

One obvious method of increasing the number of possible applications has been to prepare cards punched with the values of numerical functions which can be combined with other values by multiplication or addition. Of this type are exponential and logarithmic functions.

The first note indicates the usefulness of punched-card tables of the exponential function. From the relationship,  $\exp(a + b) = (\exp a)(\exp b)$ , one sees that this function is readily adapted to calculation by punched-card machines, any desired degree of accuracy being easily attained. Thus, the master set of cards might contain values of  $\exp \pm n$  over the range  $n = 0.00(0.01)12.00$ . A second set of 100 cards for  $n = 0.0000(0.0001)(0.0099)$  would then multiply the original range a hundredfold.

One of the most important problems in the use of punched cards is the verification of the numbers punched on them. For this purpose the author suggests the use of the formula

$$\sum_{k=0}^{N-\Delta k} e^{-k} \frac{1 - e^{-N}}{1 - e^{-\Delta k}}.$$

Partial sums of the cards are compared with the values of this function computed for suitably chosen values of  $\Delta k$ .

The second note suggests the use of punched-card tables of  $\log x$ . The authors suggest that the verification of the cards be made by comparing partial sums of the cards with computed values of  $\log n!$  from the formula:

$$\sum_{k=1}^n \log k = \log n!.$$

Tables such as those of J. BOCCARDI: *Tables logarithmiques des factorielles jusqu'à 10,000!*, Cavaillon, 1932, or F. J. DUARTE: *Nouvelles tables logarithmiques à 36 décimales*, Paris, 1933, are available for this purpose.

H. T. D.

## NOTES

29. EARLY DECIMAL DIVISION OF THE SEXAGESIMAL DEGREE.—In *MTAC*, p. 33, 100 (corrigenda), 129–130, it has been already noted that decimal division of the degree was used by RUFFI in a Latin codex of about 1450, as well as by BRIGGS, in tables prepared before 1633, the idea having been suggested to him by Viète in a work published in 1600. The decimal division of the degree was also advocated by SIMON STEVIN (1548–1620) of Bruges in 1585, in his Flemish work, 1. *De Thiende*, published at Leiden. A facsimile of this 36-page booklet is given in H. BOSMANS, *La 'Thiende' de Simon Stevin*, Antwerp and The Hague, 1924; a French edition of *De Thiende, La Disme*, occupied p. 132–160 of Stevin's 2. *La Pratique D'Arithmétique*, also published at Leiden in 1585; there are copies of this work in the Plimpton Library of Columbia University, the Royal Library of Belgium, University of Liège, etc. A facsimile of this edition was published by G. SARTON in "The first explanation of decimal fractions and measures (1585). Together with a history of the decimal idea and a facsimile (no. XVII) of Stevin's Disme," *Isis*, v. 23, 1935, p. 230–244. A French edition, ed. by A. Girard, appeared also in a new edition of 3. *L'Arithmétique*, Leiden, 1625, p. 823–849, and in 4. S. STEVIN, *Les Oeuvres Mathématiques*, Leiden, 1634, p. 206–213. There were other Flemish editions, 5–6, in 1626 (in the Brown University Library), and in 1630.

This was the first work to set forth the theory of decimal fractions. It was translated into English by 7. ROBERT NORTON, "engineer and gunner" (d. 1635, *Dict. Nat. Biog.*), son of the poet THOMAS NORTON (1532–1584),

under the title *Disme: The Art of Tenths, or, Decimall Arithmetike, Teaching how to performe all Computations whatsoever, by whole Numbers without Fractions, by the foure Principles of Common Arithmeticke . . . Invented by the excellent Mathematician Simon Steuin*, London, 1608; there are copies of this work in the British Museum, the Cambridge University Library, and in the Harvard University Library (a copy formerly in the W. A. White Library). According to the *Cambridge Bibliography of English Literature*, there were other editions, 8(?) in 1614 and, by HENRY LYTE, in 1619. The last statement is certainly incorrect in the case of Lyte's independent work (with a similar title), of which copies are in the Huntington Library, Pasadena, and the British Museum. See further SARTON, l.c., p. 158f. Norton supplied tables of interest and measurement, and instructions in decimal arithmetic to Robert Recorde's *The Grovnd of Artes*, London, 1623. See also H. Bosmans, "La 'Thiende' de Simon Stevin," *Rev. d. Questions Sci.*, s. 3, v. 27, 1920, p. 109-139.

The first of two paragraphs in "Article V, Des Compvttations Astro-nomiques" of the "Appendice" of *La Disme* (1585), with its illustration of Stevin's decimal notation (by "commencemens" he referred to the integral part of the decimal) is as follows:

Aians les anciens Astronomes parti le circle en 360 degrez, ils voioient que les computations Astronomiques d'icelles, avec leurs partitions, estoient trop labourieuses, pourtant ils ont parti chasque degré en certaines parties, & les mesmes autrefois en autant, &c. à fin de pouuoir par ainsi tousiours operer par nombres entiers, en choissisans la soixantiesme progression, parce que 60 est nombre mesurable par plusieurs mesures entieres, à sçauoir 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, mais si l'on peut croire l'experience (ce que nous disons par toute reuerence de la venerable antiquité & esmeu avec l'vtilité commune) certes la soixantiesme progression n'estoit pas la plus commode, au moins entre celles qui consistoient potentiellement en la nature, ains la dixiesme qui est telle: Nous nommons les 360 degrez aussi *commencemens*, les denotans ainsi 360(0), & chascun degré ou 1(0) se diuïsera en 10 parties egales, desquelles chascune sera 1(1), puis chasque 1(1) en 10(2), & ainsi des autres, comme le semblable est fait par plusieurs fois ci deuant.

[In 4 places near the end of this quotation we have, for typographical reasons, printed (0), (1), (2), instead of the original notation of the figures 0, 1, 2 surrounded by circles.]

R. C. A.

30. FIRST PUBLISHED COMPOUND INTEREST TABLES.—These were 1. Simon Stevin's *Tafelen van Interest, Midtsgaders De Constructie der seluer*, [Tables of interest together with their construction], Antwerp, 1582, 92 p. A French translation from the Dutch was published in 2. Stevin's *La Pratique D'Arithmetique*, Leiden, 1585, and an edition 3. by Girard in 1625 was reprinted in 4. Stevin's *Oeuvres Mathematiques*, Leyden, 1634, p. 185-206. A facsimile of the original work is given in 5. C. M. WALLER ZEPER, *De oudste intrestafels in Italië, Frankrijk en Nederland met een herdruk van Stevins "Tafelen van Interest,"* Diss. Leiden, Amsterdam, 1937.

Stevin tells us in his preface (dated Leiden July 16, 1582) that the inventor of these tables was JEAN TRENCHANT, who gave specimens of them

in his *L'Arithmetique de Ian Trenchant departie en troys livres, Ensemble vn petit discours des changes, avec L'art de calculer aux Getons*, Lyons, 1558. See G. SARTON, "JEAN TRENCHANT, French mathematician of the second half of the sixteenth century," *Isis*, v. 21, 1934, p. 207-209; and H. BOSMANS, Soc. Sci. de Bruxelles, *Annales*, v. 33, 1909, part 1, p. 184-192.

Since the description by A. DeMorgan of Stevin's Tables (*Supplement to the Penny Cyclopaedia*, v. 2, London, 1846, p. 604) contains an interesting suggestion we shall quote this: "it is pretty certain that these tables of compound interest *suggested decimal fractions*, the account of which speedily follows them. They are constructed as follows:—Ten millions being taken as the base (or *root*, as Stevinus calls it), and a rate, say five per cent., being chosen, the present value of ten millions due at the end of 1, 2, &c., up to 30 years, are put in a column, to the nearest integer. By their sides are sums of their values, which give the present values of the several annuities of ten million, as follows:

Table d'Interest de 5 pour 100.		
1	9523810	9523810
2	9070295	18594105
3	8638376	27232481
4	8227025	35459506
. . . . .	. . . . .	. . . . .
30	2313774	153924494 <sup>1</sup>

The rates are from 1 to 16 per cent., and also for 1 in 15, 1 in 16, &c., to 1 in 22,<sup>2</sup> or, as the French say, denier quinze, denier seize, &c. At the end is a direction to dispense, when convenient, with some of the last figures."

"There is thus a virtual use of decimal fractions preceding the formal one. The same thing happens in the tables of RICHARD WITT, [*Arithmetical questions, touching the Buying or Exchange of Annuities* . . . , London, 1613] . . . which we believe to be the first English tables of compound interest, and the first English work (except the translation of the *Disme* of Stevinus) in which decimals were used."

R. C. A.

<sup>1</sup> Apparently De Morgan is here quoting from Stevin's *Oeuvres*. It should be 153724494 as in the 1582 and 1585 editions.—EDITOR.

<sup>2</sup> The table for 1 in 20 is not given since it would be the same as the 5% table.—EDITOR.

31. FIRST PUBLISHED MORTALITY TABLE.—At a time when the city of Breslau is so much in the war news it is appropriate here to recall that mortality records of this city were the basis of a paper, written by EDMOND HALLEY (1656-1742) over 250 years ago, and published in R. So. London, *Phil. Trans.*, 1693, no. 196, p. 596-610, and no. 198, p. 654-656. It is entitled "An estimate of the degrees of the mortality of mankind, drawn from curious tables of the births and funerals at the city of *Breslaw*; with an attempt to ascertain the price of *annuities* upon lives." Halley reprinted this in his *Miscellanea Curiosa*, London, v. 1, 1705; second ed. 1708; third ed., 1726, p. 280-301. This was also reprinted in *Institute of Actuaries, J.*, v. 18, 1874, p. 251f and 262f. Here we have the beginnings of statistical science and the first published mortality table, in the form since followed, to show the annual movement of a population, the problem of survivorship, the average

duration of one or more lives, and the money values depending thereon. See C. WALFORD, *Insurance Cyclopaedia*, London, v. 5, 1878, p. 616–618; v. 1, 1871, "Breslau table of mortality," p. 360–363; H. Böckh, "Halley als Statistiker. Zur Feier des zweihundertjährigen Bestehens von Halleys Sterblichkeitstafel," *L'Institut International de Statistique, Bull.*, v. 7, 1893, p. 1–24; A. LOEWY, *Versicherungsllexikon*, ed. by A. Manes, third ed., Berlin 1930, cols. 742–744, 998–1000; E. J. FARREN, "On indirect methods of acquiring knowledge.—The method of history.—The first table of mortality," *Assurance Mag.*, v. 1, 1851, p. 40f.

Perhaps the inclusion of the following item is warranted, in not being too far removed from the main topic of the note. In *Math. Gazette*, v. 14, Dec. 1929, p. 574f, I published a Note on "A letter of DeMoivre and a theorem of Halley." The Note consisted mainly of a letter from De Moivre (1667–1754) to Halley which I copied from *The Mathematical Magazine and Philosophical Repository*, 1761 (see *Math. Gazette*, v. 14, p. 387, and v. 15, p. 215f) where it is stated that the letter, which is undated, was never before published. The theorem gives the rate of interest for an annuity, in a compact formula,  $1 + b - \sqrt{b^2 - 2by}$  (for explanation of the notation, see the source). I asked, "Can any reader state where Halley's theorem is to be found or otherwise assist in determining an approximate date for the letter?" A long and very interesting reply from F. P. White, of St. John's College, Cambridge, appeared in the *Math. Gazette*, v. 15, Oct. 1930, p. 213f. The formula was first given by Halley in his chapter, "Of compound interest and annuities,"<sup>1</sup> in the first edition (1705) of Sherwin's *Mathematical Tables*. Hence De Moivre's letter, which was found among Halley's papers after his death in 1742, may have been written before 1706.

Halley was in 1703 the successor of Wallis as Savilian professor of geometry at Oxford, and in 1721 followed Flamsteed as astronomer royal. Due to his efforts the publication in 1687 of Newton's *Principia* was achieved. Among his contributions to astronomy was the discovery in 1682 of the comet, with a period of 76 years, last seen in 1910. He was also the translator from the Arabic and restorer of works of Apollonius of Perga (Oxford, 1706–1710).

R. C. A.

<sup>1</sup>It was reprinted in *Assurance Mag. and J.*, v. 9, 1861, p. 259–269.

32. GUIDE (MTAC no. 7), SUPPL. 1.—In F. BERGER, "Über den Temperaturverlauf in einem Zylinder von endlicher Länge beim Abkühlen und Erwärmen," *Z. f. angew. Math. u. Mech.*, v. 11, 1931, p. 53, there are 8 values (multiplied by 80), to 4–5D, of roots of the equation  $xJ_1(x) = .53J_0(x)$ . On p. 50 are graphical solutions of the first five roots. In GERALD PICKETT, "The effect of Biot's modulus on transient thermal stresses in concrete cylinders," *Am. So. Testing Materials, Trans.*, v. 39, 1939, p. 916, a table is given of the first six roots of

$$xJ_1(x) = BJ_0(x),$$

$B = .1, .5, 1, 3, 10, 100, 1000, 10\ 000$ . The first five roots are to 3D and the sixth to 2D.

In H. A. EINSTEIN, "Der Geschiebbetrieb als Wahrscheinlichkeits-

problem," diss. Eidgen. techn. Hochschule, Zürich, Versuchsanstalt für Wasserbau, *Mitteilung*, 1937, p. 93, there are graphs of the functions  $e^{-x}I_0(x)$  and  $e^{-x}I_1(x)$  on a logarithmic scale in which  $x$  ranges from .1 to about 100. T. II gives 3D for  $e^u I_0(u)$ ,  $u = 2(xT)^{\frac{1}{2}}$ , and 4D for exp.  $[-(x^{\frac{1}{2}} - T^{\frac{1}{2}})^2]e^{-u}I_0(u)$ ,  $T = 0.895, 1.689, 3.14, 6.05, 11.48, 18.57$ ;  $\sqrt{x}$  has various values from 0 to 6.18, a different set for each value of  $T$ .

In I. E. GARRICK, "On some reciprocal relations in the theory of non-stationary flows," U. S. Nat. adv. Comm. f. Aeronautics (NACA), *Techn. Reports*, no. 629, 1938, p. 348, for Theodorsen's function,

$$C(k) = H_1^{(2)}(k)/[H_1^{(2)}(k) + iH_0^{(2)}(k)] = F(k) + iG(k),$$

there are graphs of

$$F(k) = [J_1(J_1 + Y_0) + Y_1(Y_1 - J_0)]/[(J_1 + Y_0)^2 + (Y_1 - J_0)^2]$$

$$G(k) = -[Y_1 Y_0 + J_1 J_0]/[(J_1 + Y_0)^2 + (Y_1 - J_0)^2],$$

$0 \leq k \leq 2$ , and also of  $\sqrt{F^2 + G^2}$  and  $\tan^{-1}(G/F)$ ,  $0 \leq k \leq 1$ . A table with 4D for  $k = .1(.1).6(.2)1, 2, 4, 6, 10$  is given by T. THEODORSEN in the paper "General theory of aerodynamic instability and the mechanism of flutter," NACA, *Techn. Reports*, no. 496, 1934, p. 426,  $J_0(k)$ ,  $J_1(k)$ ,  $Y_0(k)$ ,  $Y_1(k)$ ,  $F(k)$ , and with 3-4D for  $F$ , and  $G$ , for the same values of  $k$  and also  $k = 0, .025, .05, \infty$ . On p. 418 there are graphs of  $F$  and  $G$  against  $1/k$  for  $0 \leq 1/k \leq 40$ . On p. 350 Garrick gives a table with 4D of Wagner's function  $k_1(x)$  for  $x = 0(.5)1(1)10, 20, \infty$ , and also a graph (p. 347), where  $k_1(x) = (2/\pi) \int_0^\infty \sin(xt)F(t)dt/t = 1 + (2/\pi) \int_0^\infty \cos(xt)G(t)dt/t$ . These values of  $k_1(x)$  are taken from a paper by H. WAGNER, "Über die Entstehung des dynamischen Auftriebes von Tragflügeln," *Z. angew. Math. u. Mech.*, v. 5, 1925, p. 31.

In MAX JACOB, "Influence of nonuniform development of heat upon the temperature distribution in electrical coils and similar heat sources of simple form," A.S.M.E., *Trans.*, v. 65, 1943, p. 597, there is a table of the function

$$\frac{J_0(xy) - J_0(x)}{1 - J_0(x)} \quad \begin{array}{l} x = 0, .1, 1, 2, 2.3, 2.4048; \\ y = 0(.1)1. \end{array}$$

In Fig. 3 the values  $x = 0, 1, 2$  and  $2.4048$  are used.

In C. WEBER, "Zum Zerfall eines Flüssigkeitsstrahles," *Z. f. angew. Math. u. Mech.*, v. 11, 1931, p. 150, values are given for  $f_0(x)$ ,  $f_1(x)$ , where  $f_0(x) = K_0(x)/K_1(x)$ , and  $f_1(x) = x/[xf_0(x) + 1]$ ,  $x = [.1(.1)1(.2)2(.5)4(1)9; 3D]$ .

H. B.

**33. A NEW RESULT CONCERNING A MERSENNE NUMBER.**—On 2 December 1944 I finished the proof that the Mersenne number  $M_{167}$  is composite; see *MTAC*, p. 333. Hence there now remain only four of these numbers  $M_p$  ( $p = 193, 199, 227, 229$ ) whose character is unknown. The method of testing primality was the same as that described in *Nat. Acad. Sci., Proc.*, v. 30, Oct. 1944, p. 314-316.

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34. ORIGINATORS OF THE TERM RADIAN.—As long ago as 1910 THOMAS MUIR pointed out (*Nature*, v. 83, p. 156) that while the earliest recorded use of the term *Radian* in the *New English Dictionary* was in 1879, in the first part of the new edition of the first volume of William Thomson and P. G. Tait's *Treatise on Natural Philosophy*, "my own first use of it was in class-teaching in the College Hall at St. Andrews in 1869, and I possess a notebook, belonging to one of my students of that year, in which the word is used." He hesitated, however, in a definite choice between the terms radial, radian, rad. But he states that as a result of reading a publication of A. J. ELLIS (1814–1890) and exchanging letters with him in 1874 "the form radian was definitely adopted by me." Ellis remarks that he had used the term "Radial angle" from his Cambridge undergraduate days, but Muir stated that Ellis approved of radian as a contraction of "radi-al an-gle." From later correspondence in *Nature*, v. 83, p. 217, 459–460, it appears that, wholly independent of Muir, JAMES THOMSON (1822–1892), brother of the above-mentioned William, proposed the name Radian in July 1871 and that he used it in an examination paper at Queen's College, Belfast, on June 5, 1873, published in the college calendar for 1873–74.

Bibliographic reports on the use before 1869 of the term *Radial Angle*, as equivalent to *Radian*, are desired. This term is not listed in *N.E.D.*

R. C. A.

35. *Phil. Mag.* TABLES, SUPPL. 3 (for Suppl. 1–2, see *MTAC*, p. 201–202).—W. G. BICKLEY, "Deflexions and vibrations of a circular elastic plate under tension," s. 7, v. 15, 1933, p. 795. The table gives, to 5S, the first two roots of

$$\frac{xJ_{n+1}(x)}{J_n(x)} = -\frac{\sqrt{(x^2 + c^2)}I_{n+1}\sqrt{(x^2 + c^2)}}{I_n\sqrt{(x^2 + c^2)}}$$

for  $n = 1, c = 0, 1, 2, 5, 10, 20$ ; the values of  $x$  and  $x^2(x^2 + c^2)$  are given, and for  $n = 2$  the same quantities are given for the first root. This item was overlooked in the *Guide*, *MTAC* 7.

H. B.

36. ZEROS OF THE BESSEL FUNCTION  $J_\nu(x)$ .—If we denote, as usual, the  $k$ -th positive zero of  $J_\nu(x)$  by  $j_{\nu,k}$  then the symmetric function

$$\sigma_{2n}(\nu) = \sum_{k=1}^{\infty} (j_{\nu,k})^{-2n}$$

is, for each positive integer  $n$ , a rational function of  $\nu$ . It was first used by RAYLEIGH<sup>1</sup> for the calculation of  $j_{0,1}$  and  $j_{1,1}$  and later by AIREY<sup>2</sup> and others for many values of  $j_{\nu,1}$ . These functions  $\sigma_{2n}(\nu)$  are also important as coefficients of the meromorphic functions

$$\frac{1}{2}J_{\nu+1}(X)/J_\nu(X) = \sum_{n=1}^{\infty} \sigma_{2n}(\nu)X^{2n-1}$$

$$\frac{1}{2}J_\nu(X)/J_{\nu+1}(X) = (\nu + 1)X^{-1} - \sum_{n=1}^{\infty} \sigma_{2n}(1 + \nu)X^{2n-1}.$$

This last expansion, in effect, was given as far as  $n = 4$  by JACOBI<sup>3</sup> in 1849. This is doubtless the first appearance of these functions  $\sigma$ . Later writers have given lists of these functions as follows:

author		range of $n$
RAYLEIGH <sup>1</sup>	(1874)	1(1)5, 8
GRAF & GUBLER <sup>4</sup>	(1898)	1(1)5
NIELSEN <sup>5</sup>	(1904)	1(1)5
KAPTEYN <sup>6</sup>	(1906)	1(1)6
FORSYTH <sup>7</sup>	(1920)	1(1)3
WATSON <sup>8</sup>	(1922)	1(1)5, 8

As a by-product of a recent investigation the first dozen of the functions were computed and are given below. They have interesting properties which may be discussed elsewhere. We need only the following explanations here. If we use  $[x]$  to denote, as usual, the greatest integer  $\leq x$ , and if we define the polynomial  $\pi_n(\nu)$  by

$$\pi_n(\nu) = \prod_{k=1}^n (k - \nu)^{[n/k]}$$

then the function  $\phi_n(\nu)$  defined by

$$\sigma_{2n}(\nu) = 2^{-2n} \phi_n(\nu) / \pi_n(\nu)$$

is a polynomial of degree

$$d = 1 - n + \sum_{k=2}^n \left[ \frac{n}{k} \right].$$

If we write

$$\phi_n(\nu) = a_0^{(n)} + a_1^{(n)}\nu + \dots + a_d^{(n)}\nu^d,$$

then the coefficients  $a_h^{(n)}$  for  $n \leq 12$  are given in the following table.

$h$	$n = 1$	2	3	4	5	6	7	8	9
0	1	1	2	11	38	946	4580	202738	3786092
1				5	14	1026	4324	311387	6425694
2						362	1316	185430	4434158
3						42	132	53752	1596148
4								7640	317136
5								429	33134
6									1430

  

$h$	$n = 10$	11	12
0	261868876	1992367192	2381255244240
1	579783114	4028104212	7315072725560
2	547167306	3458238276	10093635442028
3	287834558	1649756012	8275251041478
4	92481350	479550668	4491836314618
5	18631334	87264812	1701744523728
6	2305702	9748732	461790600920
7	160850	614228	90534103564
8	4862	16796	12743301316
9			1257916872
10			82812812
11			3271518
12			58786

Thus for  $n = 6$  we have

$$\sigma_{12}(\nu) = \frac{42\nu^3 + 362\nu^2 + 1026\nu + 946}{2^{12}(\nu + 1)^6(\nu + 2)^3(\nu + 3)^2(\nu + 4)(\nu + 5)(\nu + 6)}.$$

D. H. L.

<sup>1</sup> RAYLEIGH, London Math. So. *Proc.*, s. 1, v. 5, 1874, p. 119–124; *Scientific Papers*, v. 1, 1899, p. 192, 195. The entry for  $n=8$  is due to A. CAYLEY; see also his *Collected Papers*, v. 9, 1896, p. 20.

<sup>2</sup> J. R. AIREY, *Phil. Mag.*, s. 6, v. 41, 1921, p. 200–203.

<sup>3</sup> C. G. J. JACOBI, *Astr. Nachrichten*, v. 28, 1849, cols. 93–94; *Gesammelte Werke*, Berlin, v. 7, 1891, p. 173 [for  $10i + 32$ , read  $10i + 22$ ].

<sup>4</sup> J. H. GRAF & E. GUBLER, *Einleitung in die Theorie der Bessel'schen Funktionen*, v. 1, Bern, 1898, p. 130.

<sup>5</sup> N. NIELSEN, *Handbuch der Theorie der Cylinderfunktionen*, Leipzig, 1904, p. 360.

<sup>6</sup> W. KAPTEYN, *Archives Néerlandaises d. Sci. exactes et nat.*, s. 2, v. 11, 1906, p. 149, 168.

<sup>7</sup> A. R. FORSYTH, *Mess. Math.*, s. 2, v. 50, 1920, p. 135.

<sup>8</sup> G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, Cambridge, 1922 or 1944, p. 502.

### QUERIES

13. TABLES OF INTEGRALS.—We are now interested in evaluating integrals of the following forms:  $\int_x^\infty e^{-t} dt/t^n$ ,  $\int_x^\infty e^{-t^2} dt/t^{2n}$ . Are there published tables of these functions?

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EDITORIAL NOTE: Among many tables of  $\int_x^\infty e^{-t} dt/t = -Ei(-x)$  reference may be given to NYMTP, *Tables of Sine, Cosine and Exponential Integrals*, 2v., 1940, for  $x = [0(.0001)1.9999; 9D]$ ,  $[0(.001)10; 9S]$ ,  $[10(.1)15; 14D]$ . There are useful Bibliographies in the volumes. When  $n$  is a positive integer  $\int_x^\infty e^{-t} dt/t^n$  may be made to depend upon  $Ei(-x)$ . For the cases  $n = +2(-1) - 2$  tables were published by W. L. Miller & T. R. Rosebrugh, R. So. Canada, *Proc. and Trans.*, series 2, section III, v. 9, 1903, p. 80–101, for  $x = [.1(.001)1(.01)2; 9D]$ . There are also tables (p. 80–81) of  $-\int_x^\infty e^{-t} dt/t^2 + 1/x + \ln x$ , and  $-\int_x^\infty e^{-t} dt/t - \ln x$ , for  $x = [0(.001).1; 9D]$ . In the case of the second integral, when  $n = 0$  we have the error function of which the most extensive table is that of A. A. MARKOV, *Table des Valeurs de l'Intégrale*  $\int_x^\infty e^{-t^2} dt$ , St. Petersburg, 1888, for  $x = [0(.001)3(.01)4.8; 11D]$  with  $\Delta^2$ ; see *MTAC*, p. 136. However a more extensive table of the closely related function  $H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$  has been published in NYMTP, *Tables of Probability Functions*, v. 1, 1941,  $x = [0(.0001)1(.001)5.6; 15D]$ . This table can be used to evaluate the above integral by means of the relation  $\int_x^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2} [1 - H(x)]$ . Are there other tables of the first function than for  $-2 > n > 2$ , and of the second for  $n \neq 0$ ?

### QUERIES—REPLIES

14. TABLES OF  $N^{3/2}$  (Q 5, p. 131; QR 8, p. 204; 11, p. 336; 13, p. 375).—We have ms. tables, to 10S, as follows for:

$$N = 100(1)1000, 1000(10)10\ 000, 1005(10)1565, \text{ and also} \\ N = [1.0001(.0001)1.0099; 9D].$$