Then, to the same approximation,

$$u_{n+1}/u_n \doteq x(m - \mu)/m, \qquad \text{where} \qquad m = n + \nu,$$

and so

$$\sum_{\nu=0}^{\infty} u_{N+\nu} = u_N C(m, \mu; x).$$

A good approximation to the tail of the series is thus obtained.

The function is tabulated to 4D for $4m = 40(1)44, 80(1)84$; $4\mu = 4(1)20$; $x = .8(.02).9(.01)1$.

No differences are given, since it is assumed that the table will generally be used without interpolation. One page is devoted to each value of x and contains the two double-entry tables corresponding to the two series of values of m.

The method of calculation *should* result in the last figures not being in error by more than .6, but the system of checking adopted was not capable, throughout the whole range, of guaranteeing an accuracy of more than one unit.

The general method of computation was the repeated application of the recurrence formulae

$$C(m+1, \mu; x) = \frac{m}{(m-\mu)x}\{C(m, \mu; x) - 1\}$$

and

$$C(m, \mu; x) = (\mu - 1)^{-1}\{(m-1) - (m-\mu)(1-x)C(m, \mu - 1; x)\}$$

due regard being paid to the loss of figures inherent in this method. Initial values were obtained from the formulae

$$C(1, \mu; x) = (1-x)^{\mu-1}; \mu \neq 1,$$

and

$$C(2, 1; x) = -x^{-1}\ln(1-x)$$

with

$$C(m, \mu; 1) = (m-1)/(\mu-1).$$

JOHN TODD & D. H. SADLER

MECHANICAL AIDS TO COMPUTATION

In the introductory article, "Admiralty Computing Service," there are reviews of publications **37**, *Electronic Differential Analyser;* **40**, *Rangefinder Performance Computer;* **112**, *The Fourier Tranformer.*

There is an interesting biographical sketch, and a portrait of HOWARD HATHAWAY AIKEN (1900–), in *Current Biography*, v. 8, no. 3, March 1947.

In *Wisconsin Engineer*, v. 51, Dec. 1946, p. 10–12 is an article by WALTER GRAHAM, "Do you know your slide rule?", in which the author explains the slide rule solution of equations of the type $x^x = k$, $a^x = x^b$, $\tan x = kx$, and $\sin x = kx$. See *MTAC*, v. 1, p. 203, Q **8** and v. 2, p. 194, **25**.

Dr. LOTHAR SCHRUTKA, professor of mathematics in the Technische Hochschule, Vienna, is the author of a third much revised edition of his *Theorie und Praxis des logarithmischen Rechenschiebers*, Vienna, Deuticke, 1943. xii, 101 p. The first edition appeared in 1911, and the second in 1929. There are 32 titles in the bibliography, p. 95–96, and there is a full index, p. 97–101.

H. H. AIKEN & GRACE M. HOPPER, "The Automatic Sequence Controlled Calculator," *Electrical Engineering*, v. 65, Aug.–Nov. 1946, p. 384–391, 449–454, 522–528. See *MTAC*, v. 2, p. 185f.

29[Z].—D. R. Hartree, *Calculating Machines, Recent and Prospective Developments and their Impact on Mathematical Physics.* Cambridge, University Press, 1947. 40 p. + a double plate. 12.4 × 18.1 cm. 2 shillings.

This is a lecture given on the occasion of the author's inauguration as Plummer Professor of Mathematical Physics in the University of Cambridge. Its general purpose was to acquaint his audience with the war time expansion of large scale computing units in the United States. The lecture is divided into 9 parts by the following subheadings: (1) Introduction, (2) Two classes of calculating equipment, (3) Functions of components of a digital machine, (4) The ENIAC, (5) The master programmer, (6) Example of the application of the ENIAC, (7) Prospective developments, (8) The impact of these developments on mathematical physics, (9) Conclusion.

Most of the lecture is devoted to a discussion of the ENIAC, see *MTAC*, v. 2, p. 97–110. Part 6 is a short description of an interesting boundary layer problem which the author put on the ENIAC in June 1946. It consists in solving the non-linear system of three differential equations

$$f' = h(1 + \alpha r)^{-1/2}, \qquad h'' = -fh', \qquad \beta r'' = fr' + (h')^2,$$

with the two-point boundary conditions,

$$f = h = r' = 0 \text{ at } x = 0, \qquad h = 2, r = 0 \text{ at } x = \infty.$$

Part 7 is a very brief description of the Electronic Discrete Variable Calculator (EDVAC) type of machine.

Part 8 is an interesting discussion of the way that the possibility of high speed large scale computing alters the outlook of the mathematical physicist. An ordinary system of linear equations becomes a problem in minimizing a quadratic form. A second order partial differential equation with certain boundary conditions becomes an integral equation with "built in" boundary conditions. To quote the author: "The facilities offered by these new calculating machines will at least make the formulating of physical problems in terms of integral equations and variation equations more familiar and may in time wean us from our present tendency to regard a differential equation as the basic way of formulating the mathematics of physical problems."

D. H. L.

30[Z].—Francis J. Murray, *The Theory of Mathematical Machines.* New York, Columbia Univ. Press, 1947. vii, 116 p. 21.5 × 28 cm. Lithographed; plastic binding $3.00.

This work appears to be a set of lectures on certain mathematical aspects of computing devices. There is a wide variety of both devices and aspects. Each of its four parts is divided into four or five short chapters of a few pages each. There are about 200 original line drawings that add much to the interest of the volume. Unfortunately these are not numbered so that sometimes the reader is in doubt as to which drawing is being referred to in the text. The book is lithographed from typewriting and is very neat.

The reviewer has found it difficult to give a short account of the actual contents of the book. There is a great deal of detail (not indicated in the chapter headings) on some topics and very little on others. On the whole the book is devoted to continuous or analogue devices almost entirely; there is only the briefest mention of high speed digital computers. There is a great deal of space devoted to electronics but only one example of an electronic counter, a soft tube prewar type. There is much material of an electro-mechanical nature but no mention is made of the possibility of using relays for computing. The mathematical discussion ranges in depth from the identity

$$4xy = (x + y)^2 - (x - y)^2$$

to Hilbert space. The four parts may be described as follows:

Part I, Digital machines, is disappointingly brief (12 p.) and is divided into counters, adders, multipliers, and "the punch card machine." The discussion is largely devoted to mechanical parts used in desk calculators such as the Leibniz wheel and Napier's bones. There is a description of the Hollerith card sorter.

Part II, Continuous operators, sounds the keynote of the text: Numerical quantities can be represented by physical magnitudes. The magnitudes discussed range from linear displacements to the phase angle of alternating currents. There is a good treatment of linear networks. Among multiplying devices there is a description of "square" gears, variably wound potentiometers, and rectifiers. Integrators and differentiators, both mechanical and electrical, are treated in great detail. The rest of Part II is devoted to the theory of amplifiers, servomechanisms, selsyn units and other electrical devices and their uses in mathematical machines.

Part III, The solution of problems, is devoted to composite machines for solving systems of linear equations, ordinary and partial differential equations. These machines include the network analyzer, differential analyzer and two electronic computers for linear equations. One of the latter has been designed by the author and is fully described. The mathematical treatment here is particularly interesting. The reader will find this material under "Adjusters" (p. 84–94, unfortunately the book has no index).

Part IV, Mathematical Instruments, is concerned with planimeters, integrometers, harmonic analyzers and cinema-integraphs. There is a page and a half of bibliography arranged topically. This does not include a large number of references inserted in the text.

The reader, whether he be interested in mathematical machines from a technical or a purely mathematical point of view, will find something interesting on every page. It is to be hoped that a second volume dealing with the theories of the many other interesting devices developed during the war may be eventually forthcoming.

D. H. L.

NOTES

73. The Checking of Functions Tabulated at Certain Fractional Points.—Many functions involving a parameter ν, in particular Bessel functions $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$, $K_\nu(x)$, etc., besides being tabulated for $\pm$ integral values of ν, as well as for $\nu = 0$, are often given for non-integral values of ν between -1 and 1, especially for $\nu = \pm \frac{3}{4}, \pm \frac{2}{3}, \pm \frac{1}{2}, \pm \frac{1}{3}$ and $\pm \frac{1}{4}$. When it is desired to perform the equivalent of a differencing check upon these or related functions (e.g. the zeros of these functions) considered as a function of ν for fixed x, due to the irregular interval in ν, it is necessary to take the divided differences. For any fixed set of n ν's, it is possible to obtain coefficients of f_ν for the last, i.e. $(n - 1)$th, divided difference which should vanish if the function behaves as a polynomial of the $(n - 2)$th degree in ν. Thus an error ϵ in any entry f_ν (this includes rounding errors) will usually show up by being multiplied by the coefficient of f_ν.

The coefficients which are given below are for three important cases likely to arise in practice, especially with Bessel functions:

(a) 7th divided difference for f_ν involving the 8 points $\nu = \pm \frac{3}{4}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{1}{4}$.
(b) 10th divided difference for f_ν involving the 11 points $\nu = \pm 1, \pm \frac{3}{4}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{1}{4}, 0$.
(c) 10th divided difference for f_ν involving the 11 points $\nu = \pm \frac{3}{4}, \pm \frac{2}{3}, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, 0$.