able to use the chart, would have been interesting. The question is not trivial. The forthright method of multiplying all the roots by some small factor may not prove effective. In fact when the coefficients are thus made small the S curve is nearly coincident with the D curve and the two do not have well defined intersections.

In dealing with problems in which a large number of quintic equations with more or less the same distribution of coefficients occur it is evident that a chart of this kind would eliminate a good deal of exploratory work and make the whole process quite methodical.

D. H. L.

[1] This family is essentially the same as that given by L. I. HEWES and H. L. SEWARD: *The Design of Diagrams for Engineering Formulas and the Theory of Nomography*, New York, McGraw-Hill, 1923, p. 68, in their nomogram for the complete cubic equation.

NOTES

21. COMPUTING A TABLE OF LOG $\Gamma(x)$.—Incidental to some required computation on a problem in ballistic theory it seemed necessary to prepare a table of the logarithms of the Γ function for the range $x = [1(.01)16.99; 7D]$. This table, available on punched cards and also in typed form exhibiting the first, second and third differences, was made as follows:

The values of $\log \Gamma(x)$ for $x = 1, 1.01, \cdots, 1.99$ were punched from the 7-place table of the *Smithsonian Physical Tables*, Washington, 1934, p 64–65. The Vega 7-place table of logarithms on punched cards allowed the use of the relation

$$\log \Gamma(x) = \log \Gamma(x-1) + \log(x-1), \tag{1}$$

for the range $x = 2, 2.01, \cdots, 2.99$, to get $\log \Gamma(x)$ for $x = 2, \cdots, 2.99$. With these newly computed values the relation (1) was again used for the range $x = 3.00, 3.01, \cdots, 3.99$, giving $\log \Gamma(x)$ for this new range. This process was applied 15 times.

In the range $16 \leqq x < 17$ exactly 15 logarithms were added to the original values of $\log \Gamma(x)$, namely:

$$\log \Gamma(x) = \log(x-1) + \log(x-2) + \cdots + \log(x-15) + \log \Gamma(x-15).$$

Since the error in each term is $< \frac{1}{2}\, 10^{-7}$, the error of any entry in our table will never reach *.000 000 8*. Because of compensation of errors the table would most likely be accurate if we rounded it off to 6 places. Of course it is absolutely accurate if rounded off to 5 places.

The whole operation was so well suited to punched-card methods that it required about a day's work for two operators.

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EDITORIAL NOTE: While this table is not new, its method of computation and its listing as on I.B.M. punched cards seemed worthy of record. W. T. RUSSELL's table of $\log \Gamma(x)$ for $x = [1(.01)50.99; 7D]$ was published in *Tracts for Computers*, no. IX (1923), ed. by K. Pearson. The Smithsonian table referred to above for $x = [1(.001)2; 7D]$ was taken from the table by LEGENDRE in his *Exercices de Calcul Intégral*, v. 2, 1816, p. 85–95, where there is a table of $\log \Gamma(x)$ for $x = [1(.001)2; 12D]$, Δ^3. A reprint of this table appeared in O. SCHLÖMILCH, *Analytische Studien*, part 1, Leipzig, 1848. The range for such a table was extended for $x = [2(.1)70(1)1200; 10D]$, with second, fourth, and sixth differences, in *Tracts for Computers*, no. VIII (1922), ed. by E. S. PEARSON. In his *Tables of the Higher Mathematical Functions*, Bloomington, Ind., 1933, p. 195–255, H. T. DAVIS has the following tables of $\log \Gamma(x)$: $x = [1(.0001)1.1; 10D]$, Δ^2; $x = [1(.001)2; 12D]$, Δ^3; $x = [2(.01)11; 12D]$, δ^2, δ^4; $x = [11(.1)101; 12D]$. $\text{Log}\, \Gamma(x+1)$ for $x = [1(1)1200; 18D]$ is given in two places, namely: (a) C. F. DEGEN, *Tabularum ad faciliorem et breviorem Probabilitatis computationem utilium Enneas*, Copenhagen, 1824; and (b) PETERS and STEIN, *Zehnstellige Logarithmentafel*, v. 1, *Anhang*, Leipzig, 1922, p. 61–68. A table for $\log \Gamma(x+1)$ for $x = [1(1)3000; 33D]$ was given by F. J. DUARTE in his *Nouvelles*

Tables de Log n!, Geneva and Paris, 1927. From such tables it is readily checked that there are 17 unit-errors in last figures of the Wrinch table (*MTAC*, p. 138–139). Of errors in Degen, Duarte notes (*l.c.*, p. iv) that there were 29 cases of end-figure errors of more than a half unit (all doubtless correct in the Peters and Stein edition), and the following two misprints:

log 1093! = 2848.446401 417732 51(7)660
log 1180! = 3114.2(0)8341 583736 442917.

For the figures in parentheses 2 and 8 should be respectively substituted.

22. EARLIEST TABLES OF *S* AND *T*.—A quotation from a letter of C. W. MERRIFIELD in *MTAC*, p. 85, suggested that the first tables of *S* and *T* were in John Newton's Trigonometry of 1658, but R.C.A. had not identified them. They occur in the *Trigonometria Britanica*, in the last section, with its own title page, *Canones Logarithmorum pro sinvbvs & Tangentibvs, ad tres primos Quadrantis Gradus, & partes Graduum Millesimas*, Londini, Ex Officina Leybourniana, MDCLVIII.

Newton gives what are now called *S* and *T* functions, to 8D, for 0(0°.001)3°. His unit is, then, 0°.001, and the (ten-added) characteristic plus first 3 decimal places are always 6.241, or 6.242 (two columns for tangents), only the 4th to 8th decimals being printed, except at head of column. First differences of *S* and *T* are also given, rising to 132 and 264 units approximately of the 8th decimal at 3° for *S* and *T* respectively. (See the right-hand column at each of the 12 openings.) Newton calls our *S* and *T* "Diff. 1ae.", and the first difference of *S* and *T* is in a column headed "2ae.", doubtless an abbreviation for "Diff. 2ae."

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23. A NEW RESULT CONCERNING A MERSENNE NUMBER.—In *Scripta Mathematica*, v. 3, 1935, p. 112–119, there is an article by R. C. ARCHIBALD on "Mersenne's numbers," $M_p = 2^p - 1$, (p prime). This article tabulates all results known in 1935 (except one) for these numbers $p = 2$ to $p = 257$. Archibald has pointed out that the one result he had overlooked was that announced by POWERS in London Math. So., *Proc.*, v. 12, Mar. 13, 1913, p. iii, that $2^{257} - 1$ has no factor <10,017,000. Until recently it was not known whether 6 of the 55 Mersenne numbers were prime or composite. On August 11, 1944, I completed the proof that the smallest of these 6 numbers, M_{167}, is composite. Details concerning my computations are about to appear in Nat. Acad. Sci., *Proc.* Our present knowledge of the M_p is tabulated below.

p	Character of M_p
2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127	Prime
11, 23, 29, 37, 41, 43, 47, 53, 59, 67, 71, 73, 79	Completely factored
113, 151, 179, 223, 233, 239, 251	Incompletely factored but two or more prime factors known
83, 97, 131, 163, 173, 181, 191, 197, 211	Only one prime factor known
101, 103, 109, 137, 139, 149, 157, 241, 257	Composite but no prime factor known
167, 193, 199, 227, 229	Character unknown

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24. OPTIMUM-INTERVAL PUNCHED-CARD TABLES.—The article of this title by HERGET & CLEMENCE, *MTAC*, p. 173–176, raises a point of some interest, namely, that in making a table that is to be interpolable linearly, it is permissible to modify the *function* itself if the values are to be entered on punched cards. It has for some time been realized that suitably modified second or higher differences considerably reduce labour in the application of Bessel's or Everett's interpolation formulae, and modified differences, particularly modified second differences, are now well established.[1] For a printed table, however, even though designed to be interpolable, it seems never to have been questioned that function values must not be altered, and rightly, since many users need only tabular values. With a punched-card table, however, where interpolation by mechanical means is the rule, the precise values used are unseen and only the interpolated result is relevant, so that modification of the function values may well be convenient.

Herget & Clemence have, however, failed to make full use of this principle. In their illustration, a table of $1/x$, they have aimed at a linear table by choosing an interval so that $\Delta'' \leqslant 4$, with the resulting effect of less than 5 units in the extra 7th, or 'guard,' decimal. This effect is *always of the same sign* throughout each interval in any table; for $1/x$, since Δ'' is positive and the interpolation coefficient negative, this means that values obtained by ordinary linear interpolation are always in excess of the true value (apart from rounding-off errors). Thus interpolated values may be improved, on the average, if the function values, already modified by Herget & Clemence, are further modified by subtraction of $\Delta''/16$, half the maximum Δ'' effect. By this further modification the error maximum is reduced by half, and errors are no longer systematically of the same sign.

Instead of having a reduced maximum error, the extra modification may be used to allow increased intervals; in fact, for $1/x$, the max. Δ'' is $2\omega^2x^{-3} = 8$, or $x^3 = \omega^2/4$. Taking values of ω as on p. 174, we have the approximate arguments for change of interval as follows:

ω	2	3	4	5	10	20	30	40	50
Arg.	1	1.310	1.590	1.850	2.930	4.700	6.100	7.400	8.600

This results in 924 cards for the table—a substantial improvement on the 1368 suggested by Herget & Clemence.[2]

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[1] See, for example, L. J. COMRIE, in B.A.A.S., *Mathematical Tables*, v. 1, p. xi–xii, London, 1930; *Interpolation and Allied Tables*, reprinted from the *Nautical Almanac* for 1937, p. 928–929, London, 1936. In these accounts the term 'modified differences' is given a special meaning, now accepted fairly widely. This use is not the same as that of HERGET & CLEMENCE, who use the term in place of the more familiar first *divided* differences (first used by A. DE MORGAN, *Differential and Integral Calculus*, p. 550, London, 1842); A. C. AITKEN has suggested the symbol $\blacktriangle$ for these (see H. FREEMAN, *Mathematics for Actuarial Students, Part II, Finite Differences, Probabilty & Elementary Statistics*, Cambridge, 1939, p. 41.

[2] It may be remarked that comparison with the 'usual' table of 9000 entries overestimates the value of the Optimum-Interval method. A six-figure table for 1000(1)3700(10)-10000 is linearly interpolable in the usual way, has intervals 1 and 10 only, and has only 4330 entries. The improvement remains striking.

25. A ROOT OF THE EQUATION $10 \log x = x$.—It is hard to find anything in ordinary mathematics which L. EULER did not in some way discuss. In N **20** (p. 203) we gave various results of the nineteenth century in connection with "coincidental logarithms." But already in the eighteenth century Euler had solved the problem, "Quaeratur numerus praeter 10, cujus logarithmus tabularis aequetur decimali parti ipsius numeri." (EULER, *Institutiones Calculi Differentialis*, St. Petersburg, 1755, p. 566–567; *Opera Omnia*, s. 1, v. 10, Leipzig and Berlin, 1913, p. 437–438). Euler found the number to be 1.37128857, agreeing exactly, for 7D, with what we recorded.

R. C. A.

QUERIES

10. ROUNDING-OFF NOTATION.—In RMT **157** (p. 192) attention is drawn by H.T.D. to a special use of the sign + by A. N. LOWAN and J. LADERMAN in their "Table of Fourier coefficients." In this paper the authors state that "A plus sign after an entry [to 10D] indicates that the eleventh decimal place is 5 or larger." This new use of the + seems, perhaps, a little unfortunate in view of (i) the variety of means already in use for indicating whether the final digit has been, or should be, raised or not, and (ii) T. N. THIELE's Rule, mentioned below, in which the same symbol + is used with a different meaning. It seems also worth while to note that the phrase used by Lowan and Laderman gave no hint, at any rate to the present writer, that the tenth decimal had not been raised in the usual way; it is easy to verify that, as H.T.D. notes, the tenth decimal has *not* been increased, but a hint at least sufficient to raise doubt seems essential.

As a single example of other methods for distinguishing whether the final digit should be, or has been, rounded off see *Four-Place Mathematical Tables with Forced Decimals* by F. S. CAREY and S. F. GRACE, Longmans, Green, London, 1927. In these, for some of the tables "Decimals in italics are in excess of the true value, those in thick type are in defect." Ordinary type is used when no discrimination is made.

The rule of T. N. THIELE was used by H. ANDOYER in his *Nouvelles Tables Trigonométriques Fondamentales* (*Logarithmes*), 1911, (*Valeurs naturelles*), 1, [Sines], 1915; 2, [Tangents], 1916; 3, [Secants], 1918, Hermann, Paris. Andoyer describes the rule in these words: "Si le nombre formé par les décimales qui suivent la dernière est compris entre 000 . . . et 250 . . . inclus, la dernière décimale est conservée telle quelle, sans aucun signe: si ce même nombre est compris entre 250 . . . et 750 . . ., la dernière décimale est encore conservée telle quelle, mais suivie du signe +; si enfin ce même nombre est supérieur ou égal à 750 . . ., la dernière décimale est forcée d'une unité, sans aucun signe." J. R. AIREY uses a colon ':' instead of '+' to give the same information, see, for instance, his table of the Lommel-Weber functions $\Omega_0(x)$ and $\Omega_1(x)$ in B.A.A.S., *Report*, 1924, p. 280.

In RMT **157** H.T.D. also refers to the use of 'high' and 'low' dots by L. M. MILNE-THOMSON & L. J. COMRIE in their *Standard Four-Figure Mathematical Tables* (London, Macmillan, 1931); the present writer finds this device both useful and practical.

What other devices have been used to give an accuracy beyond that which can be obtained by a bare final digit? What useful purpose is served by such of those devices as require extra printing space which could not be as well or better served by giving an extra figure, possibly correct only within 1 or 2 units, instead of the more usual $\frac{1}{2}$ unit (or sometimes 0.52 or 0.6 unit)?

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