EDITORIAL NOTE: In the tables of N. SAMOĬLOVA-ĬAKHONTOVA, RMT **50**, p. 6, is a table of u for $\phi = 0(1°)\ 90°$, and $k^2 = [0(.01)1;\ 5D]$, Δ. The tables of L. M. MILNE-THOMSON, *Die elliptischen Funktionen von Jacobi*, Berlin, Springer, 1931, include a table of $sn(u, k^2)$, for $k^2 = 0(.1)1$ and $u = 0(.01)3$, with Δ; also for $k^2 = 1$, and $u = 3(.1)6.5$.

28[L].—GEORGE WELLINGTON SPENCELEY (1886–). *Tables of the seven elliptic functions A, D, (Jacobi Theta Functions) F, E, sn, cn, dn,* prepared during the years 1938–43, with the seasonal assistance of N.Y.A. students, Miami University, Oxford, Ohio.

These tables were inspired by, and follow precisely, the pattern designed by GREENHILL, and carried out by R. L. HIPPISLEY (*Smithsonian Mathematical Formulae and Tables of Elliptic Functions*, Washington, D. C., first reprint, 1939). Hippisley's tables, to 10D, consist essentially of the four elliptic functions A, D, F, E, computed for modular angle $\theta = 5°(5°)80°(1°)89°$. His F column is the traditional value; his E column is the "periodic part of $E(\varphi)$"; his A, D columns are "normalized Jacobi Theta Functions."

The author's tables are more complete in that they are computed to 15D for modular angle $\theta = 1°(1°)89°$, and also include the sn, cn, dn functions. In these tables the E column has its traditional value. The *ms* shows 15D for all columns, with an error not greater, it is believed, than $\pm 2 \times 10^{-15}$. The original intention of the author was to cut to 12D on publication.

G. W. SPENCELEY

MECHANICAL AIDS TO COMPUTATION

11[H, Z].—D. P. ADAMS, "The quintic 'hypernom' for the equation $x^5 + Ax^3 + Bx^2 + Cx + D = 0$, a graphical method for finding the roots of polynomial equations through the fifth degree," *J. Math. Phys.*, M.I.T., v. 22, 1943, p. 78–92. 17.6 × 25.5 cm.

This paper describes a chart for the approximate determination of the roots of the reduced quintic

$$x^5 + Ax^3 + Bx^2 + Cx + D = 0.$$

By direct use of the chart, the real positive roots not exceeding 3.5 may be located in case the coefficients A, B, C, D do not exceed 10 in absolute value. By the simple expedient of changing the signs of B and D, the negative roots greater than -3.5 likewise may be determined. The author's statement that root values lie between 0 and 10 appears to be an error.

The chart consists of

(1) Three vertical linear scales for A, B, and C at the side margins,
(2) A family of curves D = constant,[1]
(3) Two families of vertical lines (L lines and M lines) bearing indices ranging from 1 to 32,
(4) Two identical horizontal non-linear "root" scales at the upper and lower margins.

The chart is unusual in that an auxiliary "S" curve must be plotted on it, a curve characteristic of the particular equation under investigation (depending, in fact, on A, B, and C only). Perhaps this feature accounts for the name "hypernom." The intersections (if any) of this S curve with the curve of the D family corresponding to the constant term of our equation determine the positive roots of the equation. The plotting of the S curve (which is preferably done on tracing paper placed over the chart), is fairly simple and consists in determining a series of points by drawing parallel lines across the grid of L and M lines.

The chart is intended as an aid to Horner's or Newton's method. The paper has six illustrative problems showing the use of the chart in various cases. Well-known methods of finding the 2 or 4 complex roots, once the real roots of the quintic have been found, are discussed. In all these examples the residual cubics and quartics happen to have coefficients not exceeding 10 in absolute value and roots less than 3.5. Some treatment of the question of quickly transforming quintic equations with large roots or large coefficients in order to be able to use the chart, would have been interesting. The question is not trivial. The forthright method of multiplying all the roots by some small factor may not prove effective. In fact when the coefficients are thus made small the S curve is nearly coincident with the D curve and the two do not have well defined intersections.

In dealing with problems in which a large number of quintic equations with more or less the same distribution of coefficients occur it is evident that a chart of this kind would eliminate a good deal of exploratory work and make the whole process quite methodical.

D. H. L.

[1] This family is essentially the same as that given by L. I. HEWES and H. L. SEWARD: *The Design of Diagrams for Engineering Formulas and the Theory of Nomography*, New York, McGraw-Hill, 1923, p. 68, in their nomogram for the complete cubic equation.

NOTES

21. COMPUTING A TABLE OF LOG $\Gamma(x)$.—Incidental to some required computation on a problem in ballistic theory it seemed necessary to prepare a table of the logarithms of the Γ function for the range x = [1(.01)16.99; 7D]. This table, available on punched cards and also in typed form exhibiting the first, second and third differences, was made as follows:

The values of $\log \Gamma(x)$ for $x = 1, 1.01, \cdots, 1.99$ were punched from the 7-place table of the *Smithsonian Physical Tables*, Washington, 1934, p 64–65. The Vega 7-place table of logarithms on punched cards allowed the use of the relation

$$\log \Gamma(x) = \log \Gamma(x-1) + \log (x-1), \tag{1}$$

for the range $x = 2, 2.01, \cdots, 2.99$, to get $\log \Gamma(x)$ for $x = 2, \cdots, 2.99$. With these newly computed values the relation (1) was again used for the range $x = 3.00, 3.01, \cdots, 3.99$, giving $\log \Gamma(x)$ for this new range. This process was applied 15 times.

In the range $16 \leqq x < 17$ exactly 15 logarithms were added to the original values of $\log \Gamma(x)$, namely:

$$\log \Gamma(x) = \log (x-1) + \log (x-2) + \cdots + \log (x-15) + \log \Gamma(x-15).$$

Since the error in each term is $< \frac{1}{2}\, 10^{-7}$, the error of any entry in our table will never reach *.000 000 8*. Because of compensation of errors the table would most likely be accurate if we rounded it off to 6 places. Of course it is absolutely accurate if rounded off to 5 places.

The whole operation was so well suited to punched-card methods that it required about a day's work for two operators.

BALLISTIC RESEARCH LABORATORY

Aberdeen Proving Ground, Md.

EDITORIAL NOTE: While this table is not new, its method of computation and its listing as on I.B.M. punched cards seemed worthy of record. W. T. RUSSELL's table of $\log \Gamma(x)$ for x=[1(.01)50.99; 7D] was published in *Tracts for Computers*, no. IX (1923), ed. by K. Pearson. The Smithsonian table referred to above for x=[1(.001)2; 7D] was taken from the table by LEGENDRE in his *Exercices de Calcul Intégral*, v. 2, 1816, p. 85–95, where there is a table of $\log \Gamma(x)$ for x=[1(.001)2; 12D], Δ^3. A reprint of this table appeared in O. SCHLÖMILCH, *Analytische Studien*, part 1, Leipzig, 1848. The range for such a table was extended for x=[2(.1)70(1)1200; 10D], with second, fourth, and sixth differences, in *Tracts for Computers*, no. VIII (1922), ed. by E. S. PEARSON. In his *Tables of the Higher Mathematical Functions*, Bloomington, Ind., 1933, p. 195–255, H. T. DAVIS has the following tables of $\log \Gamma(x)$: x=[1(.0001)1.1; 10D], Δ^2; x=[1(.001)2; 12D], Δ^3; x=[2(.01)11; 12D], δ^2, δ^4; x=[11(.1)101; 12D]. $\text{Log}\,\Gamma(x+1)$ for x=[1(1)1200; 18D] is given in two places, namely: (a) C. F. DEGEN, *Tabularum ad faciliorem et breviorem Probabilitatis computationem utilium Enneas*, Copenhagen, 1824; and (b) PETERS and STEIN, *Zehnstellige Logarithmentafel*, v. 1, *Anhang*, Leipzig, 1922, p. 61–68. A table for $\log \Gamma(x+1)$ for x=[1(1)3000; 33D] was given by F. J. DUARTE in his *Nouvelles*