

*S* and *T* tables, with the sexagesimal second as unit, appeared already in the first stereotyped edition of

(13s). F. CALLET, *Tables Portatives de Logarithmes*, Paris, 1795. In his *Report . . . on Mathematical Tables*, London, 1873, J. W. L. Glaisher states (p. 54), "Tables of *S* and *T* are frequently called, after their inventor, Delambre's tables." In a letter of C. M. Merrifield written to Glaisher in 1873, listing matters he wishes to bring to his friend's attention, he notes "the so-called Delambre's tables of  $\log(\sin x/x)$  and  $\log(x/\tan x)$  really John Newton in 1658." I have examined Newton's *Trigonometria Britanica (sic)*, of 1658, but as yet I have found no printed *S* or *T* tables before 1795. Delambre's dates are 1749–1822. We have already referred to the manuscript *S* and *T* tables of the *Tables du Cadastre (MTAC)*, p. 34 possibly dating from 1792 or 1793.

*S* and *T* "are required for passing from log arc to log sin and log tan, and are of particular value in geodetic calculations, where long operations have sometimes to be performed with small arcs which are usually expressed in seconds, while four or five places of the second have to be retained" (3s).

R. C. A.

MATHEMATICAL TABLES—ERRATA

8. France, Service Géographique de l'Armée, *Tables des Logarithmes à huit Décimales des Nombres entiers de 1 a 120000 et des Sinus et Tangentes de dix Secondes en dix secondes d'Arc dans le Système de la Division Centésimale du Quadrant*. Paris, 1891. Compare *MTAC*, p. 36.

In the differences and proportional parts which correspond to  $\text{Log cos } 4^{\circ}75'$  to  $5^{\circ}00'$ ,

for				read					
	49	48	47	46		51	52	53	54
1	4.9	4.8	4.7	4.6	1	5.1	5.2	5.3	5.4
2	9.8	9.6	9.4	9.2	2	10.2	10.4	10.6	10.8
3	14.7	14.4	14.1	13.8	3	15.3	15.6	15.9	16.2
4	19.6	19.2	18.8	18.4	4	20.4	20.8	21.2	21.6
5	24.5	24.0	23.5	23.0	5	25.5	26.0	26.5	27.0
6	29.4	28.8	28.2	27.6	6	30.6	31.2	31.8	32.4
7	34.3	33.6	32.9	32.2	7	35.7	36.4	37.1	37.8
8	39.2	38.4	37.6	36.8	8	40.8	41.6	42.4	43.2
9	44.1	43.2	42.3	41.4	9	45.9	46.8	47.7	48.6

	for	read
Log Sin	4°65'40"	$\bar{2}.86355936$
Log Tan	4 65 40	$\bar{2}.86472090$
Log Cot	4 65 40	1.13527910
Log Cot	34 53 60	0.21981237
Log Cos	41 28 80	$\bar{1}.90143668$

J. DE MENDIZÁBEL TAMBORÉL, Sociedad Científica "Antonio Alzate," Mexico, *Revista*, v. 5, p. 9–10, 1891.

9. Authors of frequently used works in the field of Statistics display some carelessness in the preparation of tables they publish. Here are a few illustrations (an asterisk \* denotes an exact result):

R. A. FISHER and F. YATES, *Statistical Tables for Biological, Agricultural and Medical Research*, Edinburgh, 1938. P. 33,  $n_1 = n_2 = 2$ , for 99.01, read 99.00\*; and  $n_1 = 2, n_2 = 3$ , for 30.81, read 30.82. The same mistakes occur in

G. W. SNEDECOR, *Statistical Methods applied to Experiments in Agriculture and Biology*, Ames, Iowa, Collegiate Press, third ed., 1940, p. 184. On this same page (through  $n_2 = 13$ ) are at least 53 other last figure errors of 1 to 3 units, which suggest that there may be 200 errors on the 4 pages of this table of 5% and 1% points for the *F* distribution. Five of these

53 errors occur also in FISHER and YATES. The careful worker will naturally hereafter turn to such tables as reviewed in RMT 102.

F. E. CROXTON and D. J. COWDEN, *Applied General Statistics*, New York, Prentice Hall, 1939, p. 878, has the following errors:

$n_2$	for	.05	read	for	.01	read	for	.001	read
1				4999.0		4999.5*			
2	18.999		19.00*	99.008		99.00			
3				30.815		30.817			
4	6.945		6.944	18.001		18.000*	61.238		61.246

R. C. A.

Other errors in FISHER and YATES are as follows:

P. 15, l. 10,  $A = \bar{y}$ , not  $\bar{x}$ .

P. 28, footnote to table, the formula should read,

$$z \text{ (20 percent)} = \frac{0.8416}{\sqrt{h-1}} - 0.4514 \left( \frac{1}{n_1} - \frac{1}{n_2} \right).$$

P. 42, Table XII, the entry for  $p = 72$  should be 58.1 not 58.7.

P. 48, l. 1, solution 16, the letter  $e$  in block 2 is blurred; the block letters are *adefj*.

W. G. COCHRAN

Yet other slips in FISHER and YATES are as follows:

P. 8, l. 10, for "ordinate is  $\frac{1}{2}\text{sech}^2z$ ," read "ordinate is  $\frac{1}{2}\text{sech}2z$ ."

P. 57, Table XXIII,  $n = 39$ , bottom of col. 2, for 496,388, read 4,496,388.

GERTRUDE M. COX

Univ. North Carolina,  
Raleigh, N. C.

10. LEO HUDSON, and E. S. MILLS, *Natural Trigonometric Functions Tables. Sine, Cosine, Tangent, Cotangent, Secant, and Cosecant to Eight Decimal Places. With Second Differences to ten Decimal Places, Semi-quadrantly arranged.* 1941; see RMT 80.

The sines and cosines given in this table were checked against the values appearing in the Coast and Geodetic Survey Table (see RMT 77). In the case of a discrepancy Peters' *Eight-figure Table* was referred to (see RMT 78), and finally the function was calculated to fifteen places by using Peters' *Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus* (Berlin, 1911). In this way 32 last-figure sine errors were found. One of these, at  $0^\circ 02'$ , where the eighth figure should be increased by two units, was indicated on an errata slip in the volume. The other 31 errors were of a unit in the eighth place. The first 8 cases where the eighth digit should be diminished by 1 are listed below and then the 23 cases calling for an increase by unity.

(-1)  $13^\circ 31'$ ,  $27^\circ 08'$ ,  $27^\circ 21'$ ,  $27^\circ 24'$ ,  $43^\circ 25'$ ,  $55^\circ 09'$ ,  $77^\circ 18'$ ,  $84^\circ 03'$ .

(+1)  $1^\circ 15'$ ,  $2^\circ 10'$ ,  $2^\circ 25'$ ,  $8^\circ 43'$ ,  $9^\circ 35'$ ,  $11^\circ 39'$ ,  $11^\circ 42'$ ,  $12^\circ 45'$ ,  $18^\circ 32'$ ,  $33^\circ 00'$ ,  $34^\circ 36'$ ,  $38^\circ 49'$ ,  $39^\circ 53'$ ,  $40^\circ 59'$ ,  $42^\circ 17'$ ,  $51^\circ 08'$ ,  $54^\circ 55'$ ,  $60^\circ 33'$ ,  $67^\circ 05'$ ,  $67^\circ 25'$ ,  $71^\circ 05'$ ,  $80^\circ 48'$ ,  $88^\circ 24'$ .

In comparing the column "Diff. per second" with  $1/60$  of the differences per minute of eleven-place functions interpolated from Peters' 21-place values, it is noted that the last figure of the printed difference is totally unreliable; from  $0^\circ$  to  $1^\circ$  it is wrong in 27 cases; from  $1^\circ$  to  $2^\circ$ , it is wrong in 30 cases; and from  $2^\circ$  to  $3^\circ$  in 13 cases. It is obvious therefore that the sines and cosines of this table are not to be relied on for more than seven-place accuracy, especially after using these differences with linear interpolation. Computation to "ten decimal places" is wholly out of the question. In making a test with the thought of using this table for seven-place work instead of such tables as Benson (RMT 75) or Ives (RMT 76), it was found that, after setting up a routine, it is possible, when interpolating to hundredths of a second, to save nearly 25% of the time used in locating the function in Benson to the nearest ten seconds and then interpolating. Tangents and secants have not yet been checked.

It seems rather a shame that anyone should have put in the enormous amount of time and energy required to compute these values from a series, and not attain the accuracy that was already available in Peters' *Eight-figure Table*, 1939, or in an abridgement of Andoyer, 1916. It would not be much of a job to compute the differences per second to six significant figures instead of five, using ten-place functions, interpolated either from Peters or Andoyer or Pitiscus, which would make the table far more valuable than it is in its present state.

F. W. HOFFMAN,  
689 East Avenue, Pawtucket, R. I.

A comparison of Legendre's table of sines, to 15D, for each 15' (*Traité des Fonctions Elliptiques*, v. 2, Paris, 1826, p. 252–255), readily revealed not only three of the errors noted by Mr. Hoffman, but also a similar error for 48°15'. Comparison with Andoyer's table of sines and cosines to 15D (*Nouvelles Tables Trigonométriques Fondamentales, Valeurs Naturelles*, v. 1, Paris, 1915) showed that for the sines the eighth place values should also be increased by unity at 10°31' and 32°36'. In the C.G.S. table (which Mr. Hoffman used for comparison) there were also errors in the three new cases noted.

R. C. A. and D. H. L.

11. J. Y. DREISONSTOK, *Navigation Tables for Mariners and Aviators* (H. O. no. 208), sixth ed., 1942; see RMT 103.

Tables I and IA of this volume have been recomputed at the Ladd Observatory, using 7-place logarithms and punched cards in Hollerith Machines. The comparison between values given in H. O. 208 and the newly computed values is complete only for A and C.

In table I, 1858 errata were found in A, of which 158 were of two or more units in the last place given. In table IA, 426 errata in A were noted, 9 of two or more units. 345 errata in C were located in table I, 8 of two or more units; 263 errata in C were found in table IA, 28 of two or more units in the last place given.

Thus a total of 2892 errata have been noted in A and C, 203 of which are of two or more units in the last place given. The largest error in A was 26 units in the last place; the largest in C 20 units. The largest error in a computed altitude resulting from one of these errata would be about 4.4 minutes of arc, with a corresponding error of position of 4.4 nautical miles. This largest error would probably not occur in ordinary navigation; it represents a theoretical maximum.

The list of 203 errata of two or more units in the last place are given below.

L	t	A should be	L	t	A should be	L	t	A should be	L	t	A should be	L	t	A should be	
1°	75°	58608	3°	89	140877	5°	73	51710	8°	83	76521	10°	75	51086	
	77	64667		66	38771	76	59129	84		79574	83		70469	80	61311
	78	68066		67	40485	87	99328	87		87786	86		76717	81	63426
	80	75821		77	63703	88	102764	78		60465	88		79537	82	65517
	81	80305		78	66934	89	105123	79		62953	89		80305	83	67554
	84	97486	80	74200	6°	68°	41236	80	65517	10°	75	51086			
	85	105123	82	82825	72	48863	81	68142	80		61311				
	86	114329	83	87786	77	60742	84	76083	81		63426				
	87	125836	84	93267	79	66424	86	80862	82		65517				
	88	140877	85	93288	80	69493	87	82825	83		67554				
2°	89	160767	89	125836	82	76083	88	84345	9°	64	33690	10°	84°	69493	
	68	42481	4°	73	52304	83	79574	65		35089	86		72876		
	71	48514	75	57276	84	83144	70	42914		87	74200				
	76	61211	76	59996	87	93267	71	44661		88	75199				
	80	75199	79	69310	89	97486	89	97486		89	75821				
	81	79537	80	72876	7°	65	35970	72	46475	11°	68	38270			
	82	84345	81	76717	72	48144	77	56586	78		55378				
	85	102764	82	80862	74	52360	78	58813	81		61096				
	86	110812	88	110812	80	67554	79	61096	82		62953				
	88	130680	89	114329	81	70469	80	63426	84		66424				

<i>L</i>	<i>t</i>	<i>A</i> should be	<i>L</i>	<i>t</i>	<i>A</i> should be	<i>L</i>	<i>t</i>	<i>A</i> should be	<i>L</i>	<i>t</i>	<i>C</i> should be	<i>L</i>	<i>t</i>	<i>C</i> should be	
12°	86	69310	17°	83	52360	24°	87	38771	9°	86°	6	86	19	1644	
	74	46792		88	55647	25°	81	35089	29	42	233	86	78	1166	
	76	50169		81	48357		83	35970	31	11	786	87	1	3039	
	79	55378		85	51710	29°	85	30913	42	83	132	87	4	2438	
	81	58813		86	52304	30°	74	25644	44	84	145	88	4	2614	
13°	82	60465	18°	69	33719	38°	78	19580	55	18	751	88	5	2517	
	87	66934		80	45546	42	65	13126	58	8	1132	89	1	3516	
	89	68066		81	46475	52	78	9781	60	4	1457	89	3	3039	
	69	38157		83	48144	66	76	3677	76	14	1233	89	4	2915	
	81	56586		84	48863	67	16	253.3	79	15	1306	89	5	2818	
14°	84	60742	18°	88°	50753	67	58	2526	79	17	1253	89	6	2739	
	87	63703	19°	81	44661	71	59	1761	82	1	2615	89	8	2615	
	89	64667		87	48240	71	78	2322	84	1	2739	89	10	2518	
	78	50169		88	48514	73	62	1498	85	1	2818	89	15	2345	
	82	55743	20°	74	36751	77	6	12.0	85	2	2517	89	20	2224	
15°	85	59129	21°	81	42914	77	86	1122	85	66	1099	89	58	1830	
	86	59996		72	33719	81	14	31.1	86	1	2915				
	88	61211		77	38157				86	2	2614				
	70	37708		83	42482				86	3	2438				
	78	48466		89	44522				86	17	1690				
16°	80	51086	22°	79	38270										
	86	57276		84	41236										
	89	58608		88	42481										
	70	36751		23°	85	39916									
	78	46792			87	40485									

C. H. SMILEY

12[A, D, P].—EARLE BUCKINGHAM, *Manual of Gear Design. Section one. Eight Place Tables of Angular Functions in Degrees and Hundredths of a Degree and Tables of Involute Functions, Radians, Gear Ratios, and Factors of Numbers.* New York, Machinery, 1935. 183 p. 21.2 × 27.9 cm. \$2.50.

No explanation of any kind is given of the sources, construction or checking of these tables; letters to the author asking for information have been ignored. Hence a thorough examination was necessary in order that their value could be appraised.

Pages 8–97 give 8-figure values of sine, cosine, tangent and cotangent at interval 0°.01 (= 36''). This section has been compared by Mr. Sidney Johnston with every 36th value in Peters' *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadranten*, Berlin, Reichsamt für Landesaufnahme, 1939. The corrections thus found were then confirmed by the present writer from Briggs' *Trigonometria Britannica* (1633), and afterwards analyzed to discover the mode of preparation.

All errors in the sines and cosines greater than 0.55 units in the eighth decimal are shown in Table I. The error is in units of the last decimal, in the sense True Value minus Buckingham. Of the remaining 96 end-figure errors, 7 are cases where Buckingham's value is too high, but only by a turn of the figure, 4 are too low by about 0.54 units, while the remaining 85 are too low by amounts that vary between 0.50 and an upper limit that increases steadily to 0.53 as  $\sin x$  increases from 0 to 1. The distribution of errors in  $\sin x$  at intervals of 30° is shown alongside, treating cosines as the sines of their complementary angles. Not one of the 96 end-figure errors occurs in an angle that is a multiple of 0°.05. A comparison with Gifford's *Natural Sines to every Second of Arc and Eight Places of Decimals* shows that those tables have not been the source of the values before us; and obviously they have not come from the *Trigonometria Britannica*, which would have been by far the best source.

The explanation is that a table at interval 10'' has been used. This yields values at interval 0°.05 directly, while the remaining values have been formed by linear interpolation between the appropriate 10'' values. The maximum effect of neglecting second differences

(in sines) is  $0.03 \sin x$  units of the eighth decimal and so varies from 0 at  $x = 0^\circ$  to  $0.03$  at  $x = 90^\circ$ —precisely what we found after eliminating 23 (Table I) + 7 + 4 = 34 values that appear to be attributable to lack of care in handling end figures. This accounts also for the observed increasing frequency as  $x$  increases. The only natural tables at interval  $10''$  are the *Opus Palatinum* of Rheticus (1596), the *Thesaurus Mathematicus* of Pitiscus (1613, but computed by Rheticus), and Andoyer's *Nouvelles Tables Trigonométriques Fondamentales*, Paris, Hermann, 1915. Had the former been used, it is certain that there would have been many more errors, as every table based on Rheticus contains errors that can be traced to his tables. Actually none of the errors in Tables I and II are due to the *Opus Palatinum*, which, in all the multiples of  $10''$  bordering the values in these two lists, never differs from Andoyer by more than one unit in the tenth decimal. We may take it, therefore, that Andoyer has been used.

There is only one serious error in the tangents, namely on page 20,  $\tan 6^\circ.40$ , where for  $0.11226\ 797$  we must read  $0.11216\ 797$ . Twelve end-figure errors greater than  $0.60$  units of the eighth decimal are given in Table II. Besides these there are 126 cases in which the error does not exceed  $0.60$  units, and so may be considered negligible in computations. In four of these Buckingham's value is too low, while in the remaining 122 (analyzed alongside) it is too high. Here again the total number of errors, and their increasing frequency as  $x$  increases, correspond as nearly as possible to our expectation if 10-figure values at interval  $10''$  were interpolated linearly. Three of these errors occur where  $x$  is a multiple of  $0^\circ.05$ , namely  $2^\circ.20$ ,  $16^\circ.50$  and  $16^\circ.55$ . An examination of Andoyer showed that these were three of the nine cases in which the ninth and tenth decimals are 50. In eight of these the eighth decimal has been rounded up—in five cases correctly, but in the three cases under review incorrectly. In the ninth case ( $23^\circ.50$ ) the eighth decimal has been correctly rounded

down. In all nine cases, the *Opus Palatinum* values also end in 50.

The cotangents present an interesting problem to the "error-analyst." Table III lists 27 cases in which the error is greater than a unit of the last decimal. The table alongside gives an analysis of errors not exceeding  $\pm 1.6$  units. The reason for the break at  $5^\circ.71$  is that the number of decimals increases at that point from 6 to 7. The reason for the second break will appear later. It will be realized that errors of  $0.5$  include those from  $0.50$  to  $0.55$  only. Bearing this in mind, the distributions (taking positive and negative frequencies separately) are approximately gaussian. After  $18^\circ.50$  Peters gives 8 decimals, and Buckingham 7, so this portion was also read against Briggs-Gellibrand in order to detect errors between  $0.50$  and  $0.55$  units. In no case is a cotangent that is a multiple of  $0^\circ.05$  in error, even by a turn of the figure.

The five errors marked with an asterisk could all have been easily detected by writing second differences, since first differences are already given in the tables. There can be no excuse for the neglect of this simple and elementary table-maker's precaution. It appears probable that the first three of these errors have arisen from confusion in copying. Thus:

- $2^\circ.27\ 227227$  has been copied for  $227224$
- $2^\circ.29\ 006670$  has been copied from  $00666696$
- $3^\circ.24\ 665099$  has been copied from  $66502899$

We are faced then with the fact that up to  $17^\circ$  Buckingham's tendency is to be too low, and from that point too high. The sudden switch-over at  $17^\circ$  is even more apparent from the full list of errors than it is from the summary given above. The table below gives observed and theoretical maxima and frequencies.

Error	$0^\circ.00$ to $5^\circ.71$	$5^\circ.72$ to $17^\circ.00$	$17^\circ.00$ to $45^\circ.00$
$\geq +1.0$		16	
+0.9	1	8	
+0.8	2	11	
+0.7	1	17	1
+0.6	5	32	1
+0.5	15	29	2
-0.5	4	9	64
-0.6	3	9	33
-0.7	1	1	2
-0.8		1	1
-0.9		4	
$\leq -1.0$	3	2	1
	<u>35</u>	<u>139</u>	<u>105</u>

$x$ °	Maximum obs.	comp.	obs.	No. in range comp.	o-c
17	0.79	0.74	27	34	-7
20	0.66	0.63	27	32	-5
25	0.54	0.57	21	17	+4
30	0.53	0.54	13	10	+3
35	0.52	0.52	7	7	0
40	0.52	0.52	5	5	0
45	0.51	0.52	100	105	-5

The theoretical maximum error resulting from linear interpolation of values at interval  $10''$  is, in units of the seventh decimal,

$$0.50 + 2 \cot x \operatorname{cosec}^2 x \operatorname{arc}^2 10'' \times 0.12 \times 10^7$$

in which 0.12 is the coefficient of the second difference for 0.4 and 0.6. The computed frequency is found with the aid of the difference  $\Delta'$  (taken positively) of  $\operatorname{cosec}^2 x$ , and is

$$\Delta'(\operatorname{cosec}^2 x)10^7 \operatorname{arc}^2 10'' \times 0.1 \times 80 \times 57.3 = 10.8 \Delta'(\operatorname{cosec}^2 x)$$

where

0.1 is average coefficient of second difference

80 is number of interpolates per degree.

The agreement between observation and theory supplies ample confirmation of the hypothesis of Buckingham's use of Andoyer and linear interpolation. Actually this section of the table is by far the most accurate, and has quite likely been done by a different computer.

It remains to account for the predominantly *low* values up to  $17^\circ$ . They have evidently been formed, not by interpolating Andoyer, but by taking the reciprocals of tangents formed by linear interpolation. These tangents would tend to be too *high* (as we found) and their reciprocals too *low*. The fact that a small number are slightly too high may possibly be the result of rounding off the tangents to nine decimals before reciprocating. The maximum error of such a procedure would be, in units of the seventh decimal,  $0.006 \cot x \pm 0.005 \cot^2 x$ , apart from any error arising from the final rounding off, i.e., a possible  $\pm 0.5$ . This gives errors up to  $+1.1$  and  $-0.9$  at  $5^\circ.7$ ,  $+0.70$  and  $-0.62$  at  $10^\circ$ , and  $+0.57$  and  $-0.53$  at  $17^\circ$ . Actually the observed errors exceed these limits.

In six cases up to  $17^\circ$  the tangents and cotangents of the same angle are in error. In each case the errors are of opposite sign, the most striking examples being at  $4^\circ.59$ ,  $14^\circ.88$  and  $14^\circ.92$ .

One interesting fact emerges. In Andoyer's tangents the sixth decimal is one unit too low at  $12^\circ 15' 10''$ ,  $20''$ ,  $30''$  and  $40''$ , but Buckingham is correct at  $12^\circ.26$ . Similarly the sixth decimal of Andoyer's value of  $\cot 40^\circ 43' 20''$  is a unit too high, but Buckingham is correct at  $40^\circ.72$ . The errors would, of course, be evident from the differences, since Buckingham's tables are linear at these points.

The above somewhat lengthy analysis affords an excellent example of the way in which errors in tables give clues to the sources from which the tables are derived, and the methods used in computing them. It is, however, far better that the author himself should give this information.

Page 98 is a table for converting minutes into decimals of a degree. In every case where the end figure is 6, it should be 7.

Pages 100-129 give the function involute  $x$  or  $\tan x - x$  at interval  $0^\circ.01$  up to  $60^\circ$ . 12 decimals are given up to  $0^\circ.5$ , 10 to  $1^\circ$ , 8 to  $37^\circ$ , and 7 to  $60^\circ$ . This table was examined by Mr. S. Johnston, by using the relation  $\tan x - \operatorname{inv} x = x$ , a process that would not detect errors of a unit or less in the last decimal. Some further examination was also made by Mr. J. C. P. Miller. Apart from a trivial omission of leading figures at  $6^\circ.03$ , there are two errors:

Page 104 inv  $9^\circ.15$  for 6160 read 7160

Page 106 inv  $13^\circ.01$  for 8468 read 8470

Pages 132-146 give, to 8 decimals, the radian equivalent of  $0^\circ(0^\circ.01)45^\circ$ . It should, of course, have been formed by taking multiples of  $1^\circ = 0^\circ.01745 32925 19943$ . Actually multiples of  $0.01745 329$  were first taken up to  $18^\circ$ . At this stage comparison with  $\pi/10$

showed a defect of just over  $4\frac{1}{2}$  units in the eighth decimal. Instead of tracing the cause, and correcting it, the values at  $17^{\circ}\cdot97$ ,  $17^{\circ}\cdot98$ ,  $17^{\circ}\cdot99$  and  $18^{\circ}\cdot00$  were "fudged" by increasing them by 1, 2, 3 and 4 units respectively in the last decimal, to prevent a sudden discontinuity in the differences. Thereafter increments of 0.00017 45329 were again added, up to the end of the table at  $45^{\circ}$ , where the error has risen to more than seven units. Thus the value given for  $45^{\circ}$  is 0.78539 809, whereas  $\pi/4$  is 0.78539 81634. It would be difficult to match this example of incompetent table-making.

Pages 148–169, described as "Brocot's Tables of Gear Ratios," give, to eight decimals, the values of all proper fractions (in their lowest terms) whose denominators do not exceed 120. Table IV gives a list of 27 errors found by Mr. S. Johnston and by Mr. J. C. P. Miller, who made a partial examination. Eight of these are of a unit in the last figure, and are thus of no engineering consequence. Every one of these errors and omissions could easily have been detected by the simple process of seeing that  $N/D$  and  $(D - N)/D$  added precisely to 1. It was observed that, with one exception (argument 44/87 for 44/47),  $D$  is always greater than 60 in the error list. As *Machinery's Handbook* gives a similar table, also called Brocot's, but with  $D$  not exceeding 60, the inference is that Buckingham computed the values for  $D$  greater than 60, and introduced the errors now found. Compare RMT 87.

Pages 172–183 give all the factors of *all* numbers up to 6009. A comparison by Mr. S. Johnston with the Br. Ass. Adv. Sci., *Mathematical Tables*, v. 5. (London, 1935) showed that two numbers on page 182 given as primes are really composite, namely  $5183 = 71\cdot73$  and  $5461 = 43\cdot127$ . These errors do not occur in any other table that I know.

TABLE I—Sines and Cosines

page	col.	$x$	for	read	error
8	sin	0.02	906	907	+0.6
8	cos	0.04	975	976	+0.6
10	cos	1.43	857	856	-1.0
11	sin	1.54	484	483	-0.6
15	cos	3.64	264	265	+0.9
20	cos	6.27	827	828	+0.6
21	cos	6.71	026	027	+1.2
22	cos	7.46	576	574	-1.6
25	sin	8.52	462	463	+1.4
26	sin	9.04	397	396	-0.7
27	sin	9.96	060	061	+1.0
34	sin	13.36	871	872	+1.1
42	sin	17.06	299	298	-0.6
47	cos	19.76	704	702	-1.7
54	sin	23.25	387	386	-1.4
79	cos	35.87	856	855	-0.7
80	cos	36.31	495	494	-0.6
81	cos	36.56	353	352	-0.7
85	sin	38.99	475	474	-0.6
89	cos	40.77	709	708	-0.8
89	cos	40.87	618	619	+0.7
92	sin	42.11	610	611	+0.8
97	sin	44.76	867	866	-0.6

TABLE II—Tangents

page	$x$	for	read	error
12	2.27	971	972	+0.6
17	4.59	242	243	+0.9
29	10.59	428	427	-0.6
37	14.88	569	568	-0.8
37	14.92	323	324	+1.0
45	18.83	216	215	-0.8
45	18.84	700	699	-0.7
62	27.49	523	524	+1.1
75	33.79	894	893	-0.9
81	36.98	692	691	-0.7
85	38.82	553	552	-0.7
87	39.66	736	735	-0.6

TABLE III—Cotangents

page	$x$	for	read	error
12	2.07	078	077	-1.3
12	2.27	227	224	-3.2*
12	2.29	670	667	-3.0*
14	3.24	099	029	-7.0*
14	3.42	251	250	-1.0
15	3.51	153	158	+5*
17	4.59	027	026	-1.3
21	6.66	885	886	+1.4
21	6.74	093	095	+2.4*
21	6.87	905	906	+1.2
23	7.88	360	361	+1.1
23	7.89	621	622	+1.0
24	8.31	853	854	+1.2
24	8.39	749	750	+1.0
25	8.64	140	141	+1.1
26	9.27	493	495	+1.6
26	9.28	305	307	+1.6
26	9.43	438	437	-1.4
28	11.19	992	993	+1.4
31	11.82	967	968	+1.0
34	13.24	731	730	-1.2
34	13.41	132	133	+1.1
35	13.52	037	038	+1.1
37	14.88	626	627	+1.2
38	15.36	019	020	+1.2
40	16.18	125	126	+1.0
94	43.36	504	503	-1.4

\* These values, and the corresponding differences, should be corrected, even if the remaining errors are considered negligible.

TABLE IV—Brocot's Tables of Gear Ratios

page	<i>N</i>	<i>D</i>	for	read	authority
148	3	106	.02830 187	... 189	J
148	3	86	Omitted	.03488 372	M; J
148	3	70	Omitted	.04285 714	M; J
149	7	79	.08860 760	... 759	J
150	9	67	.13432 856	... 836	J
151	13	72	.18055 555	... 556	J
152	22	101	.21789 178	.21782 178	J
153	28	103	.27184 465	... 466	J
154	32	113	.28318 585	... 584	J
155	38	107	.35513 919	.35514 019	J
156	33	85	.38823 530	... 529	J
156	46	117	.39316 293	... 239	J
156	41	[100]	<i>D</i> = 00	<i>D</i> = 100	M
157	41	97	.42268 042	... 041	J
157	46	103	Omitted	.44660 194	M; J
157	31	72	.43055 555	... 556	J
157	39	79	.49367 087	... 089	J
157	50	101	.49504 951	... 950	J
159	41	79	Omitted	.51898 734	M; J
161	67	105	.63803 524	.63809 524	J
162	59	89	.66291 135	.66292 135	J
165	62	79	.78481 083	... 013	J
166	61	73	.83561 484	... 644	J
166	98	117	.83760 601	... 684	J
167	95	[119]	<i>D</i> = 119	<i>D</i> = 109	M
168	44	[87]	<i>D</i> = 87	<i>D</i> = 47	M
168	107	112	.95535 710	... 714	J

L. J. C.

In *Math. Gazette*, v. 26, Dec. 1942, p. 226–230, J. C. P. Miller has an article entitled “The decimal subdivision of the degree,” which is also a review of Buckingham’s book. Many of the facts stated above were first published in this article; for example, besides the 7 errors credited to *M* in Table IV, 12 more of the others were also published in his own review.—EDITOR.

## UNPUBLISHED MATHEMATICAL TABLES

In *MTAC*, p. 27, we referred to an unpublished ms. of the late A. J. C. CUNNINGHAM giving the complete factorization of  $n^2 + 1$  for  $1 \leq n \leq 15,000$ . Through L. J. Comrie we were informed by a letter, dated 5 May 1943, from A. E. Western, custodian of the Cunningham mss. of the London Mathematical Society, that this ms., as well as others, and many of the Society’s books, housed in the library of University College, London, were destroyed by an enemy air raid.

4[L].—PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, *Spherical Bessel Functions*. Ms. in possession of the Project.

The Spherical Bessel Functions

$$Q_n(x) = \sqrt{\frac{\pi}{2x}} J_{n+1/2}(x)$$

occur in a wide variety of problems of wave motion, potential theory, heat conduction and quantum mechanics. The Project’s preliminary manuscript is of the functions  $Q_n(x)$  for  $n = 0, \pm 1, \pm 2, \dots, \pm 21$  and  $x = [0(0.01)10; 8S-10S]$ , with second and fourth central differences. It is contemplated to extend this table for values of  $n$  ranging from  $-20$  to  $-35$ ,  $n = 20$  to  $n = 35$  and for  $x = [10(0.1)30; \text{about } 7S]$ .