICAL TYPE ON GL_2 ”

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Corollary 3 of [4] is not known unconditionally, as cohomological automorphic forms on GL_2 over an imaginary quadratic field are not known to satisfy the Ramanujan conjecture. We shall briefly describe the reason for this and discuss what information Theorem 1 of [4] does give in the case of imaginary quadratic fields.

Let K be an imaginary quadratic field with nontrivial automorphism c , and let π be a cuspidal automorphic representation of $GL_2(\mathbb{A}_K)$ with unitary central character ω . Suppose that $\omega = \omega^c$ and that π_∞ has Langlands parameter $W_{\mathbb{C}} = \mathbb{C}^\times \rightarrow GL_2(\mathbb{C})$ given by $z \mapsto \text{diag}(z^{1-k}, \bar{z}^{1-k})$ for some integer $k \geq 2$ (i.e. so that π is any cohomological representation up to twist). It is then known (see Theorem 1.1 of [1]) that for any ℓ one may associate a continuous irreducible representation $\rho : \text{Gal}(\bar{K}/K) \rightarrow GL_2(\bar{\mathbb{Q}}_\ell)$ to π such that the characteristic polynomial of $\rho(\text{Frob}_v)$ agrees with the Hecke polynomial of π_v at all places v which do not divide ℓ and at which K/\mathbb{Q} , π , and π^c are unramified. However, because ρ is constructed via an ℓ -adic limiting process, it is not known to arise from a motive and so is not known to be pure.

To construct ρ , one first makes a theta lift from π to a holomorphic limit of discrete series representation Π on Sp_4/\mathbb{Q} as in [3]. Weissauer [6] has proven that if Π' is a holomorphic discrete series representation of Sp_4/\mathbb{Q} which is not a CAP representation, then one may associate a Galois representation to it which is pure and locally compatible with Π' at all unramified places, so that Π' satisfies Ramanujan wherever it is unramified. These results are not known for the limit of discrete series representation Π , and to associate a Galois representation to it one must apply techniques of Taylor [5] which are similar to those used by Deligne and Serre to associate Galois representations to classical weight 1 modular forms. These involve multiplying a holomorphic form in Π by a well understood regular holomorphic form of large weight, applying the results of Weissauer and recovering ρ from these products by an ℓ -adic limiting process. Any Archimedean information about the Frobenius eigenvalues of ρ is lost during this, and neither does one know that ρ arises from a motive. As a result, the algebraic information we obtain about π is insufficient to deduce Ramanujan for it.

We may still draw interesting conclusions from Theorem 1 of [4] in the imaginary quadratic case. Cohomological forms on GL_2/K which are base changes from \mathbb{Q} will satisfy Ramanujan, and so Theorem 1 establishes their equidistribution as their weight becomes large. Moreover, the experimental results of [2] suggest that all

Received by the editors June 16, 2011 and, in revised form, September 13, 2011.

2010 *Mathematics Subject Classification*. Primary 11F41, 11F11; Secondary 11F75.

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but finitely many forms of fixed level and growing weight on GL_2/K are obtained from base change and CM constructions, so that the question of whether a general cohomological form on GL_2/K satisfies Ramanujan does not seem to matter in practice.

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