C. CONCORDIA, S. B. CRARY & E. E. PARKER, "Effect of prime-mover speed governor characteristics on power system frequency variations and tie line power swings," *Electrical Engineering*, 1941, p. 559–567, discussion p. 734f; A.I.E.E., *Trans.*, v. 60.

C. CONCORDIA, C. N. WEYGANDT, & H. S. SHOTT, "Transient characteristics of current transformers during faults," *Electrical Engineering*, 1942, p. 280–285, discussion p. 469; A.I.E.E., *Trans.*, v. 61.

C. CONCORDIA, H. S. SHOTT & C. N. WEYGANDT, "Control of tie line power swings," *Electrical Engineering*, 1942, p. 306–313, discussion p. 395; A.I.E.E., *Trans.*, v. 61.

J. G. BRAINERD & H. W. EMMONS, "Effect of variable viscosity on boundary layers, with a discussion of drag measurements," *J. Appl. Mech.*, v. 9, 1942, p. A1–6; A.S.M.E., *Trans.*, v. 64.

S. H. C.

NOTES

40. CORRECT, BUT—!.—In H. LEVY & L. ROTH, *Elements of Probability*, Oxford, 1936, p. 80, is the following footnote: "For example, if $n = 10$, the error in replacing $(1 - 1/n)^{-n}$ by e does not affect the sixth decimal place." This is a footnote to the word "large" in the text statement, "If n is sufficiently large, $(1 - 1/n)^{-n}$ is approximately e. . . ."

The correct values of $(.9)^{-10}$ and e, to 9D, are respectively as follows: 2.867 97<u>1</u> 991 and 2.718 28<u>1</u> 828.

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EDITORIAL NOTE: It may be noted that LEVY & ROTH evidently did not carry out the necessary computations; perhaps they had in mind the well-known fact that when e is expressed as the infinite sum of reciprocals of successive factorials one needs to take only a few terms in order to obtain a fair approximation to the value of e. Indeed, if one confines one's self to $n = 10$, and carries out computations to 9D, then

$$e \doteq 1 + 1/1! + 1/2! + 1/3! + \cdots + 1/10!,$$

and the sum of the terms is 2.718 281 801. In other words, "if $n = 10$, the error in replacing $\sum_{n=0}^{10} 1/n!$ by e does not affect the sixth [or even the seventh] decimal place."

41. EARLY DECIMAL DIVISION OF THE SEXAGESIMAL DEGREE (see N **29**, p. 400f).—In our previous note on this topic we listed seven or eight editions of *De Thiende*, 1585, by Simon Stevin, including Norton's English translation. We forgot to give a reference to the English edition, 8 or 9, of VERA SANFORD, in *A Source Book in Mathematics*, ed. by D. E. SMITH, New York, 1929, p. 20–34. This was translated from no. 4, Girard's French edition of 1634.

R. C. A.

42. FIRST MORTALITY TABLE (see *MTAC*, p. 402f).—A facsimile of R. So. London, *Phil. Trans.*, 1693, p. 600, including Halley's first mortality table, is printed in *Isis*, v. 23, 1935, p. 16.

G. SARTON

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43. FRANCIS WILLIAM NEWMAN (1805–1897).—Newman was a younger brother of J. H. NEWMAN (1801–1890), the English cardinal. He had a brilliant career at Oxford where he obtained a double first in classics and mathematics in 1826. After serving as professor of classical literature in Manchester New College, "the celebrated Unitarian seminary long established at York and the parent of Manchester College, Oxford," from 1840 until 1846, he was appointed professor of Latin at University College, London, where he remained until 1869. The range of his publications was most extraordinary, for example: *Lectures on Logic* (1838), *Lectures on Political Economy* (1851), an English translation of Homer's Iliad (1856), a volume of Longfellow's Hiawatha, somewhat abridged, translated into Latin (1862), *A Handbook of Modern Arabic* (1866), *Translations of English Poetry into Latin Verse* (1868), *A Dictionary of Modern Arabic* (2 v., 1871), *Libyan Vocabulary* (1882), *Comments on the Text of Aeschylus* (1884), *Anglo-Saxon Abolition of Negro Slavery* (1889), and works in history, theology, morals, politics, in addition to publishing nearly a score of mathematical papers and four mathematical books.

These papers are listed in the Royal Society's *Catalogue of Scientific Papers*, the first in 1836, and the last two, in 1883 and 1889, which contain the following important mathematical tables:

I. "Table of the descending exponential function to twelve or fourteen places of decimals," Cambridge Phil. So., *Trans.*, v. 13, part 3, 1883, p. 145–241.
II. "Table of the exponential function e^x to twelve places of decimals," *idem*, v. 14, part 3, 1889, p. 237–249. This is a table for e^x, x = (a) [.1(.1)3; 16D], (b) [.001(.001)2; 12D]. It was presented to the So. in 1887.

T. I was presented to the So. in 1876 and there is an account of it in Cambridge Phil. Soc., *Proc.*, v. 3, 1876, p. 24. It is a table of e^{-x}, x = (a) [0(.1)37; 18D], (b) [0(.001)15.349; 12D], (c) [15.350(.002)17.298; 14D], (d) [17.3(.005)27.635; 14D]. Concerning (a) C. E. Van Orstrand remarks in his "Tables of the exponential function and of the circular sine and cosine to radian argument," Nat. Acad. Sci., Washington, D. C., *Memoirs*, v. 14, no. 5, 1921, p. 6, "The 18-place table is hardly the equivalent of a 16-place table, as the original computation included only 18 decimals." On p. 11, Van Orstrand draws attention to the following errors in the table:

x	*For*	*Read*
3.5	301 0738 34223 18502	301 9738 34223 18501
26.1	... 22985	... 22895
26.4	... 27424	... 24725
26.9	... 72200	... 77201

The first of Newman's mathematical books was *The Difficulties of Elementary Geometry, especially those which concern the Straight Line, the Plane and the Theory of Parallels*, London, 1841, viii, 143 p. In the introduction is the following statement: "This book consists of extracts from one which was intended to form a continuous system of elementary geometry; but . . . the author . . . has determined on selecting those parts which are either wholly new, or wanting in the common treatises."

The other three mathematical books were published after Newman had become an octogenarian. The first of these appeared in two volumes, paged continuously, *Mathematical Tracts*, Part I, 1888, ii, p. 1–80; Part II, 1889, iv, p. 81–139; both parts published in Cambridge, by Macmillan & Bowes.

The following tables are to be found in the *Tracts:*

P. 55–68.—values of A^{-n}, to 20D, $A = 2(1)60$, and the odd numbers 61 to 77, $n = 1, 2, 3, \cdots$, "continued until A^{-n} is about to vanish. For $A = 2$, and 3, only the odd values of n are used, up to 29, instead of to the necessary respective values $n = 65$ and $n = 41$ in order to reach the limit defined above for A^{-n}.
P. 69–79.—values of A^n to 12D, powers of $A = .02(.01).5$, except .1, $n = 1, 2, 3, \cdots$, continued until A^n is about to vanish.
P. 84–85.—$\tanh x$, $x = [.01(.01)1; 12D]$, corrected by J. C. ADAMS.
P. 109.—$1/x - \operatorname{csch} x$, $1 - \operatorname{sech} x$, $\coth x - 1/x$, $\tanh x$, $x = [.9(-.1).1; 16D]$.
P. 110–113.—$\operatorname{csch} x$, $\operatorname{sech} x$, $x = [1(.1)12.6; 16D]$.
P. 114–116.—$\coth x - 1$, $1 - \tanh x$, $x = [1(.1)9.3; 16D]$.
P. 117–118.—$\operatorname{csch} x - 2e^{-x}$, $2e^{-x} - \operatorname{sech} x$, $x = [1(.1)7.4; 16D]$.
P. 119–120.—$\coth x - 1 - 2e^{-2x}$, $2e^{-2x} - 1 + \tanh x$, $x = [1(.1)6.1; 16D]$.
P. 121–123.—$\sigma(x) = -\ln(1 - e^{-2x})$, $k(x) = \ln(1 + e^{-2x})$, $x = [1(.1)9.1; 16D]$.
P. 124.—$\ln \coth x$, $x = [1(.1)6; 16D]$.
P. 126.—"$\phi(\rho) = -\log(1 - c^2 \sin^2 \beta)^{\frac{1}{2}}$ in Legendre's Elliptic scale," $\rho = [1(.1)6.4; 16D]$. After this table Newman wrote, "Carefully as I have worked at this table for $\phi(\rho)$ I must confess that I myself distrust it, because I have no check on error and am sadly aware how a tired brain may blunder."
P. 129.—$\ln \coth x + \ln \coth 3x + \ln \coth 5x + \cdots$ and $D(x) = \ln \coth x - \ln \coth 2x + \ln \coth 3x - \ln \coth 4x + \cdots$, $x = [1(.1)6.3; 16D]$.
P. 131.—$D(2x) + D(4x) + D(6x) + \cdots$, $x = [1(.1)6.1; 16D]$.
P. 132.—$\sigma(2x) + \sigma(4x) + \sigma(6x) + \cdots$, $x = [1(.1)4.6; 16D]$.
P. 133.—$D(2x) - D(4x) + D(6x) - D(8x) + \cdots$, $x = [1(.1)6.3; 16D]$.
P. 134.—$D(x) + D(3x) + D(5x) + \cdots$, $x = [1(.1)6; 16D]$.
P. 135–139.—e^{-x}, $x = [.1(.1)37; 18D]$. Also T. I (a), 1883, above.

Newman's next mathematical work, published when he was 84 years of age, was

Elliptic Integrals, Cambridge, Macmillan & Bowes, 1889. xvi, 200 p. On p. 131 is the same table as on p. 126 of the *Tracts*. On p. 132–133 is a table of F′ taken from A. M. LEGENDRE, *Traité des Fonctions Elliptiques*, v. 2, Paris, 1826, p. 289–290.

During the last five years of his life Newman was totally blind. His last published book appearing just before this period, was

The Higher Trigonometry. Superrationals of Second Order, Cambridge, Macmillan & Bowes, 1892. ii, 117 p.
P. 7.—"The late Professor Jarrett introduced the notation $\underline{|n}$ for $(1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots n)$." This was THOMAS JARRETT (1805–1882), professor of Arabic at the University of Cambridge, in a publication of 1830. The notation $n!$ seems to have been introduced by C. KRAMP, in 1808. See F. CAJORI, *A History of Mathematical Notations*, v. 2, Chicago, 1929, p. 69, 72f.
P. 12.—$x \cot x = 1 - 2H_1x^2 - 2H_2x^4 - 2H_3x^6 \cdots - 2H_nx^{2n} - \cdots$; values of H_n, $n = [6(1)16; 16D]$. $H_n = 2^{n-1} B_n/(2n)!$. In this connection Newman refers to "the hideous numbers of Bernoulli."
P. 38.—$S_n = 1 + 2^{-n} + 3^{-n} + \cdots$; Newman reprints Legendre's table of $S_n - 1$, $n = [2(1)35; 16D]$, given both in the above mentioned work of

1826, p. 432, and in his *Exercices de Calcul Intégral*, part 4, Paris, 1814, p. 65. J. W. L. GLAISHER gave a table of S_n, $n = [2(1)107; 32D]$ in *Quart. J. Math.*, v. 45, 1914, p. 148–150; this table is reprinted in H. T. DAVIS, *Tables of the Higher Mathematical Functions*, Bloomington, Indiana, v. 2, 1935, p. 244, 218. Comparison of Glaisher's table with Legendre's showed that in the latter there were errors in last figures; there should be a unit decrease for $n = 5$, and unit increases for $n = 7, 10, 11, 16$. Setting $T_n = 1 + 3^{-n} + 5^{-n} + \cdots = (1 - 2^{-n})S_n$, Newman gave a table of $T_n - 1$, $n = [2(1)23; 16D]$. J. W. L. Glaisher gave a table of T_n, $n = [2(1)67; 32D]$, in *Quart. J. Math.*, v. 45, 1914, p. 151–152; this table is reprinted in H. T. Davis, *idem*, p. 245. Comparison of Glaisher's table with Newman's revealed the following Newman errors: $n = 13$, *for* ...6218, *read* ...4218; $n = 17$, *for* ...8400, *read* ...8395; also 5 other last figure unit errors. EULER gave a table for S_n, $n = [2(1)16; 16D]$ in his *Institutiones Calculi Differentialis*, St. Petersburg, 1755, p. 456–457; also in his *Opera Omnia*, s. 1, v. 10, Leipzig, 1913, p. 349, where errors in the first printing are corrected. Euler gave also a table of T_n, $n = [2(2)44; 23D]$ in his *Introductio in Analysin Infinitorum*, 1748, p. 150–151; also in his *Opera Omnia*, s.1, v. 8, Leipzig, 1922, where errors in the first edition are corrected.

P. 39.—$2^{-n} - 3^{-n} + 4^{-n} - \cdots$, (a) $n = [2(1)34; 16D]$, (b) $n = [2(2)18; 20D]$. J. W. L. Glaisher gave a table of this function $n = [1(1)107; 32D]$ in *Quart. J. Math.*, v. 45, 1914, p. 156–158. Comparison of this table with Newman's revealed the following discrepancies in the latter's: in (a) $n = 3$, *for* ...5932..., *read* ...5732...; $n = 11$, *for* ...324 7126..., *read* ...285 6501...; also 9 end-figure errors of from 1 to 6 units; in (b), $n = 4$, *for* ...764 082..., *read* ...754 082...; $n = 10$, *for* ...83 8435, *read* ...84 3436; also 7 last-figure errors of 1 to 5 units.

P. 45.—$1 - V_n = 3^{-n} - 5^{-n} + 7^{-n} - \cdots$, $n = [1(1)20; 13D]$. After this table Newman states "Having no check on trivial error, contingent on a tired brain, I have to speak diffidently of this little table." PETERS & STEIN give a table of $1 - V_n$, $n = [1(1)53; 32D]$, in PETERS, *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, *Anhang*, p. 94. Comparison of these tables showed that in Newman's table there were 3 errors, namely: 2 end-figure unit errors, and at $n = 4$, *for* ...5488..., *read* ...5448.... Euler gave a table of V_n, $n = [3(2)13; 7D]$, in his *Opuscula Analytica*, St. Petersburg, v. 2, 1785, p. 251. Newman quotes also a table of $(2/n)(1 - V_n)$, $n = [1(2)9; 25D]$, $[11(2)17; 24 - 18D]$ by C. GUDERMANN, *Theorie der Potenzial- oder cyklisch-hyperbolischen Functionen*, Berlin, 1833, p. 72; also in *J. f. d. reine u. angew. Math.*, v. 6, 1830, p. 194.

P. 64–65, 103–104.—LnN, N prime, 11 to 97; 13D, with 14 last-figure errors, from 1 to 70 units. Also (p. 103–104) $\phi(x) = x + 3^{-2}x^3 + 5^{-2}x^5 + 7^{-2}x^7 + \cdots$, and $\psi(x) = 2^{-2}x^2 + 4^{-2}x^4 + 6^{-2}x^6 + \cdots$, $x = [.01(.01).5; 12D]$; these tables were checked by J. C. Adams.

Spence's integral is $L(x) = \int_1^x \ln x\, dx/(x - 1)$, and

$$L(1 + x) = \int_1^x \ln(1 + x)dx/x = x - 2^{-2}x^2 + 3^{-2}x^3 - 4^{-2}x^4 + \cdots = \phi(x) - \psi(x)$$

$$-L(1 - x) = x + 2^{-2}x^2 + 3^{-2}x^3 + 4^{-2}x^4 + \cdots = \phi(x) + \psi(x).$$

There are tables of $L(1 + x)$ and $-L(1 - x)$, $x = [0(.01).5; 12D]$. Spence's integral occurs in an essay by WILLIAM SPENCE (1777–1815); see his *Mathe-*

matical Essays, edited by JOHN F. W. HERSCHEL, with a biographical sketch of the author by John Galt, London, 1819, or in the first edition of *An Essay on the Theory of the Various Orders of Logarithmic Transcendents* . . ., London and Edinburgh, 1809. ALAN FLETCHER has made a study of the tables of Spence's integral by Spence (p. 24), Newman, and by E. O. POWELL (see *MTAC*, p. 189), in *Phil. Mag.*, s. 7, v. 35, Jan. 1944, p. 16–17. Fletcher shows that there are at least "nine gross errors, all due to faulty addition or subtraction by Newman, and not to errors in the basic values of Adams." The study served "to give confidence in the accuracy of Mr. Powell's table." More than one hundred values in this table, or more than a quarter of the whole were examined. The 7-place value needs decreasing by unity at $x = 1.29$ and increasing by unity at 1.42; also three rounding-off errors of a unit in the seventh decimal place were found between .50 and .67. Powell's claim that the seventh decimal place "should not usually be in error by more than one unit," "therefore appears to be amply justified."

P. 85–87.—Clausen's integral, $\mathrm{Cl}\, x = -\int_0^x \ln (2 \sin \tfrac{1}{2}x)dx = \sin x + 2^{-2} \sin 2x + 3^{-2} \sin 3x + \cdots$, tabulated for $x = [1°(1°)180°;\ 16D]$, is reprinted from Clausen's article in *J. f. d. reine u. angew. Math.*, v. 8, 1832, p. 300.

P. 101–102.—$\tanh x$, $x = [.01(.01)1;\ 12D]$, corrected by J. C. ADAMS; same table as p. 84–85 of *Tracts*, part II.

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Newman's character is vividly drawn by Carlyle in his life of Sterling (1851, part III, chap. 1) of whose son Newman was guardian: "an ardently inquiring soul, of fine University and other attainments, of sharp-cutting, restlessly advancing intellect, and the mildest pious enthusiasm." Material concerning Newman and his works may be found in the following sources:

F. HARRISON, *Realities and Ideals, Social, Political, Literary and Artistic*, New York, 1908, p. 371–377.

J. MCCABE, *Biographical Dictionary of Modern Rationalists*, London, 1920.

R. GARNETT, *Encyclopædia Britannica*, eleventh ed., v. 19, 1911; his quotation from Carlyle is highly unreliable. *Allibone's Critical Dict. of English Literature*, Philadelphia, v. 2, 1870 and *Supplement*, v. 2, 1892. More than four columns of the British Museum, *Catalogue of Printed Books*, 1892, are filled with a list of his writings.

I. G. SIEVEKING, *Memoir and Letters of Francis W. Newman*, London, 1909. His mathematical work is not mentioned here. The frontispiece is a reproduction of a daguerreotype of Newman, taken in 1851; he appears also in a reproduction of a sketch of the Newman family by Maria R. Giberne. There are also a reproduction of a photograph taken in middle life, and two views of a bronze bust, by Mrs. Georgina Bainsmith, presented to the University of London in 1907. A reproduction of a photograph of Newman may also be found in *Illustr. London News*, v. 111, 1897, p. 521.

Since the volumes we have been discussing are not often found in public libraries it may be noted that Newman's *Difficulties of Elementary Geometry* is in the British Museum (B.M.), Cornell University, and University of Michigan (U.M.); copies of *Elliptic Integrals* are in Brown Univ. (B.U.), B.M., Boston Public Library (B.P.L.), and the Mittag-Leffler Library at

Djursholm; *Higher Trigonometry* may be seen in B.M., B.P.L., and Columbia Univ.; Part I of *Mathematical Tracts* is in B.U., and U.M., but both parts are in B.P.L., University of California, and the Mittag-Leffler Library. The Mathematical Association, England, has all four of Newman's mathematical works, including both editions of his *Difficulties of Elementary Geometry*, each printed in 1841, but by different printers. The copy of Spence's rare *Mathematical Essays* in B.P.L. was acquired by Nathaniel Bowditch soon after publication, possibly a presentation copy from the editor, and the first Essay, to which we refer, contains many marginal notes in Bowditch's handwriting. Both of the Spence volumes mentioned are in the New York Public Library, and the 1809 *Essay* is in University of California, Berkeley.

R. C. A.

44. TABLE OF $\frac{1}{2}Wx/V$.—A 5-place table of this function is given (*MTAC*, p. 256) in E. B. ROSA & F. W. GROVER, "Formulas and tables for the calculation of mutual and self-inductance," U. S. Bureau of Standards, *Bulletin*, v. 8, no. 1, 1912, table XXII, p. 226–228; $W = \text{ber}\, x \text{bei}' x - \text{bei}\, x \text{ber}' x$, $V = \text{ber}'^2 x + \text{bei}'^2 x$, $x = 0(.1)5(.2)10(.5)15(1)26(2)50(10)100$, with δ^2. A 3-place adaptation of this table, with Δ, is given on p. 162 of H. B. DWIGHT, *Electric Coils and Conductors*, New York, McGraw-Hill, 1945. Dwight tells us that table XXII was calculated by Grover, and that the adaptation was with his permission.

R. C. A.

QUERIES

15. INTEGRAL AND FUNCTIONAL TABLES.—Are there any tables of $\int_0^z e^{-t^2}\, dt$ and of $e^{-z^2}\int_0^z e^{t^2}\, dt$, where z is complex? Or of $\int_0^x e^{-t^2} \sin at\, dt$, and $\int_0^x e^{-t^2} \cos at\, dt$, where x is real?

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EDITORIAL NOTE: In 1930 RONALD M. FOSTER, of the American Telephone and Telegraph Co. (now of the department of mathematics at Polytechnic Institute of Brooklyn), prepared tables of $\text{erfc} z = (2/\pi^{1/2})\int_z^\infty e^{-t^2} dt$, and of e^{z^2} erfc$z$, $z = x + iy$, for $x = 0(1)3$, $y = 0(1)3$, to 5S. From this material he computed a rather large number of values by a simple method of numerical integration along selected rays in the complex plane. These results were then used to draw contour lines, so that the real and the imaginary parts of the error function could be read off in a rough sort of way for a limited range. Charts I and II (50.7 × 50.7 cm.) are of the real and complex parts of erfc$z$, $0 < x < 2$, $0 < y < 2$; Chart III (38 × 38 cm.) is of real and imaginary parts of erfc$z$, $0 < x < 3$, $0 < y < 3$; Chart IV (38 × 38 cm.) is for absolute value and angle of erfc$z$, $0 < x < 3$, $0 < y < 3$. None of this material has been published. A particular case of the first integral of the Query, $z = \frac{1}{2}\pi^{\frac{1}{2}}(1 + i)u$, may be reduced to functions already tabulated, *MTAC*, p. 250, since we then have $\int_0^z e^{-t^2} dt = (\frac{1}{2}\pi i)^{\frac{1}{2}}[C(u) - iS(u)]$.

QUERIES—REPLIES

17. ROOTS OF THE EQUATION TAN $x = cx$ (Q **8**, p. 203; QR **10**, p. 336). —In A. T. MCKAY, "Diffusion for the infinite plane sheet," Phys. So. London, *Proc.*, v. 44, 1932, p. 22–23, there are tables of real roots, x_n, $n = [1(1)4; 4D]$, of this equation for $c = \pm \tan \lambda$, $\lambda° = 0(5°)90°$. In the case of $c = + \tan \lambda$, there are no roots x_1 for $\lambda < 45°$.

R. C. A.