120[L, S].—M. KOTANI, A. AMEMIYA and T. SIMOSE, I. "Tables of integrals useful for the calculations of molecular energies," Phys. Math. So., Japan, *Proc.*, s. 3, v. 20, 1938, extra no. 1, 70 p. II. *idem.*, v. 22, 1940, extra no. 1, 28 p. 18.8 × 25.8 cm.

To some extent the Tables of I form a continuation of the work of S. Ikehara who compiled the tables of the functions of Zener and Guillemin [1] used in the work of Rosen.[2] The tables were used by T. Invi in his work [3] on the hydrogen molecule and he cooperated with the authors. He says that the integrals occurring in his work can be reduced to some basic integrals tabulated by Bartlett, Furry and Rosen [4] but their tables are not sufficiently extensive. Values for the functions $A_n(x)$, $B_n(x)$ in which $\exp(-xt)t^n$ is integrated over the ranges, 1, ∞ and −1, 1 respectively, are given for x = [0.25(0.25)8; 10D] this range being extended in the first case by 8.0(0.5)13(1)16(2)20(1)22, 24. These tables go far beyond the tables with 6S used by Rosen though these latter go up to $n = 20$ instead of $n = 15$ but for $n > 10$ are for x = 1(1)10, 12, 14 only. In the 4-place tables of A. Unsöld [5] n has only the values 1, 2, 3 and x = 0(.1).2, .5(.5)2, 3. In the work of R. D. Misra [6] $2n$ = − 9(1)6 and x ranges from π to 4π, 7D being given while in a second table $2n$ = − 1(1)6 and $4x$ runs from 3π to 11π. Values of $A_n(x)$ for $n = -3, -2, -1$ were given by E. Gold [7] in 1909.

The paper also contains a table of the exponential integral $-Ei(-x)$, for x = [0.5(0.5)18(1)23, 25; 18D], and other values of x, 24–48, p. 15 of II. The integrals involving Legendre functions, Table 4, are given, for x = [0.5(0.5)1(0.25)7; 8D]. In the tables of double or triple entry, 5–6, the order of the Legendre functions ranges from 0 to an integral value not exceeding 8 while the other integers have no more than 4 values. In the Tables of II 13 integrals for hydrides are tabulated. P. 16–28 are occupied with lists of errata in I.

H. B.

[1] C. ZENER and V. GUILLEMIN, "The B-state of the hydrogen molecule," *Phys. Rev.*, s. 2, v. 34, 1929, p. 999–1009.
[2] N. ROSEN, "Normal state of the hydrogen molecule," *Phys. Rev.*, s. 2, v. 38, 1931, p. 2099–2114 (see especially p. 2109–2110).
[3] T. INVI, "A contribution to the theory of the hydrogen molecule," Phys. Math. So. Japan, *Proc.*, s. 3, v. 20, 1938, p. 770–779.
[4] J. H. BARTLETT and W. H. FURRY, "Valence forces in lithium and beryllium," *Phys. Rev.*, s. 2, v. 38, 1931, p. 1615–1622.
[5] A. UNSÖLD, "Zur Deutung der Intensitätsverteilung in den Fraunhoferschen Linien, II. Teil: Die Intensität der Linienflügel-approximative Lösung der Schwarzschildschen Integralgleichung für eine beliebig geschichtete Atmosphäre," *Zeit. f. Astrophysik*, v. 4, 1932, p. 339–357.
[6] R. D. MISRA, "On the stability of crystal lattices," Cambridge Phil. So., *Proc.*, v. 36, 1940, p. 173–182.
[7] E. GOLD, "The isothermal layer of the atmosphere and atmospheric radiation," R. So. London, *Proc.*, 82A, 1909, p. 62.

MATHEMATICAL TABLES—ERRATA

On the pages indicated Errata are listed for tables by the following: J. R. AIREY (p. 72), A. GRAY and G. B. MATHEWS (p. 74), D. F. E. MEISSEL (p. 74), A. G. WEBSTER (p. 70), and R. W. WILLSON and B. O. PEIRCE (p. 74). In this issue we have referred to Errata in RMT **113** (JAHNKE and EMDE).

13. L. M. MILNE-THOMSON, *Standard Table of Square Roots. The square roots to eight significant figures of all four-figure numbers, with printed differences*, London, Bell, 1929.

The following error was discovered by Mr. T. WHITWELL:

Difference following 239.7, *for* 3329, *read* 3229.

This error was found because Mr. Whitwell uses a precaution that I have always advocated when doing linear interpolation with a calculating machine. The preceding tabular value is set, then the difference, which is added (or subtracted) and the number in the product register compared with the next tabular value. Any error of setting, or in the difference, or in the direction of turning, is immediately revealed by the lack of agreement between the machine and table. The interpolation is, of course, completed by changing the decimal part of the multiplier from ciphers to the required interpolating factor.

24 May 1943 L. J. C.

14. J. T. PETERS, *Sechsstellige Tafel der trigonometrischen Funktionen*, · · ·, Berlin, 1929.

In April 1939 Mr. Peters reported that a second edition of this volume was about to be published, and asked me to send him a list of errors which I had noted. I sent him at once the following:

P. 49 Sec. 4°10′30″, *for* 2261 *read* 2661
P. 55 Argument following 5°14′20″, *for* 50″ *read* 30″
P. 61 Sec 6°17′00″, *for* 0.006043 *read* 1.006043
P. 129 Sec 17°39′50″, *for* 8464 *read* 9464
P. 234 Headings cosec and cotg to be transposed.

L. J. C.

15. H. M. PARKHURST, *Astronomical Tables, comprising Logarithms from 3 to 100 decimal places . . . Revised edition*, 1876; see RMT **86**.

It has been already noted (p. 20) that in this volume Parkhurst gives log N, to 102D, for 90 values of $N \leq 109$. Although the results given are to 102D both Tables III and IX (+II and XIII) in which they occur are headed "Logarithms to 100 decimal places." Thus accuracy to more than 100D is not claimed. I can now report concerning the exactness of the printed results for everything except the digits in the 101st and 102d decimal places for the logarithms of 73, 89. It was found that to 100D all of the values of log N, except 3, were entirely correct and also that in the following 9 of these cases the results were correct to 102D: 7, 9, 11, 21, 33, 63, 67, 74, 94. For values of N greater than 19 the first 15 figures of log N are given in Parkhurst's Table XIII. The fact that these figures which are omitted from Tables III and IX are all correct was established by the Editor who compared the data with the Peters-Sharp logs. (A reproduction of Table XIII was not immediately available to me.) We tested the 20-place Table II independently [N = 1, 2, · · ·, 19], and detected no error.

The three non-terminal errors in log N were as follows: In log 51 the 9 in the 82d decimal place should read 8, the correct environment being . . . 8$\underline{8}$ 386 . . . . In log 83 there is an error of +1 in the 91st decimal place which affects three consecutive figures. Parkhurst's ". . . 181$\underline{40}$ $\underline{0}$1079 . . ." should read . . . 181$\underline{39}$ $\underline{9}$1079 . . . . In log 91 Parkhurst's 3 in the 57th decimal place should read 4 since the correct environment is . . . 0$\underline{4}$825 . . . .

In order to correct the values of the 79 remaining log N to 102D, the digits after the 100th have to be decreased or increased by amount (−15 to +15) for the N's which are indicated as follows: (−15) 71; (−11) 82; (−10) 41; (−9) 39; (−8) 97; (−7) 52, 76, 78; (−6) 26, 38, 57, 64, 91; (−5) 13, 32, 48, 59, 65, 86; (−4) 19, 43, 58, 72, 106, 109; (−3) 8, 16, 24, 36, 56, 87, 88, 95; (−2) 4, 6, 12, 29, 44, 53, 54, 103; (−1) 2, 3, 14, 18, 22, 27, 28, 42, 66, 68, 81, 84, 107; (+1) 5, 15, 17, 34, 37, 45, 47, 51, 62, 75, 77, 105; (+2) 25, 35, 49, 55, 79, 83, 93; (+3) 31, 46, 85; (+4) 69; (+5) 23; (+15) 61.

The following account of a device for determining simultaneously the number and decimal positions of all the non-terminal errors in a table of logarithms of integers may be interesting because it was new to me and it is very effective as compared with the more obvious but crude scheme of computing a complete table of higher approximation in order to test a table having a smaller number of decimal places.

The basic idea is that

$$\log \left[\prod_{j=1}^{k} (n+j)\right] = \log [(n+k)!] - \log [n!].$$

The device consists in adding the logarithms of the table under investigation and then comparing the sum with standard values of the logarithms of the factorials. At the beginning of a table $n = 0$. In the special case of Parkhurst's Tables III and IX the values of $\log N$ are omitted for N equal to all multiples of 10 and for $N = 92, 96, 98, 99$. The missing logarithms were supplied from my own master values after conversion to the base 10. More specifically, the characterizing value of log (100!) was computed by the formula $12 + 14 \log 2 + 7 \log 3 + 3 \log 7 + \log 11 + \log 23 +$ (sum of Parkhurst's log's inclusive of log 97). Comparison of this sum with the converted value of my published 153-place approximation to log (100!), as obtained from Stirling's series for $\log \Gamma(x)$, showed only three discrepancies. As indicated earlier the numerical differences had the common value 1 and they occurred respectively in the 57th, 82d and 91st decimal columns. The above comparison also showed that the algebraic sum of the two undetermined terminal errors equals $+15.1$ in terms of unity in the 102d decimal place.

The determination of the second coördinate of the separate errors was effected by comparison of Parkhurst's logarithms with juxtaposable photostatic copies of the 61-place Peters-Sharp table, of A. Grimpen's 84-place table for prime N's, and with my own multifariously checked log's. For values of N greater than 100 the complete comparisons were made of necessity with my own converted log's.

The omnibus exploratory test explained above is a necessary but not a sufficient condition for perfection in a table since the theoretical probability of the miraculous occurrence of mutually compensating non-terminal errors is not zero. This qualification applies to the present concrete case of Parkhurst's tables only for the logarithms of composite values of N in the range of figures from the 60th (end of Sharp's table) to the 79th decimal place, a region within which it was not considered necessary to attain purely theoretical sufficiency by multiplying every figure of my natural logarithms by the common modulus.

H. S. Uhler

Yale University

Final note. On June 9, 1943 I finished calculating to about 121D the natural logarithms of 73 and 89, and in converting these data to the base 10 in an interval including the 100th dec. place. It was found that the respective pentads beginning at the 101st place are 10647 and 63094, hence Parkhurst's terminal pairs "23" and "66" are too high by about 12 and 3 in terms of unity in the 102nd place of decimals. Finally as an omnibus check on all of my base 10 sequences, which had been used as standards of comparison for Parkhurst's terminal figures, I computed from them a section of log (100!) on the basis of the exponents of the 25($>$1) prime factors of 100!. The section thus obtained, which begins at the 96th dec. place, reads . . . 16858,35310,985 . . . and it agrees absolutely with the figures in the approximation to log (100!) as derived earlier from my coefficients of Stirling's series.

H. S. Uhler

16. Project for the Computation of Mathematical Tables, *Tables of Probability Functions*, v. 2, 1942. Compare RMT **91**.

P. 46, *for* 0.2335, *read* 0.2235
P. 63, *for* 0.0370, *read* 0.3070.

D. H. L.

17. A. H. H. Tallqvist, *Tafeln der abgeleiteten und zugeordneten Kugelfunctionen erster Art.* Soc. Scient. Fennicae, *Acta*, v. 33, no. 9, 1908, 67 p. Compare RMT **118.**

$P_2^2(0.19)$, *for* 0.8917, *read* 2.8917
$P_4^4(0.64)$, *for* 46.6000768, *read* 36.6000768
For $P_7^3(0.40)$, $P_7^3(0.41)$ the respective correct values, -118.19932808 and -121.11152590, are interchanged.

Z. Mursi, *Tables of Legendre Associated Functions*, Cairo, 1941, preface.

18. Financial Publishing Company, *Monthly Bond Values . . .*, second ed., Boston, 1941. Compare RMT **117**.

The following list of errors in this work was supplied by the Company, and is complete, according to their latest information, except for a few last-figure unit errors, which are of no consequence in a table of this kind. The Company has remarked that since computation is made arithmetically from one coupon to the next, a single original mistake may appear, as below, in several or all coupon rates.

page	coupon	yield	yrs.	mos.	for	read
3	1%	3.00	1	3	97.562848	97.563248
5	1	.45	3	5	101.862678	101.862638
47	1¼	3.00	1	3	97.866993	97.867493
49	1¼	.45	3	5	102.709357	102.709307
91	1½	3.00	1	3	98.171138	98.171738
93	1½	.45	3	5	103.556036	103.555976
135	1¾	3.00	1	3	98.475283	98.475983
137	1¾	.45	3	5	104.402716	104.402646
143	1¾	2.05	9	11	97.019650	97.319650
179	2	3.00	1	3	98.779428	98.780228
181	2	.45	3	5	105.249395	105.249315
223	2¼	3.00	1	3	99.083573	99.084473
225	2¼	.45	3	5	106.096075	106.095985
267	2½	3.00	1	3	99.387718	99.388718
269	2½	.45	3	5	106.942754	106.942654
311	2¾	3.00	1	3	99.691863	99.692963
313	2¾	.45	3	5	107.789433	107.789323
442	3½	2.85	0	8	100.323488	100.423488
887	6	3.15	5	8	104.680108	114.680108
926	3	.45	3	5	108.636113	108.635993
938	3¼	.45	3	5	109.482792	109.482662
950	3½	.45	3	5	110.329472	110.329332
962	3¾	.45	3	5	111.176151	111.176001
974	4	.45	3	5	112.022830	112.022670
986	4¼	.45	3	5	112.869510	112.869340
999	4½	.45	3	5	113.716189	113.716009
1015	4¾	.45	3	5	114.562869	114.562679
1030 [1]	5	.90	1	7	106.420496	106.430496
1031	5	.45	3	5	115.409548	115.409348
1047	5¼	.45	3	5	116.256227	116.256017
1063	5½	.45	3	5	117.102907	117.102687
1079	5¾	.45	3	5	117.949586	117.949356
1095	6	.45	3	5	118.796266	118.796026
1117	0		7	10	values at 2.00 and 2.25 transposed	

[1] This particular error does not appear in all books. In the middle of the press run a super-efficient pressman decided to improve a blurred figure, and in doing so substituted a "2" for a "3."

19. J. P. L. Bourguet, "Sur les intégrales Eulériennes et quelques autres fonctions uniformes," *Acta Mathematica*, v. 2, 1883, p. 261–295.

In this paper on the gamma function, among the six tables, to 16D, are two (p. 288–289) for coefficients in the following expansions of $1/\Gamma(x)$:

$$1/\Gamma(x) = x(x+1)(1 + B_1x + B_2x^2 + B_3x^3 + \cdots), \quad \text{and}$$
$$1/\Gamma(x) = x + C_2x^2 + C_3x^3 + \cdots.$$

These tables are reprinted in H. T. Davis, *Tables of the Higher Mathematical Functions*, Bloomington, Indiana, v. 1, 1933, p. 185.

Besides checking these coefficients, it was thought desirable to compute the remaining few coefficients that are nonvanishing in the 16th decimal place. Corrections and new values for the B_n's are accurate to much within 3/4 of a unit in the 16th place, and for the C_n's (which are obtained by addition of B_n's) to much within $1\frac{1}{2}$ units in the 16th place.

Errors in the last four figures of the B_n's:

n	for	read	difference
7	6044	6042	−2
11	0224	0290	+66
12	5466	5469	+3
13	3152	3149	−3
18	0028	0030	+2
19	2202	2207	+5

Addition to B_n's: $B_{23} = +13$, and $B_{24} = +1$ units in the 16th place.

Errors in the last four figures of the C_n's:

n	for	read	difference
12	7741	7807	+66
13	4758	4821	+63
14	2314	2320	+6
15	8420	8417	−3
19	3425	3427	+2
20	7826	7823	−3
21	6962	6968	+6

Addition to C_n's: $C_{24} = -54$, $C_{25} = +14$ and $C_{26} = +1$ units in the 16th place.

E. Isaacson and H. E. Salzer

Mathematical Tables Project,
New York City

20. R. A. Davis, *Table of Natural Sines and Radians*, Oakland, California, 1941. Compare UMT 7.

This table lists $\sin^{-1} x$ in radians for $x = [0(.001)1; 6D]$. It was proofread against the Project's manuscript and no error larger than two units in the last place was found in Davis. But the errors show a trend: Of the first 450 values there are 265 entries which are too small by a unit in the last place. The only other last place which is a unit too small occurs at .907. Among the last ninety entries from .910 to 1, are 34 values which are too large by a unit in the last place. The only other last place which is a unit too large is at .416. In addition, the last place was two units too small for the following arguments: .053, .055, .057, .058, .061 − .064, .110, .116 − .119, .163, .171, .176, .178, .179, .181, .182, .185, .186, .189, .236, .258, .353, .354, .396, and .441. Also the last place was two units too large for .998. Davis uses a + or − sign after the last place from .450 to .900. In that interval, we allowed him a unit leeway. Thus if his value was larger by a unit in the last place followed by a minus sign or smaller by a unit followed by a plus sign, it was not noted. Also if his last place value agreed with ours, we ignored the plus or minus sign that sometimes followed. Under these conditions no error was found from .450 to .900.

I. I. Rhodes and H. E. Salzer

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