A. J. C. CUNNINGHAM, "On binal fractions," *Math. Gazette*, v. 4, 1908, p. 266–267, gives the complete period of $1/n$ to base 2, for $n < 100$; and tables to the same extent for the bases 3 and 5, in "On tertial, quintal, etc. fractions," *Math. Gazette*, v. 6, 1911, p. 111–116. The first 16 digits in the period of $1/n$, for $n = 1(1)1728$, to base 12, may be obtained from G. S. TERRY, *Duodecimal Arithmetic*, London, New York, Toronto, Longmans, 1938, p. 59–70.

45[G, I].—H. E. SALZER, *Coefficients in Polynomials.* Two mss. in the possession of the author at NYMTP.

The first ms. gives the coefficients of the various powers of p in the polynomials $q(q^2 - 1^2)(q^2 - 2^2) \cdots (q^2 - n^2)$ where $q = 1 - p$, and $n = 1(1)10$.

The second ms. gives the exact values of the coefficients of the eleventh to the twentieth Laguerre polynomials.

H. E. SALZER

EDITORIAL NOTE: For Laguerre polynomials compare *MTAC*, v. 2, p. 31.

MECHANICAL AIDS TO COMPUTATION

18[Z].—MAX BORN, R. FÜRTH & R. W. PRINGLE, "A photoelectric transformer," *Nature*, v. 156, 22 Dec. 1945, p. 756–757.

The following is the first paragraph of this preliminary account of the transformer, from the department of mathematical physics, University of Edinburgh: "The 'Fourier transform' $g(y)$ of a function $f(x)$, usually defined by the integral

$$g(y) = \frac{1}{\sqrt{2\pi}} \int_a^b f(x) e^{-iyx} dx,$$

plays an important part in many problems of pure and applied physics. It represents, for example, the connexion between the intensity distribution of a wave scattered by matter of a certain density distribution, which has to be calculated in a number of acoustical and optical problems and, above all, in X-ray crystal analysis work. It further allows the resolution of a complicated oscillation into a continuous frequency spectrum of harmonic oscillations, which is required in many problems of mechanical and electrical engineering. It therefore seems of some importance to have an instrument by which the Fourier transform of a given function can be automatically and quickly produced. We have now succeeded in building up an instrument which produced the graph of the function

$$g'(y) = \int_a^b f(x) \cos (yx + \delta) dx$$

on the screen of a cathode ray oscillograph, from a mask cut out of black paper in the shape of the graph of the function $f(x)$, or from a record of this function on a plate or film in density variation. Obviously two of the functions $g'(y)$ for two values of δ, say, $\delta = 0$ and $\delta = \frac{1}{2}\pi$, are equivalent to the complex function $g(y)$ when $f(x) = 0$ for $x < a$, $x > b$."

19[Z].—V. BUSH & S. H. CALDWELL, "A new type of differential analyzer," Franklin Institute, *J.*, v. 240, Oct. 1945, p. 255–326; 51 figures and illustrations.

The first proposal for a machine to solve differential equations appears to be due to Lord Kelvin, in 1876, but the present activity in the construction of such devices was initiated by the independent work of V. Bush, and others at the Massachusetts Institute of Technology, starting in 1925. The first machine capable of handling a fairly wide class of differential equations was described by Bush in 1931, and named the "Differential Analyzer." Since that time many similar machines have been built in this country, in England, Norway and Germany.

Engineers and physicists reduce many of their problems to differential equations, and families of particular solutions of those equations have proved to be of great service. The Massachusetts Institute of Technology and the Rockefeller Foundation joined forces in 1935 in an effort to produce a more flexible and accurate differential analyzer for the calculation of such solutions. The Differential Analyzer, which resulted from this program, was placed in service in 1942, and has been in use throughout the war. In November 1945, it was opened to the public for scientific work in general including that of industry.

The natural or canonical form of differential equation for the Analyzer differs somewhat from the forms usually chosen in the analytic study of differential equations. It may be written (the following notation is not used by the authors)

$$x_j = K_j \int_{x_{j0}}^{x_i} x_k dx_i + x_{j0},$$

$$x_p = A_{pq}x_q + C_p, \quad x_r = x_s + x_t + D_r, \quad x_u = f_u(x_v) + E_u,$$

where i, j, k, p, q, r, s, t, u, v range over suitable restricted integral values. These equations represent the types of mechanism available for the solution.

The variables, x, are represented in the Analyzer by the angular positions of shafts, the shafts being continuously rotated during the solution of a problem. The first equation of the canonical set is "solved" by devices called "integrators," each of which consists of a wheel rolling on the surface of a rotating disk. It is interesting to note that the variable of integration may be any of the variables in the problem, and in any one problem, integrals may be taken with respect to several distinct variables. This fact is frequently relied upon to simplify equations.

The remaining equations of the canonical set are mechanized by means of gear boxes, (multiplication by constants A_{pq}), "adders," commonly known as differentials (addition) and function tables, on which graphs of the functions f_u are plotted.

The constants, C_p, D_r, E_u and x_{j0} must be set into the machine at the start of each problem. Their magnitudes are assigned by the initial conditions of each required solution. They are introduced to the machine at scales which are selected with due regard to the ranges over which the variables will run during the solution, the ranges available in the equipment, and the accuracy with which the solution is desired.

The M.I.T. Analyzer represents an advance over previous analyzers to some extent in its increased accuracy, but to a far greater extent in the speed with which new problems can be set up. The accuracy of such complicated mechanisms is difficult to evaluate, even if an agreement can be reached on the definition of the term. Errors in any continuous computer, of which the Analyzer is an example, depend critically on the state of maintenance of the parts, on dirt, temperature, wear, etc. etc., as well as upon the wisdom used in the choice of scale factors and speeds of solution employed.

The errors in the solution depend, of course, upon the type of differential equation being solved, some equations being sensitive to inaccuracies in the elements and others being very stable.

In view of the facts noted, it is dangerous to quote any figures, but in order to give the uninformed a rough idea, it may be said that the errors in a trajectory are likely to be of the order of one part in ten thousand, with a very liberal factor one way or the other.

The speed with which new problems can be set up on the machine, once they have been reduced to the form of codes on punched tapes, is such that only about 15 minutes of machine time is required. This contrasts favorably with the one or two days required to set up the previous analyzers.

The class of differential equations which can be handled with the Analyzer is somewhat wider than that heretofore treated, because the number of computing elements—integrators, gear boxes, etc.—is greater. There are 18 integrators available. This does not ordinarily mean that 18th order equations can be solved, since some of the integrators are likely to be needed for the generation of special functions. For example, if a product x_jx_k appears in the original differential equation, it is replaced by

$$x_jx_k = \int x_j dx_k + \int x_k dx_j,$$

which requires two integrators to generate.

In addition to such functions as may be defined by subsidiary differential equations, the Analyzer can accept problems involving three arbitrary functions of one variable each. There is no means for dealing with functions of two or more variables, unless these functions can be reduced to the solutions of subsidiary differential equations, or to functions of one variable. There are in existence plans for four "function units" which will accept values tabulated at discrete intervals and interpolate continuously between them. These units are not yet in use.

Results of the computation are presented either graphically or in the form of typed tables.

Special techniques and schematic symbolisms have been developed for use in reducing mathematics to machine connections and are described by Bush and Caldwell. The symbols are simple and suggest the mechanical units they are intended to represent. While not logically necessary, the schematic or short-hand notation is important practically, as an aid to the rapid and easy reduction of problems to a form acceptable to the analyzer.

The design and construction of continuous computers having the accuracy and flexibility of the M.I.T. Analyzer, demands a high order of ingenuity and mechanical skill. Some of the outstanding problems met and solved by the Massachusetts Institute of Technology group and their associates are described by Bush and Caldwell. Among these are the design of data-transmission and servo units, the development of codes and related equipment for the rapid set-up of problems, and the conversion of continuous variables (shaft rotations) into digital form, suitable for printing.

The M.I.T. Analyzer is finding and will continue to find a field of usefulness in the solution of the differential equations of engineering and physics, especially as engineers delve further into situations which demand the solution of non-linear relations.

G. R. Stibitz

20[Z].—International Business Machines Corp., *IBM Automatic Sequence Controlled Calculator*, New York, I.B.M., 590 Madison Avenue, 1945. 6 p. (21.5 × 28 cm.) + folding plate (83 × 28 cm.). Copies may be procured on application to the publisher.

This publication contains some notes on the development of this electro-mechanical calculator installed and constantly working in the Cruft Memorial Laboratory at Harvard University; it was officially turned over to the University on August 7, 1944, and has been, and still is, exclusively used by the U. S. Navy. The folding plate gives a complete view of the front side of the machine, and various statistics concerning the number of elements in the machine and the material used in its construction.

21[Z].—William E. Morrell, "A slide rule for the addition of squares," *Science*, n.s., v. 103, 25 Jan. 1946, p. 113–114.

The author describes a slide rule which he constructed, with appropriate square and square-root scales, for evaluating such an expression as $d = (x^2 + y^2 + z^2)^{\frac{1}{2}}$.

NOTES

For a Note on Harry Bateman see RMT **289**.

51. Early Decimal Division of the Sexagesimal Degree (see N **29**, p. 400f; N **41**, p. 454).—In our previous notes on this topic we listed 8 or 9 editions of *De Thiende*, 1585, by Simon Stevin; but we forgot to give a reference to the Dutch edition, 9 or 10, of Ezechiel De Decker, *Eerste Deel van de Nievwe Telkonst . . . Noch is hier achter byghevoeght de Thiende*