

EDITORIAL NOTE.—Part of this value is the first in a table of ten “coincidental logarithms” listed by D. M. KNAPPEN,¹ solutions (8 of them irrational) of the equations $10^n \log x = x$, $n = 1, 2, \dots, 10$. To 50D it agrees with the result of Mr. Wrench; in the remaining three places there is disagreement, 650 instead of 707. For $n = 8$ the value given for x , to 66D, is as follows:

$$\begin{array}{r} 8.95191\ 59982\ 67846\ 23159\ 99873\ 77688\ 49072- \\ 72932\ 33707\ 59495\ 47848\ 84326\ 51391\ 4 \quad \times 10^8. \end{array}$$

[Upon bringing this result to the attention of Mr. Wrench he verified its accuracy through 64D and then added the following 16 digits: 0 73919 87374 90256.] Knappen states that these remarkable numbers were called “constants” by OLIVER BYRNE, who certainly discussed such numbers in at least two publications,² of 1849 and 1864. In each of these places Byrne considered the problem “to find a number whose common logarithm is composed of the same digits and in the same order as itself.” In the first he gives the ten values, each to 15D, but the last two or three figures of eight of these are incorrect. Similar remarks apply to the seven values given in the second publication, where Byrne recklessly writes “no known development but the dual will establish” them. As yet we have not been able to connect with Byrne, the 53 to 66D values given by Knappen. Perhaps some reader can supply this information.

¹ D. M. KNAPPEN, *Math. Mag.*, Washington, D. C., v. 1, p. 202—the number containing this page is dated Oct. 1884, although published in August 1887.

² O. BYRNE, (a) *Practical, Short, and Direct Method of Calculating the Logarithm of any given Number, and the Number corresponding to any given Logarithm*. London and New York, 1849, p. viii, 14–15, 52, 65–76. Compare J. HENDERSON, *Bibliotheca Tabularum Mathematicarum*, Cambridge, 1926, p. 121–122.

(b) *Dual Arithmetic. A New Art. . . . New issue with a Complete Analysis*, London, 1864, *Analysis*, p. 75–78.

QUERIES

8. ROOTS OF THE EQUATION $\tan x = cx$.—As early as 1748 in v. 2 of his *Introductio in Analysin Infinitorum*, Lausanne, p. 318–320, EULER solved the problem “Invenire omnes Arcus, qui Tangentibus suis sint aequales,” by deriving a general formula, from which he found the first 10 non-negative roots of $\tan x = x$ ($c = 1$). The second of these roots was given in the form “ $3.90^\circ - 12' 32'' 48'''$ ” [4.49340834 instead of 4.49340964], and the tenth as “ $19.90^\circ - 1' 55' 16''$.” These results may be regarded as correct to 5D. $J_{3/2}(x) = 0$ is equivalent to Euler’s equation $\tan x = x$. In 1886 LOMMEL listed the first 17 non-negative roots, to 6D (Akad. d. Wiss., Munich, *Math.-naturw. Abt., Abhandlungen*, v. 15, p. 651). This list, abbreviated to 4D, was published in the first, second and fifth editions of JAHNKE and EMDE (see RMT 113), and in E. P. ADAMS, *Smithsonian Mathematical Formulae . . .*, Washington, D. C., 1939 (1922), p. 84. The first 36 roots, to 4D, are given in K. HAYASHI, *Fünfstellige Funktionentafeln*, Berlin, 1930, p. 52. The 18th to the 36th roots, to 6D, are given in L. SILBERSTEIN, *Bell’s Mathematical Tables*, London, Bell, 1922 (= *Synopsis of Applicable Mathematics with Tables*, New York, Van Nostrand, 1923), p. 97. In 1827 CAUCHY found the first 4 roots to 7D when $c = 1$, the first 5, to 6D, when $c = 4$, and the first 4, to 4–6D, when $c = 8/5$ (“Théorie de la propagation des ondes à la surface d’un fluide pesant d’une profondeur indéfinie,” Acad. d. Sci., Paris, *Mémoires prés. par divers Savants*, s. 2, v. 1, p. 198–209; *Oeuvres*, s. 1, v. 1, Paris, 1882, p. 204–212). Compare UMT 22. What other solutions of $\tan x = cx$ are known?

H. B. and R. C. A.

9. SYSTEM OF LINEAR EQUATIONS.—Where may one find an adequate treatment of the method of GAUSS and SEIDEL for solving a system of n

linear equations in n unknowns by successive approximations? The discussion given in WHITTAKER and ROBINSON, *The Calculus of Observations* (London, 1924, and third ed., 1940, p. 255–256), is not satisfactory. The part purporting to show that the process always improves a trial solution suffers the following simple exception:

$$2x + y = 1, \quad x + 3y = -1.$$

Here the initial solution $x = 1/2, y = -1/3$ is not improved by replacing x by $2/3$ as required by the process.

D. H. L.

QUERIES—REPLIES

8. TABLES OF $N^{3/2}$ (Q5, p. 131).—Another table for three-halves powers of numbers to more than three places is T. 70, p. 290 of J. T. FANNING, *A Practical Treatise on Hydraulic and Water-Supply Emergency*, tenth ed., New York, 1892, where $N = [0.04(0.01)0.20(0.02)1.0(0.1)4; 4D]$.

H. B.

CORRIGENDA

- P. 2, l. 31, for Reply to Query 6, read Reply-to-Query 6. P. 6, l. 6, for v. 4, read v. 14. P. 9, 76 for CHAPMAN, read CHAPIN. P. 14, l. 5 from bottom, for 0.001, read 0.0001. P. 15, l. 6, add Also, p. 224–224c, $\sin x$, $\cos x$ to 10D, $\log \sin x$, $\log \cos x$ to 5D, $x = 0(.1)10, 0(1)100$. P. 15–16, omit references to HAYASHI tables of $\sin \frac{1}{2}x\pi$, $\cos \frac{1}{2}x\pi$, l. 13–14 from bottom of p. 15; also to tables of KOLKMEIJER and BUERGER, top of p. 16. P. 16, l. 8 from bottom, for Spoon, read Spon. P. 18, l. 1 and 2 from bottom, for 6D, read 6D-7D. P. 19, l. 3 from bottom, for $x, \dots 3D]$, read $x = [0.00(0.01)1.0(0.1)10(1)-100(10)1000; 3D]$. P. 20, footnote, l. 6, after "109." insert With the aid of the entries presented the logarithms of all numbers $N = 1(1)109$ are readily found. P. 47, 90, l. 3, for State, read City. P. 69, 2, l. 3, for Houghton, read Haughton; 3, l. 1, for 12S, read 10S–12S; 5 and 6, for with differences, read with first differences.
- P. 70, 8, l. 2, for 10D, read 9D–10D; l. 4, for $0(1/2)(13/2)$, read $0(\frac{1}{2})6\frac{1}{2}$; l. 5, for $\frac{1}{2}x\pi$ read $\frac{1}{2}\pi$, [this was a mistake in the Report]; 10, l. 3, for by degrees, read at three-degree intervals; 12, l. 3, for $80^\circ 1$, read $80^\circ.1$. P. 73, 44, l. 2, Ei in roman, not ital.; 49, l. 4, for $0.0(0.1)10.0$ read $0.0(0.1)7(1)10$. P. 74, 52, l. 20, for J_k^0 and J_k^1 , read I_k^0 and I_k^1 ; 56, l. 4, for 120, read 12.0. P. 96, in UMT 9, totals, make the following changes: 390 for 391; Poulet 65 (for 68); Escott 233 (for 235); and add Poulet and Gérardin 4 (1929). P. 109, l. 17–18, for $J_1(17)$, read $J_1(x_{17})$; l. 20–22, for these lines read, the roots of $J_1(x)N_1(kx) - J_1(kx)N_1(x) = 0$ on p. 204 of nos. 3–5, p. 274 of no. 2, and p. 162 of no. 1, the first three roots for the value $k = 2$ should be 3.1917, 6.3116, and 9.4446 according to values given in MUSKAT, . . . P. 108, l. 17, for Debye, read Debye.
- P. 125, l. 20–23, for numbers, read figures. P. 138, 26, l. 4, for $J_{+\frac{1}{2}}(x)$, read $J_{\pm\frac{1}{2}}(x)$; for uncertain fourth, read approximate fifth; l. 5, for $J_{+\frac{1}{2}}(x)$, read $J_{\pm\frac{1}{2}}(x)$; l. 5–6, for uncertainties, read approximate fifths; l. 7, for $\frac{1}{2}(n+1)$, read $\frac{1}{2}/(n+1)$. P. 140, no. 38, for ∂x , and ∂x^2 , read ∂v and ∂v^2 . P. 143, l. 4 from bottom, for einen, read einem. P. 145, for line 8, read: place tables for A with $D = 0.0000(0.0001)2.000(0.001)4.00(0.01)6.94$; and for S with $D = 0.3000(0.0001)2.000(0.001)4.00(0.01)6.94$. P. 157, l. 16–17, for $B_n^{(n)}(0)$ and $B_n^{(n)}(1)$, read $B_n^{(n)}(0)/n!$ and $B_n^{(n)}(1)/n!$. P. 161, l. 11, delete "P. 54, F(35°, 30°), for 0.6220, read 6200." P. 161, l. 13, for 1035, read 1037. P. 164, l. 11 from bottom, eliminate the second "10;". P. 168, l. 26, for Küster, read Küstner. P. 169, l. 27, read Physical; l. 6 from bottom, for *kkadratov*, read *kvadratov*; l. 4 from bottom, read *Izdatyel'stvo*.