a correction

$$\epsilon_n = -(\tan x_n^0 + a x_n^0)/(\sec^2 x_n^0 + a).$$

The corrected values $x_n^0 + \epsilon_n$ are about as accurate as we need them.

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23[L].—HANSRAJ GUPTA, *Table of Liouville's function and its sum.* Ms. in possession of the author, Government College, Hoshiarpur, Punjab, India.

Liouville's function $\lambda(n)$ may be defined for the positive integer argument $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_t^{\alpha_t}$ to be $+1$ or -1 according as $m = \alpha_1 + \alpha_2 + \cdots + \alpha_t$ is even or odd. This table gives $\lambda(n)$ and also the sum

$$L(n) = \lambda(1) + \cdots + \lambda(n)$$

for $n \leqslant 20\,000$. The function $L(n)$ is clearly the excess of the number of those integers n which have an even number of prime factors over the number of integers having an odd number of prime factors. The reason for computing this table is to verify or refute an important conjecture of PÓLYA (Deutsche Math.-Ver. *Jahresb.*, v. 28, 1919, p. 38) to the effect that this "excess" is really negative or zero for $n > 1$. The table shows that the conjecture is correct as far as $n = 20\,000$ but does not show that the conjecture has safely passed its worst trials. In fact one finds that $L(48\,512) = -2$. Data, taken from this table, on the behavior of $L(n)$ have been published by the author, Indian Acad. Sci., *Proc.*, v. 12A, 1940, p. 407–409. Current interest in Pólya's conjecture (which implies the Riemann Hypothesis) has been heightened by the recent results of A. E. INGHAM, *Amer. J. Math.*, v. 64, 1942, p. 313–319. The corresponding problem for the companion function $\mu(n)$ of MÖBIUS and its sum is also unsolved and has been the subject of much tabulation; see *Guide to Tables in the Theory of Numbers*, Nat. Res. Council, *Bull.* no. 105, Washington, D. C., 1941, p. 9.

D. H. L.

NOTES

17. DESIRABLE MATHEMATICAL TABLES FOR REPUBLICATION.—The Office of Alien Property Custodian has licensed, during the past several months, the reprinting of scientific and technical books, of enemy origin, which are not available in a quantity sufficient to meet the demands of the wartime operations of science and industry. In RMT **79, 113, 128,** references have been made to three volumes of this kind. Before definite decision can be made regarding the licensing of additional Mathematical Tables for republication, it is necessary for the Custodian to be informed about the extent of the need of such tables and to receive suggestions of specific titles for consideration. This can be accomplished if suggestions of specific significant tables, or any inquiries, are sent by individuals to the undersigned at Division of Patent Administration, Office of Alien Property Custodian, Washington, D. C.

H. H. SARGEANT, Chief

18. *Phil. Mag.* TABLES, SUPPL. 1 (see *MTAC*, p. 135–141).—In H. C. PLUMMER, "The numerical solution of a type of equation," s. 7, v. 32, 1941, p. 505–512, roots are found of the following transcendental equations, among others, of the type $\tan x = xf(x)$, where $f(x)$ is a single-valued function:

I. For $\tan x = x/(1 - x^2)$, which relates to electrical waves on a sphere (J. J. THOMSON, *Notes on Recent Researches in Electricity and Magnetism*, Oxford, 1893, p. 373), are given the first 5 positive roots to 4D. The first 3 of these roots agree with those of L. SILBERSTEIN, *Bell's Mathematical Tables*, London, Bell, 1922, p. 97. Silberstein's fourth root is appreciably in error. Before either of these results had been published, H. LAMB gave the first 6 positive roots to 4D in London Math. So., *Proc.*, v. 13, p. 202, 1882.

II. $\tan x = 2x/(2 - x^2)$, an equation found in connection with discussion of sound waves, or the free oscillations of gas, in a rigid spherical shell (RAYLEIGH, *The Theory of Sound*, v. 2, 1878, p. 232; second ed., 1896, p. 265). Five positive roots to 4D are given. For this equation also LAMB gave (*l.c.*) 6 positive roots to 4D.

III. Another equation considered by Rayleigh (*ibid.*, v. 2, 1896, p. 266) is $\tan x = x(x^2 - 9)/(4x^2 - 9)$ for which he gives one non-zero solution to 4D; Plummer finds four positive roots to 4D.

IV. The equations $\cos x \cdot \cosh x = \pm 1$, occur in connection with the lateral vibration of bars (RAYLEIGH, *ibid.*, v. 1, 1877, p. 222–224; second ed., 1896, p. 278). Four positive solutions, to 6D, are found for each of these equations. In this connection Rayleigh's errors in the first edition are corrected in the second. The oversight was noted by A. G. GREENHILL (*Mess. Math.*, v. 16, 1886, p. 119) who has given another interesting application of the same equations. More roots and general formulae are given in E. P. ADAMS, *Smithsonian Mathematical Formulae*, Washington, D. C., 1939 (1922), p. 86.

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19. *Phil. Mag.* TABLES, SUPPL. 2.—In H. CARRINGTON, "The frequencies of vibration of flat circular plates fixed at the circumference," s. 6, v. 50, 1925, p. 1261–1262, the 16 roots, < 16, of $J_{n+1}(x) \cdot I_n(x) + I_{n+1}(x) \cdot J_n(x) = 0$, for $n = 0, 1, 2, 3$, are given to 5S. The first ten roots for each of these values of n are given by J. R. AIREY, "The vibrations of circular plates and their relation to Bessel functions," Phys. So. London, *Proc.*, v. 23, 1911, p. 227; for $n = 0$ these roots are given to 4D, but for the others, to 3D. Carrington's first root for $n = 0$ was 3.1961; according to Airey this should be 3.1955. Of the 10 roots given in JAHNKE and EMDE (1933), p. 283, and 1938–43 editions, p. 234, 8 are erroneous to the extent of 1 to 20 points in the last decimal places.

R. C. A.

20. A ROOT OF THE EQUATION $10 \log x = x$.—In *Mathematical Gazette*, v. 15, p. 367, 1931, C. V. BOYS gives what purports to be a 60D approximation to the irrational root of this equation. I checked this value and found it correct to only 38D. By Newton's method I deduced a value of x to 66D and then in a second calculation determined the logarithm of this last approximation. It developed that my result was correct to 65D. Incidentally I employed the rational approximation $x = 2^7.5.7/3^3.11^2$. The correct value for x, to 65D, is as follows:

1.37128 85742 38623 53686 13621 06299 68995 88428 54404 84225-
70704 08723 85385.

J. W. WRENCH, JR.

EDITORIAL NOTE.—Part of this value is the first in a table of ten "coincidental logarithms" listed by D. M. KNAPPEN,[1] solutions (8 of them irrational) of the equations $10^n \log x = x$, $n = 1, 2, \cdots, 10$. To 50D it agrees with the result of Mr. Wrench; in the remaining three places there is disagreement, 650 instead of 707. For $n = 8$ the value given for x, to 66D, is as follows:

8.95191 59982 67846 23159 99873 77688 49072-
72932 33707 59495 47848 84326 51391 4 $\times 10^8$.

[Upon bringing this result to the attention of Mr. Wrench he verified its accuracy through 64D and then added the following 16 digits: 0 73919 87374 90256.] Knappen states that these remarkable numbers were called "constants" by OLIVER BYRNE, who certainly discussed such numbers in at least two publications,[2] of 1849 and 1864. In each of these places Byrne considered the problem "to find a number whose common logarithm is composed of the same digits and in the same order as itself." In the first he gives the ten values, each to 15D, but the last two or three figures of eight of these are incorrect. Similar remarks apply to the seven values given in the second publication, where Byrne recklessly writes "no known development but the dual will establish" them. As yet we have not been able to connect with Byrne, the 53 to 66D values given by Knappen. Perhaps some reader can supply this information.

[1] D. M. KNAPPEN, *Math. Mag.*, Washington, D. C., v. 1, p. 202—the number containing this page is dated Oct. 1884, although published in August 1887.

[2] O. BYRNE, (a) *Practical, Short, and Direct Method of Calculating the Logarithm of any given Number, and the Number corresponding to any given Logarithm.* London and New York, 1849, p. viii, 14–15, 52, 65–76. Compare J. HENDERSON, *Bibliotheca Tabularum Mathematicarum*, Cambridge, 1926, p. 121–122.

(b) *Dual Arithmetic. A New Art. . . . New issue with a Complete Analysis*, London, 1864, *Analysis*, p. 75–78.

QUERIES

8. ROOTS OF THE EQUATION $\tan x = cx$.—As early as 1748 in v. 2 of his *Introductio in Analysin Infinitorum*, Lausanne, p. 318–320, EULER solved the problem "Invenire omnes Arcus, qui Tangentibus suis sint aequales," by deriving a general formula, from which he found the first 10 non-negative roots of $\tan x = x (c = 1)$. The second of these roots was given in the form "$3.90° - 12° 32' 48''$" [4.49340834 instead of 4.49340964], and the tenth as "$19.90° - 1° 55' 16''$." These results may be regarded as correct to 5D. $J_{3/2}(x) = 0$ is equivalent to Euler's equation $\tan x = x$. In 1886 LOMMEL listed the first 17 non-negative roots, to 6D (Akad. d. Wiss., Munich, *Math.-naturw. Abt.*, *Abhandlungen*, v. 15, p. 651). This list, abbreviated to 4D, was published in the first, second and fifth editions of JAHNKE and EMDE (see RMT **113**), and in E. P. ADAMS, *Smithsonian Mathematical Formulae . . .* , Washington, D. C., 1939 (1922), p. 84. The first 36 roots, to 4D, are given in K. HAYASHI, *Fünfstellige Funktionentafeln*, Berlin, 1930, p. 52. The 18th to the 36th roots, to 6D, are given in L. SILBERSTEIN, *Bell's Mathematical Tables*, London, Bell, 1922 (= *Synopsis of Applicable Mathematics with Tables*, New York, Van Nostrand, 1923), p. 97. In 1827 CAUCHY found the first 4 roots to 7D when $c = 1$, the first 5, to 6D, when $c = 4$, and the first 4, to 4–6D, when $c = 8/5$ ("Théorie de la propagation des ondes à la surface d'un fluide pesant d'une profondeur indéfinie," Acad. d. Sci., Paris, *Mémoires prés. par divers Savants*, s. 2, v. 1, p. 198–209; *Oeuvres*, s. 1, v. 1, Paris, 1882, p. 204–212). Compare UMT **22**. What other solutions of $\tan x = cx$ are known?

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9. SYSTEM OF LINEAR EQUATIONS.—Where may one find an adequate treatment of the method of GAUSS and SEIDEL for solving a system of n