FARLEY (publs. of 1840–56), HERSCHELL E. FILIPOWSKI (publs. of 1849–57), EMMA GIFFORD (1861–1936), PETER GRAY (1807?–1887), EDWARD LINDSAY INCE (1891–1941), HENRY SHERWIN (publs. of 1705–1741), ROBERT SHORTREDE (1800–1868), JOHN SPEIDELL (publ. of 1619), MICHAEL TAYLOR (1756–1789)? None of them are listed in Ball's Collection of portraits (E. M. HORSBURGH, *Modern Instruments and Methods of Calculation*, London, 1914). Karl Pearson tells us (*Logarithmetica Britannica*, part IX, 1924) that he vainly sought a portrait of Briggs. The *Dict. Nat. Biog.* contains sketches of Briggs and Gray, and there is biographical material about Cunningham in (a) Poggendorff, (b) London Math. Soc., *Jn.*, v. 3, 1928 (by A. E. Western), and about Ince in (a) *Who's Who 1941*, (b) London Mathem. Soc., *Jn.*, v. 16, 1941 (by E. T. Whittaker), (c) Edinb. Math. Soc., *Proc.*, s. 2, v. 6, 1941 (by A. W. Young), (d) *Nature*, v. 148, 1941 (by A. C. Aiken). Where may one find biographical data concerning the other persons listed?

R. C. A.

## QUERIES—REPLIES

**28.** TABLES OF TAN$^{-1}(m/n)$ (Q**14**, v. 1, p. 431; QR**18**, v. 1, p. 460; **20**, v. 2, p. 62; **24**, p. 147).—In *MTAC*, v. 2, p. 63, footnote 2, J. C. P. MILLER suggested the following problem: Determine all positive integers $N$ which are fundamental in the sense that there is no relation of the form

$$\tan^{-1} N = \sum_i \lambda_i \tan^{-1} n_i \tag{1}$$

(where the $\lambda_i$ are integers and the $n_i$ are positive integers *less than* $N$).

This problem can be completely solved using methods of elementary number theory (C. F. GAUSS, *Werke*, v. 2, 1863, and 1876, p. 477 and 523, and the papers by C. STØRMER and others quoted in *MTAC*, v. 2, p. 28). It can be shown that $N$ is fundamental if and only if $N^2 + 1$ has a prime factor which is not a factor of a number $n^2 + 1$ with $n < N$. An effective construction for the relation of the form (1) in the case when $N$ is not fundamental can be given; a table giving all such relations with $N$ satisfying $N^2 + 1 < 100\,000$ has been prepared. The positive integers which are fundamental form a minimal "integral" basis for the set $\{\tan^{-1} n\}$ and therefore also for the set[1] $\{\tan^{-1}(m/n)\}$. The connection between this "integral" basis and the "rational" basis which was apparently known to GAUSS[2] can be made clear by the following remark. Corresponding to a prime $p = 2$ or $4n + 1$, Gauss has $\tan^{-1}(a/b)$ where $a$ and $b$ are determined uniquely by the conditions $a \geqslant b > 0$ and $a^2 + b^2 = p$ while we have $\tan^{-1} N$ where $N$ is the least positive integer such that $N^2 + 1$ is a multiple of $p$. A table has been prepared, covering all such primes $p < 500$, expressing $\tan^{-1}(a/b)$ in terms of $\tan^{-1} N$ and $\tan^{-1} m$ with $m < N$. These results will be submitted to the London Math. So., *Jn.*

The reduction of $\tan^{-1}$ 1 40333 78718 was attempted in order to test the reduction algorithm. This is reducible since the prime factors of 1 40333 78718$^2$ + 1 all occur as factors of $n^2 + 1$ with $n <$ 1 40333 78718. Use of the factorisation given by Gauss (*Werke*, v. 2, p. 481) led to a contradiction which was found to be due to an extra factor 13 in the decomposition given by Gauss. The entry in his table corresponding to 1 40333 78718 should read

$$5 \cdot 5 \cdot 13 \cdot 17 \cdot 17 \cdot 61 \cdot 61 \cdot 61 \cdot 61 \cdot 73 \cdot 73 \cdot 157 \cdot 181$$

and the required reduction is $\tan^{-1} 1\,40333\,78718 = -\tan^{-1} 28 - 2\tan^{-1} 27 + \tan^{-1} 19 - 4\tan^{-1} 11 - \tan^{-1} 5 - 2\tan^{-1} 4 - \tan^{-1} 2 + 20\tan^{-1} 1.$

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[1] D. H. LEHMER, *Duke Math. Jn.*, v. 4, 1938, p. 323–340.
[2] C. F. GAUSS, *l.c.*, p. 523. See C. STØRMER, *Archiv f. Math. og Naturv.*, v. 19, 1896, no. 3, p. 1–96, especially p. 77.

## CORRIGENDA

V. 1, p. 360, l. − 4, *for* $x = 1(2)6(1)10$, *read* $x = 1(1)6(2)10$.

V. 2, p. 68, 275, *for* PEDERSON, *read* PEDERSEN.

V. 2, p. 195, l. 6, *for* $\frac{1}{2}(t - \pi)$, *read* $-\frac{1}{2}(t + \pi)$; p. 228, last line, *delete* "266, *read* 288," *and replace by* "Lewin, *read* Levin."