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Zeros of Certain Bessel Functions of Fractional Order

The following tables contain the zeros of $J_\nu(x)$ for $x \leq 25$, where $\nu = \pm 1/3, \pm 2/3, \pm 1/4, \pm 3/4$. These zeros were obtained by inverse interpolation in a thirteen-place manuscript of these functions, computed by the NYMTP. The accuracy of the zeros to 10D is guaranteed, and the two additional places have a high probability of being correct.

| s | $j_{+1/4,s}$ | $j_{-1/4,s}$ |
|-----|----------------------------|----------------------------|
| 1 | 2.78088 77239 95 | 2.00629 96717 90 |
| 2 | 5.90614 26988 43 | 5.12306 27427 46 |
| 3 | 9.04238 36635 83 | 8.25795 11756 42 |
| 4 | 12.18134 15289 55 | 11.39646 76969 87 |
| 5 | 15.32136 98260 12 | 14.53629 98843 38 |
| 6 | 18.46192 72456 89 | 17.67675 35868 47 |
| 7 | 21.60278 44489 13 | 20.81754 94222 32 |
| 8 | 24.74382 77961 28 (24.740) | 23.95855 34952 86 (23.955) |

| s | $j_{+3/4,s}$ | $j_{-3/4,s}$ |
|-----|----------------------------|----------------------------|
| 1 | 3.49100 83741 08 | 1.05850 82594 04 |
| 2 | 6.65263 55231 22 | 4.28405 38127 24 |
| 3 | 9.80161 23591 40 | 7.44045 44040 05 |
| 4 | 12.94703 48891 39 | 10.58817 91486 60 |
| 5 | 16.09096 95281 99 | 13.73311 84505 74 |
| 6 | 19.23414 17604 82 | 16.87681 75138 75 |
| 7 | 22.37687 15748 16 (22.384) | 20.01985 75839 86 |
| 8 | | 23.16250 59340 75 (23.169) |

| s | $j_{+1/2,s}$ | $j_{-1/2,s}$ |
|-----|----------------------------|----------------------------|
| 1 | 2.90258 62484 17 | 1.86635 08588 74 |
| 2 | 6.03274 70572 66 | 4.98785 32314 35 |
| 3 | 9.17050 66694 64 | 8.12426 53819 40 |
| 4 | 12.31019 37716 45 | 11.26351 48254 28 |
| 5 | 15.45064 89678 17 | 14.40377 58801 36 |
| 6 | 18.59148 63361 81 | 17.54451 06557 21 |
| 7 | 21.73254 11617 47 | 20.68550 48061 24 |
| 8 | 24.87373 14228 06 (24.871) | 23.82665 62470 57 (23.824) |

| s | $j_{+2/3,s}$ | $j_{-2/3,s}$ |
|-----|----------------------------|----------------------------|
| 1 | 3.37561 06526 94 | 1.24304 62596 19 |
| 2 | 6.53025 59365 13 | 4.42912 06776 99 |
| 3 | 9.67658 06352 38 | 7.57945 84465 30 |
| 4 | 12.82060 86784 66 | 10.72474 69244 99 |
| 5 | 15.96368 38809 06 | 13.86837 45833 31 |
| 6 | 19.10627 35045 92 | 17.01125 45001 33 |
| 7 | 22.24858 23933 60 (22.253) | 20.15373 45371 51 |
| 8 | | 23.29597 58670 60 (23.300) |

For $x > 25$, ten or more decimal places in the zeros may be obtained from the well-known formula (five terms) for the roots of Bessel functions, given below; see, for example, G. N. WATSON, *Treatise on the Theory of Bessel Functions*, 1922 and 1944, p. 506; the sixth term was supplied by W. G. BICKLEY,¹ *Phil. Mag.*, s. 7, v. 34, 1943, p. 40:

$$j_{\nu,s} = \beta - \frac{\mu - 1}{2^3\beta} - \frac{(\mu - 1)(7\mu - 31)}{3 \cdot 2^7 \cdot \beta^3} - \frac{(\mu - 1)(83\mu^2 - 982\mu + 3779)}{15 \cdot 2^{10} \cdot \beta^5} - \frac{(\mu - 1)(6949\mu^3 - 153855\mu^2 + 1585743\mu - 6277237)}{105 \cdot 2^{15} \cdot \beta^7} - \frac{(\mu - 1)(70197\mu^4 - 24\,79316\mu^3 + 480\,10494\mu^2 - 5120\,62548\mu + 20921\,63573)}{315 \cdot 2^{18} \cdot \beta^9} - \dots,$$

where $\beta = (s + \frac{1}{2}\nu - \frac{1}{4})\pi$; $\mu = 4\nu^2$. The first five terms of the above expression will yield at least 10 decimals for roots greater than those given here. The values of β corresponding to the highest roots given here are noted in parenthesis; it is apparent that for x close to 25, β approximates the root to two decimals at least.

The author desires to express his appreciation of assistance rendered by several members of the NYMTP in checking these values.

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¹ It may be noted that in Bickley's article the fifth term of the formula has an erroneous number 6277327, for 6277237.