and pitch P connects two chain gears, the number of whose teeth are T and t. The tables enable one to find the distance S between the centers of the gears. The formula on which the tables are based is:

$$S = \tfrac{1}{2}P(D - d)\csc x$$

where

$$D = \csc \pi T^{-1} \qquad \text{and} \qquad d = \csc \pi t^{-1},$$

and x is the least positive solution of

$$\cot x + cx = a, \quad \text{where}$$
$$c = \pi^{-1}(T - t)/(D - d) \quad \text{and} \quad a = [N - \tfrac{1}{2}(T + t)]/(D - d)$$

The table is one of double entry giving the factor $\csc x$ to 6, 5 and 4D in terms of c and a. The value of c in a typical case is slightly greater than unity. The present tables are for $c = 1$, 1.01 and 1.02 only. The second variable a has the following range

a = [1.75(.002)1.8(.0025)1.835(.005)1.93(.01)2.07, 2.075(.0125)2.325(.025)2.95(.05) 4.15, 4.125(.125)6.5(.25)10; 6D], [11.25(1.25)22.5(2.5)35(5)105; 5D], [100(50)300(100)1000; 4D].

The partial differences of $\csc x$ with respect to c and a are given together with the Bessel coefficient $\Delta(\Delta - 1)/4$. There is an auxiliary table of $\csc \pi T^{-1}$ for T = [10(1)160; 8D] for finding the pitch diameters d and D. Comparing this table with a similar smaller table to 4D in BRITISH STANDARDS INSTITUTION, *Specifications for Steel Roller Chains and Chain Wheels*, revised April 1934, no. 228–1934, p. 18–19, one finds in the latter table, 10 last-figure unit errors: in excess for T = 44, 46, 63, 76, and in defect for T = 11, 22, 91, 95, 107, 133; also at T = 127, *for* 49.4295, *read* 40.4295.

The present tables, if made available to machine design people, should do much to replace the crude approximations usually resorted to in dealing with this comparatively precise problem. A few handbooks give the exact formula

$$N = T + \pi^{-1}(T - t)(\tan A - A)$$

where

$$A = \arccos P(T - t)/2\pi S$$

or an equivalent formula but the reviewer has not found any other tables for obtaining S directly.

D. H. L.

MECHANICAL AIDS TO COMPUTATION

The reader is referred to the first two articles of this issue. dealing with MAC topics.

28[Z].—Akad. N., SSSR, Moscow, *Izvestiia, Otdelenie Tekhnicheskikh Nauk*, 1946, no. 8, September, p. 1065–1200. 16.7 × 25.8 cm. Entirely in Russian.

This issue is almost wholly devoted to material dealing with computation and computing mechanisms. The contents are as follows:

N. G. BRUEVICH, "The present state and problems of the theory of mechanism precision," p. 1065–1079.

I. ÎA. AKUSHSKIĬ, "An outline of punched cards machines," p. 1081–1120.

L. I. GUTENMAKHER, "Electrical models of physical phenomena and their applications in technology and physics," p. 1121–1146. In the literature list there are several references to American authors including two to BUSH.

L. LIUSTERNIK, "Problems of computational mathematics," p. 1147–1156.

L. ÎA. NEÏSHULER, "Tabulation of functions," p. 1157–1176. There are here two references (p. 1157, 1175) to *MTAC* and to U. S. A. as "the country with the greatest development in the industry of calculating machines."

CHRONICLE: M. L. BYKHOVSKIÏ, "The new differential analyzer of Bush," p. 1177–1198. An illustrated description based on the long article of V. BUSH & S. H. CALDWELL, "A new type of differential analyzer," Franklin Inst., *Jn.*, v. 240, 1945, p. 255–326; see *MTAC*, v. 2, p. 89–91.

R. C. A.

NOTES

68. DOCTOR COMRIE'S ADDRESS.—We regret that we omitted to state in connection with the L. J. C. article, published v. 2, p. 149–159, which has been much in demand, that it was the address which he delivered 31 October 1945 at the Conference on Advanced Computation Techniques (*MTAC*, v. 2, p. 65–68), as chairman of subcommittee Z of the Committee on Mathematical Tables and Other Aids to Computation.

69. GIBBS' PHENOMENON.—I feel that the Note on the sine integral in *MTAC*, v. 2, p. 195, will give the impression that any description of the Gibbs' phenomenon in which the number 1.08949 (approx.) occurs is wrong, and that this number should be replaced by 1.17898 (approx.) as the result of a new and careful evaluation of $K = (2/\pi)\mathrm{Si}\,\pi$. This is not so, as the appropriate number depends upon the way the phenomenon is described.

The series $F(t)$ represented by $\frac{1}{2}(\pi - t)$ for $o < t < \pi$, and by $-\frac{1}{2}(\pi + t)$ for $-\pi < t < o$ [not by $\frac{1}{2}(t - \pi)$ as stated in the Note if we interpret t algebraically] is an odd function with a discontinuity or jump of π from $-\frac{1}{2}\pi$ to $\frac{1}{2}\pi$ at $t = 0$. The Fourier series representing this function, i.e.,

$$\sum_{n=1}^{\infty} \sin nt/n,$$

exhibits the Gibbs phenomenon as an overshoot at each end of amount, say, δ, and, measured from the origin, the function jumps each way by an amount $\frac{1}{2}\pi + \delta$ which is given by

$$\mathrm{Si}\,\pi = \int_0^{\pi} \sin t\,dt/t = \tfrac{1}{2}\pi + \mathrm{si}\,\pi,$$

where $$\mathrm{si}\,\pi = -\int_{\pi}^{\infty} \sin t\,dt/t \sim .28114, \quad \text{so that} \quad \delta = \mathrm{si}\,\pi.$$

The phenomenon may be defined as in *MTAC* by the ratio K of $\frac{1}{2}\pi + \delta$ to $\frac{1}{2}\pi$, i.e., $K = (2/\pi)[\frac{1}{2}\pi + \mathrm{si}\,\pi] = 1 + (2/\pi)\,\mathrm{si}\,\pi \sim 1.17898$. This ratio is also that of the jump including both the overshoots to the jump itself, i.e., $\pi + 2\delta$ to π.

We can, however, define the phenomenon by the ratio K' of the jump + either overshoot to the jump itself, i.e., $\pi + \delta$ to π, so that

$$K' = (1/\pi)(\pi + \delta) = 1 + (1/\pi)\,\mathrm{si}\,\pi \sim 1.08949.$$

When the jump is not necessarily at the origin nor of amount π symmetrically disposed about the t axis, it is usual to describe the phenomenon as an overshoot at each end by an amount which is about 9% of the jump