QUERIES—REPLIES

3. TABLES TO MANY PLACES OF DECIMALS (Q **1**; QR **1**, **2**).—When are many-place values needed? By the engineer or physicist using tables for a specific numerical problem, practically never. But by the *computer of new fundamental tables* not infrequently. They may be invaluable, or even indispensable, where the nature of the computing process employed is such that many decimal places are lost in the working; from the old trouble of "differences of nearly equal quantities," and particularly in the use of recurrence formulae. Some years ago, in order to settle a question as to the possibility of interpolating for fractional values of the order n of the Bessel function $J_n(x)$ (considered as a continuous function of the order n for a fixed parameter x), *I* computed from series, to 7D, $J_n(0.1)$ and $J_n(1.0)$ for $n = 0(0.01)3$. This table can be interpolated with 6th differences. It can be extended for $n > 3$ by the recurrence formula

$$J_{n+1}(x) = (2n/x)J_n(x) - J_{n-1}(x),$$

but repeated use of this formula loses decimal places; notably so in the case $x = 0.1$ we lose at least one place at every application.

While the method is inappropriate for finding a single value, it is comparatively rapid for forming a whole table. If such a table were required up to $n = 20$ say, it would probably pay to compute the fundamental 20 or 30 values to 25 or 30 decimals, in order to make sure of accuracy in the 6th or 7th decimal after employing the recurrence formula twenty times. It would then be necessary to compute the Gamma function to "many places."

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4. TABLES TO MANY PLACES OF DECIMALS (Q **1**; QR **1**, **2**, **3**).—The evaluation of integrals by means of reduction formulae and series provides a further illustration in answer to this query. When an integral of type

$$\int_0^{\pi/4} f(\tan a)da$$

is evaluated by expanding $f(x)$ in a Maclaurin series, and integrating term by term, it is necessary to use accurate values of the integrals of this type in which $f(x) = x^n$. These integrals may be found by means of a well known reduction formula which reduces them to the cases $n = 0$ and $n = 1$. The first of these is $\frac{1}{4}\pi$ and so when n is large the value of π is needed to a large number of places for the value of the integral is equal to the difference between $\frac{1}{4}\pi$ and a quantity which is very nearly equal to $\frac{1}{4}\pi$. When $n = 1$ the integral depends on log 2 and so this number is also needed very accurately. Though the integral for a large value of n is small this integral may have a large coefficient when use is made of the Maclaurin expansion. There are innumerable cases in which the use of a reduction formula is at present the only practical means of computing an integral and so it is desirable that there should be very accurate tables of some of the functions needed for the expression of standard types of indefinite integral. In addition to the logarithm, the inverse tangent, inverse sine, inverse hyperbolic functions, integral sine and cosine, the incomplete gamma function, dilogarithm and various integrals of Bessel functions, are of frequent occurrence.

H. B.

Attention is directed to Mr. Bateman's article "Occasional need for very accurate logarithms," *Amer. Math. Mo.*, v. 32, 1925, p. 249.—EDITOR.

5. SCARCE MATHEMATICAL TABLES (Q **2**).—Since this query was sent to the printer I find that three copies of item *E*, [H. GOODWYN], *A Table of Circles*, 1823, are listed in catalogues of Edinburgh libraries, namely:

(a) *Catalogue of the Printed Books in the Library of the Faculty of Advocates*, Edinburgh and London, v. 3, 1874;

(b) *Catalogue of the Crawford Library of the Royal Observatory*, Edinburgh, 1890;

(c) *Catalogue of the Printed Books in the Library of the University of Edinburgh*, Edinburgh, v. 2, 1921.

Mr. C. R. COSENS (see above) has reported to me that a fourth copy is in the Library of the University of Cambridge. This copy, together with the other Goodwyn publications, including those mentioned *MTAC* 1, p. 22, are preserved with the following letter: "Miss Catherine Goodwyn presents to the Library of the University of Cambridge a complete set of the works of her late father, Henry Goodwyn, Esq. of Blackheath, Kent." "Royal Hill, Greenwich, Sept. 16, 1831."

While I have not yet found a library with items *C* and *D* I did find a third enlarged and greatly improved edition of these items in the Library of Massachusetts Institute of Technology with the following title: *Hütte Hilfstafeln zur I. Verwandlung von echten Brüchen in Dezimalbrüche, II. Zerlegung der Zahlen bis* 10 000 *in Primfaktoren. Ein Hilfsbuch zur Ermittelung geeigneter Zähnezahlen für Räderübersetzungen. Herausgegeben vom Akademischen Verein Hütte E. V. Berlin.* Dritte neubearbeitete Auflage. Berlin, 1922, vi, 83 p. 11.9 × 18.6 cm. Table I, the "Brocot" table, occupies p. 23–62; and Table II, factor table, p. 63–83; see RMT **87**, p. 21–22. What is especially interesting about this edition is that both of the tables were thoroughly checked by J. T. PETERS.

R. C. A.

CORRIGENDA

P. 26, MTE **4**, the editor regrets that ll. 3–6 give a wrong impression in the following three respects: (1) it should have been stated that l. 3 was an error printed on a slip pasted in the fourth edition before distribution; (2) l. 4, *for* "first," read "third" (as indicated in RMT **82**); (3) after the correct information for ll. 4–5 had been supplied by L. J. C. his signature was forged by the editor.

P. 33, for ll. 11–15, substitute the following:

1450 in a Latin codex in Munich[1], compiled by Theodericus Ruffi. The idea of the centesimal division of the degree was mainly suggested to Henry Briggs as the appropriate unit for his wonderful "sine canon"[2] (see RMT **79**), by Vieta's *Calendarij Gregoriani*, Paris, 1600, folio 29 (*Opera Mathematica*, Leyden, 1646, p. 487.)

P. 39, l. 3, *for* Andoyer items, *read* Andoyer item

P. 45, l. 12, *for* **$5.00**, *read* **$2.50**

P. 56, *for* l. 7, *read* $\ln 71 = 2 \ln 2 + \ln 3 + (\ln 5 + \ln 7)/2 + S\,(1/10081)$

P. 57, l. 4, *for* i–xxviii, *read* I–XXVIII

P. 58, heading last column, "abbreviation of"