

---

VOLUME 31

NUMBER 2

APRIL 2018

---

# JOURNAL

OF THE

ISSN 0894-0347 (print)  
ISSN 1088-6834 (online)

---

A M E R I C A N M A T H E M A T I C A L S O C I E T Y

---

**EDITORS**

Brian Conrad  
Laura G. DeMarco  
Simon Donaldson  
Pavel Etingof  
Sergey Fomin  
Assaf Naor  
Igor Rodnianski  
Shmuel Weinberger

**ASSOCIATE EDITORS**

Ian Agol  
Denis Auroux  
Andrea Bertozzi  
Roman Bezrukavnikov  
Dmitry Dolgopyat  
Alice Guionnet  
Lawrence Guth  
Lars Hesselholt  
Richard Kenyon  
Michael J. Larsen  
Ciprian Manolescu  
William P. Minicozzi II  
Anand Pillay  
Peter Sarnak  
Peter Scholze  
Amit Singer  
Benjamin Sudakov  
Burt Totaro



AMERICAN  
MATHEMATICAL  
SOCIETY

Providence, Rhode Island USA

## Journal of the American Mathematical Society

This journal is devoted to research articles of the highest quality in all areas of pure and applied mathematics.

**Submission information.** See **Information for Authors** at the end of this issue.

**Publication on the AMS website.** Articles are published on the AMS website individually after proof is returned from authors and before appearing in an issue.

**Subscription information.** The *Journal of the American Mathematical Society* is published quarterly and is also accessible electronically from [www.ams.org/journals/](http://www.ams.org/journals/). Subscription prices for Volume 31 (2018) are as follows: for paper delivery, US\$425 list, US\$340 institutional member, US\$382.50 corporate member, US\$255 individual member; for electronic delivery, US\$374 list, US\$299.20 institutional member, US\$336.60 corporate member, US\$224.40 individual member. Upon request, subscribers to paper delivery of this journal are also entitled to receive electronic delivery. If ordering the paper version, add US\$5.25 for delivery within the United States; US\$20 for surface delivery outside the United States. Subscription renewals are subject to late fees. See [www.ams.org/journal-faq](http://www.ams.org/journal-faq) for more journal subscription information.

**Back number information.** For back issues see [www.ams.org/backvols](http://www.ams.org/backvols).

Subscriptions and orders should be addressed to the American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904 USA. *All orders must be accompanied by payment.* Other correspondence should be addressed to 201 Charles Street, Providence, RI 02904-2213 USA.

**Copying and reprinting.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Permissions to reuse portions of AMS publication content are handled by Copyright Clearance Center's RightsLink® service. For more information, please visit: <http://www.ams.org/rightslink>.

Translation rights and licensed reprint requests should be sent to [reprint-permission@ams.org](mailto:reprint-permission@ams.org).

Excluded from these provisions is material for which the author holds copyright. In such cases, requests for permission to reuse or reprint material should be addressed directly to the author(s). Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.

---

The *Journal of the American Mathematical Society* (ISSN 0894-0347 (print); ISSN 1088-6834 (online)) is published quarterly by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2213 USA and is mailed from Providence, Rhode Island. Periodicals postage is paid at Providence, Rhode Island. Postmaster: Send address changes to Journal of the AMS, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.

© 2018 by the American Mathematical Society. All rights reserved.

This journal is indexed in *Mathematical Reviews*, *Zentralblatt MATH*, *Science Citation Index*®, *Science Citation Index*™-Expanded, *ISI Alerting Services*™, *CompuMath Citation Index*®, and *Current Contents*®/Physical, Chemical & Earth Sciences. This journal is archived in *Portico* and in *CLOCKSS*.

Printed in the United States of America.

⊗ The paper used in this journal is acid-free and falls within the guidelines established to ensure permanence and durability.

10 9 8 7 6 5 4 3 2 1      23 22 21 20 19 18



## IN MEMORIAM

We note with great sadness the untimely death of Maryam Mirzakhani, who died on July 14, 2017, at the age of 40. She had served as an editor of the Journal of the American Mathematical Society (JAMS) since February 2016 and as an associate editor for four years before that.

First diagnosed with breast cancer in 2013, Maryam agreed to become a JAMS editor when her cancer was thought to be in remission. Unfortunately, that proved to be a false hope, and the cancer recurred. Despite the relapse, Maryam continued her editorial work with keen insight and broad mathematical vision. She participated in the editorial conference calls to the end, sharing her wisdom and well-founded opinions.

Maryam Mirzakhani was born in Tehran, Iran, in 1977. She earned international acclaim already as a high school student, winning gold medals in the 1994 and 1995 International Mathematical Olympiads. Upon completion of an undergraduate degree at the Sharif University of Technology in Tehran, she commenced her graduate

studies at Harvard under the direction of C. McMullen. Her thesis, completed in 2004, contained several remarkable results, including the asymptotics for the number of simple closed geodesics in hyperbolic surfaces and a new proof of a conjecture by E. Witten (first proven by M. Kontsevich using a different approach). She became interested in various aspects of dynamics on the moduli spaces of quadratic and abelian differentials, a subject she had first explored in her thesis. A major highlight of this research direction was her joint work with A. Eskin and with Eskin and A. Mohammadi, classifying invariant measures and orbit closures for the natural action of  $SL(2, \mathbb{R})$  on these spaces. This classification has had a dramatic impact on the subject, with ramifications for dynamics of interval exchange systems and billiards, Teichmüller theory, algebraic geometry, and beyond.

Remarkably modest and at the same time very ambitious, a fearless researcher with a far-reaching vision, Maryam served as a role model to mathematicians around the world and to the public at large. In recognition of her extraordinary achievements, she was awarded the Fields Medal in 2014, becoming the first woman to receive this honor. Her accolades included two bestowed by the AMS, the Blumenthal Award in 2009 and the Satter Prize in 2013, as well as a Clay Research Award in 2014.

Maryam's death is a great loss to all of us and to the entire mathematical community. She is survived by her husband, Jan Vondrak, and her daughter, Anahita, as well as her parents, sister, and two brothers.

JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY  
CONTENTS

Vol. 31, No. 2

April 2018

<b>Seung uk Jang and Junehyuk Jung, Quantum unique ergodicity and the number of nodal domains of eigenfunctions</b> .....	303
<b>Piotr Przytycki and Daniel T. Wise, Mixed 3-manifolds are virtually special</b> .....	319
<b>Seok-Jin Kang, Masaki Kashiwara, Myungho Kim, and Se-jin Oh, Monoidal categorification of cluster algebras</b> .....	349
<b>Massimiliano Gubinelli and Nicolas Perkowski, Energy solutions of KPZ are unique</b> .....	427
<b>Jason McCullough and Irena Peeva, Counterexamples to the Eisenbud–Goto regularity conjecture</b> .....	473
<b>Mark Gross, Paul Hacking, Sean Keel, and Maxim Kontsevich, Canonical bases for cluster algebras</b> .....	497

## Editors

Brian Conrad  
Department of Mathematics  
Stanford University  
Stanford, CA 94305 USA  
conrad@math.stanford.edu

Laura G. DeMarco  
Department of Mathematics  
Northwestern University  
Evanston, IL 60208-2730 USA  
demarco@northwestern.edu

Simon Donaldson  
Simons Center for Geometry and Physics  
Stony Brook University  
Stony Brook, NY 11794 USA  
sdonaldson@scgp.stonybrook.edu

Pavel Etingof  
Department of Mathematics  
Massachusetts Institute of Technology  
77 Massachusetts Avenue  
Cambridge, MA 02139 USA  
etingof@math.mit.edu

Sergey Fomin  
Department of Mathematics  
University of Michigan  
530 Church Street  
Ann Arbor, MI 48109-1043 USA  
fomin@umich.edu

Assaf Naor  
Department of Mathematics  
Princeton University  
Fine Hall, Washington Road  
Princeton, NJ 08544 USA  
naor@math.princeton.edu

Igor Rodnianski  
Department of Mathematics  
Princeton University  
Fine Hall, Washington Road  
Princeton, NJ 08544 USA  
irod@math.princeton.edu

Shmuel Weinberger  
Department of Mathematics  
University of Chicago  
5734 S University Avenue  
Chicago, IL 60637 USA  
shmuel@math.uchicago.edu

## Associate Editors

Ian Agol, University of California, Berkeley  
Denis Auroux, University of California, Berkeley  
Andrea Bertozzi, University of California, Los Angeles  
Roman Bezrukavnikov, Massachusetts Institute of Technology  
Dmitry Dolgopyat, University of Maryland  
Alice Guionnet, Massachusetts Institute of Technology  
Lawrence Guth, Massachusetts Institute of Technology  
Lars Hesselholt, Nagoya University  
Richard Kenyon, Brown University  
Michael J. Larsen, Indiana University  
Ciprian Manolescu, University of California, Los Angeles  
William P. Minicozzi II, Massachusetts Institute of Technology  
Anand Pillay, University of Notre Dame  
Peter Sarnak, The Institute for Advanced Study and Princeton University  
Peter Scholze, Universität Bonn  
Amit Singer, Princeton University  
Benjamin Sudakov, ETH Zürich  
Burt Totaro, University of California, Los Angeles

## Editorial Information

Information on the backlog for this journal can be found on the AMS website starting from <http://www.ams.org/jams>.

In an effort to make articles available as quickly as possible, articles are electronically published on the AMS website individually after proof is returned from authors and before appearing in an issue.

A Consent to Publish is required before we can begin processing your paper. After a paper is accepted for publication, the Providence office will send a Consent to Publish and Copyright Agreement to all authors of the paper. By submitting a paper to this journal, authors certify that the manuscript has not been submitted to nor is it under consideration for publication by another journal, conference proceedings, or similar publication.

### Information for Authors

**Initial submission.** All articles submitted to this journal are peer-reviewed. The AMS has a single blind peer-review process in which the reviewers know who the authors of the manuscript are, but the authors do not have access to the information on who the peer reviewers are. The AMS uses Centralized Manuscript Processing for initial submission. Authors should submit a PDF file using the Initial Manuscript Submission form found at [www.ams.org/submission/jams](http://www.ams.org/submission/jams) or should send one copy of the manuscript to the following address: Centralized Manuscript Processing, JOURNAL OF THE AMS, 201 Charles Street, Providence, RI 02904-2213 USA. If a paper copy is being forwarded to the AMS, indicate that it is for the *Journal of the AMS* and include the name of the corresponding author, contact information such as email address or mailing address, and the name of an appropriate Editor to review the paper (see the list of Editors above).

The first page must contain a *descriptive title* that is short, but informative; useless or vague phrases such as “some remarks about” or “concerning” should be avoided. Although an abstract is not required upon initial submission, upon acceptance authors will be requested to supply an abstract for the electronic version of this journal. The AMS offers free worldwide access to the electronic abstracts. An abstract should be at least one complete sentence and at most 300 words. No abstracts appear in the printed journal starting in 1998. Included with the footnotes to the paper should be the 2010 *Mathematics Subject Classification* representing the primary and secondary subjects of the article. The classifications are accessible from [www.ams.org/msc/](http://www.ams.org/msc/). The Mathematics Subject Classification footnote may be followed by a list of *key words and phrases* describing the subject matter of the article and taken from it. Journal abbreviations used in bibliographies are listed in the latest *Mathematical Reviews* annual index. The series abbreviations are also accessible from [www.ams.org/msnhtml/serials.pdf](http://www.ams.org/msnhtml/serials.pdf). To help in preparing and verifying references, the AMS offers MR Lookup, a Reference Tool for Linking, at [www.ams.org/mrlookup/](http://www.ams.org/mrlookup/).

**Electronically prepared manuscripts.** Manuscripts should be electronically prepared in  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$ . To this end, the Society has prepared  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$  author packages for each AMS publication. Author packages include instructions for preparing electronic manuscripts, samples, and a style file that generates the particular design specifications of that publication series. Articles properly prepared using the  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$  style file and the `\label` and `\ref` commands automatically enable extensive intra-document linking to the bibliography and other elements of the article for searching electronically on the Web.

Authors may retrieve an author package for *Journal of the AMS* starting from [www.ams.org/jams/jamsauthorpac.html](http://www.ams.org/jams/jamsauthorpac.html). The *AMS Author Handbook* is available in PDF format from the author package link. The author package can also be obtained free of charge by sending email to [tech-support@ams.org](mailto:tech-support@ams.org) or from the Publication Division, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA. When requesting an author package, please specify the publication in which your paper will appear. Please be sure to include your complete email address.

**After acceptance.** The source files for the final version of the electronic manuscript should be sent to the Providence office immediately after the paper has been accepted for publication. The author should also submit a PDF of the final version of the paper to the Editor, who will forward a copy to the Providence office. Accepted electronically prepared manuscripts can be submitted via the Web at [www.ams.org/submit-book-journal/](http://www.ams.org/submit-book-journal/), sent via email to [pub-submit@ams.org](mailto:pub-submit@ams.org), or sent on CD to the Electronic Prepress Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA. When sending a manuscript electronically via email or CD, please be sure to include a

message indicating in which publication the paper has been accepted. Complete instructions on how to send files are included in the author package.

**Electronic graphics.** Comprehensive instructions on preparing graphics are available from [www.ams.org/authors/journals.html](http://www.ams.org/authors/journals.html). A few of the major requirements are given here.

Submit files for graphics as EPS (Encapsulated PostScript) files. This includes graphics originated via a graphics application as well as scanned photographs or other computer-generated images. If this is not possible, TIFF files are acceptable as long as they can be opened in Adobe Photoshop or Illustrator.

Authors using graphics packages for the creation of electronic art should also avoid the use of any lines thinner than 0.5 points in width. Many graphics packages allow the user to specify a “hairline” for a very thin line. Hairlines often look acceptable when proofed on a typical laser printer. However, when produced on a high-resolution laser imagesetter, hairlines become nearly invisible and will be lost entirely in the final printing process.

Screens should be set to values between 15% and 85%. Screens which fall outside of this range are too light or too dark to print correctly. Variations of screens within a graphic should be no less than 10%.

Any graphics created in color will be rendered in grayscale for the printed version unless color printing is authorized by the Managing Editor and the Publisher. In general, color graphics will appear in color in the online version.

**AMS policy on making changes to articles after publication.** Articles are published on the AMS website individually after proof is returned from authors and before appearing in an issue. To preserve the integrity of electronically published articles, once an article is individually published to the AMS website, changes cannot be made in place in the paper. The AMS does not keep author-related information such as affiliation, current address, and email address up to date after a paper is electronically published.

Corrections of critical errors may be made to the paper by submitting an errata article to the Editor. The errata article will be published electronically, will appear in a future print issue, and will link back and forth on the Web with the original article.

**Secure manuscript tracking on the Web.** Authors can track their manuscripts through the AMS journal production process using the personal AMS ID and Article ID printed in the upper right-hand corner of the Consent to Publish form sent to each author who publishes in AMS journals. Access to the tracking system is available from [www.ams.org/mstrack/](http://www.ams.org/mstrack/). An explanation of each production step is provided on the Web through links from the manuscript tracking screen. Questions can be sent to [jams-query@ams.org](mailto:jams-query@ams.org).

**Inquiries.** Any inquiries concerning a paper that has been accepted for publication that cannot be answered via the manuscript tracking system mentioned above should be sent to [jams-query@ams.org](mailto:jams-query@ams.org) or directly to the Electronic Prepress Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.





JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY  
CONTENTS

Vol. 31, No. 2

April 2018

Seung uk Jang and Junehyuk Jung, Quantum unique ergodicity and the number of nodal domains of eigenfunctions .....	303
Piotr Przytycki and Daniel T. Wise, Mixed 3-manifolds are virtually special .....	319
Seok-Jin Kang, Masaki Kashiwara, Myungho Kim, and Se-jin Oh, Monoidal categorification of cluster algebras .....	349
Massimiliano Gubinelli and Nicolas Perkowski, Energy solutions of KPZ are unique .....	427
Jason McCullough and Irena Peeva, Counterexamples to the Eisenbud–Goto regularity conjecture .....	473
Mark Gross, Paul Hacking, Sean Keel, and Maxim Kontsevich, Canonical bases for cluster algebras .....	497

Journal of the American Mathematical Society

VOLUME 31 NUMBER 2 PAGES 303–608 APRIL 2018



0894-0347(201804)31:2;1-6

