108. NBSMTP, *Tables of Sine, Cosine and Exponential Integrals*, v. 1, 1940.

P. 59, argument column, *for* 1.1405, *read* 0.1405.

109. NBSMTP, *Tables of the Exponential Function e^x*, 1939. See *MTAC*, v. 1, p. 438.

P. 168, $x = 1.6742$, *for* 5.33452 58202 12879, *read* 5.33452 58209 12879.
P. 304, $x = .2333$, *for* .79181, *read* .79191.

UNPUBLISHED MATHEMATICAL TABLES

56[B].—Great Britain, Admiralty Computing Service, *Tables of* $x^{1/4}$, $x^{-1/4}$, $x^{3/4}$, $x^{-3/4}$. Machine printed copy prepared by and in the possession of H. M. Nautical Almanac Office. Compare RMT **339**, *MTAC*, v. 2, p. 205.

Several requirements arose for quarter powers during the course of the computational work undertaken by Admiralty Computing Service at H. M. Nautical Almanac Office during 1943–1945. In the same period the Office was faced with the training of new staff with no previous computing experience. It was accordingly decided to make systematic tables of the four powers $-\frac{3}{4}, -\frac{1}{4}, +\frac{1}{4}, +\frac{3}{4}$ for a comprehensive range of argument; by this means considerable individual calculation for special investigations was avoided and the new staff provided with excellent material for elementary training in computing and tabulation.

Copy has been prepared for two tables in both of which the four functions are arranged side by side in the order $+\frac{1}{4}, -\frac{1}{4}, +\frac{3}{4}, -\frac{3}{4}$ for the range $x = 1(.01)10(.1)100(1)1000(10)10000$.

Table A. An accurate table with at least 7S with manuscript first differences written in small figures interlinearly. The number of decimals retained is:

Range	Power			
x	$+\frac{1}{4}$	$-\frac{1}{4}$	$+\frac{3}{4}$	$-\frac{3}{4}$
1–10	6	7	6	7
10–100	6	7	6	8
100–1000	6	7	5	9
1000–10000	6	7	4	9

Table B. A "working" table to 11 or 12S intended solely to give the tabulated values to the greatest accuracy to which they are available; therefore no differences are provided and the end-figure may be in error by several units. The number of decimals (D) and the error (E) in the last figure which is unlikely to be exceeded are given in the following table:

Range	Power							
x	$+\frac{1}{4}$		$-\frac{1}{4}$		$+\frac{3}{4}$		$-\frac{3}{4}$	
	D	E	D	E	D	E	D	E
1–10	10	2	10	2	10	2	10	2
10–100	10	3	10	2	9	2	11	4
100–1000	9	2	10	2	8	2	12	5
1000–10000	9	2	10	2	7	2	12	2

The original aim was to provide a table giving 7S accuracy throughout, interpolable with only trivial second difference corrections. Basic values were calculated to 10 or 11S for $x = 1(.01)3(.05)6.5(.1)10$ for powers $\pm\frac{1}{4}$; $x = 1(.01)4(.05)7.5(.1)10$ for powers $\pm\frac{3}{4}$. These were multiplied by the appropriate powers of 10 to give powers of $10x$, $100x$ and $1000x$ over the same ranges of x. Values of all 16 functions were then obtained for a uniform interval of .01 in x, over the whole range $x = 1$ to 10, by standard methods of interpolation to fifths and tenths on the National machines. The copy in each case was prepared by integrating on the National machine from differences produced by end-figure differencing of the interpolated values. Each table contains 72 foolscap pages, each with 4×50 entries, apart from differences.

JOHN TODD & D. H. SADLER

57[D].—GREAT BRITAIN, Admiralty Computing Service, *Six-figure logarithmic-trigonometrical tables*. Copy prepared by machine printing with ms. proportional parts, and in the possession of H. M. Nautical Almanac Office.

At the request of the Ministry of Supply, Admiralty Computing Service undertook to prepare copy for a six-figure table of logarithmic-trigonometric functions for use in the optical industry. The argument was to be in degrees and decimals. The obvious course of reproducing by photo-lithography PETERS, *Sechsstellige Logarithmen der trigonometrischen Funktionen von 0° bis 90° für jedes Tausendstel des Grades*, Verlag der Preussischen Landesaufnahme, Berlin 1921, was ruled out as being too expensive; it was decided to compromise by photographing the first five degrees of Peters' table and adding to it a new table at interval 0°.01.

It is this latter table, giving logarithms of the four functions sine, tangent, cotangent and cosine in the range 5°(0°.01)45°, which has been prepared. For various reasons, it was later decided not to proceed with the publication of the table.

The copy was prepared by building up from the second differences of the known end-figures; it was prepared on the National machine in a form as closely similar to Peters' table as the limitations of the machine would permit. Proportional parts were written in by hand. The copy of 80 p. is in perfect condition for the printer, but not good enough for photo-lithography.

JOHN TODD & D. H. SADLER

58[D, E].—GREAT BRITAIN, Admiralty Computing Service, *Tables of* $\operatorname{cosec}^2 x - x^{-2}$ *and* $x^{-2} - \operatorname{cosech}^2 x$. On National differencing sheets and in MSS. prepared by and in the possession of H. M. Nautical Almanac Office.

In connection with a special investigation tables were required of the two functions

$$\operatorname{cosec}^2 x - x^{-2} \quad \text{and} \quad x^{-2} - \operatorname{cosech}^2 x$$

to 14D for the range $x = 0(.001)1.6$. They have been calculated to 17D for $x = 0(.01)1.69$ and now await interpolation to tenths.

The fundamental values have been deduced from the tables of C. E. VAN ORSTRAND, Nat. Acad. Sci., *Memoirs*, v. 14, no. 5, 1921.

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59[L].—GREAT BRITAIN, Admiralty Computing Service, *Tables of* $C(m, \mu; x)$. Machine printed copy prepared by, and in the possession of H. M. Nautical Almanac Office.

These tables of the hypergeometric function

$$C(m, \mu; x) = F(1, m - \mu; m, x)$$

have been prepared at the suggestion of Dr. W. G. BICKLEY and Dr. J. C. P. MILLER, with a view to their use in the summation of certain slowly convergent series. Consider a series Σu_n, in which the ratio of consecutive terms can be written, to a good approximation for values of n greater than N, in the form:

$$u_{n+1}/u_n = x(1 - \mu n^{-1} + \mu\nu n^{-2})$$

where μ and ν are constants.

Then, to the same approximation,

$$u_{n+1}/u_n \doteq x(m - \mu)/m, \qquad \text{where} \qquad m = n + \nu,$$

and so

$$\sum_{\nu=0}^{\infty} u_{N+\nu} = u_N C(m, \mu; x).$$

A good approximation to the tail of the series is thus obtained.

The function is tabulated to 4D for $4m = 40(1)44, 80(1)84$; $4\mu = 4(1)20$; $x = .8(.02).9(.01)1$.

No differences are given, since it is assumed that the table will generally be used without interpolation. One page is devoted to each value of x and contains the two double-entry tables corresponding to the two series of values of m.

The method of calculation *should* result in the last figures not being in error by more than .6, but the system of checking adopted was not capable, throughout the whole range, of guaranteeing an accuracy of more than one unit.

The general method of computation was the repeated application of the recurrence formulae

$$C(m+1, \mu; x) = \frac{m}{(m-\mu)x}\{C(m, \mu; x) - 1\}$$

and

$$C(m, \mu; x) = (\mu - 1)^{-1}\{(m-1) - (m-\mu)(1-x)C(m, \mu - 1; x)\}$$

due regard being paid to the loss of figures inherent in this method. Initial values were obtained from the formulae

$$C(1, \mu; x) = (1 - x)^{\mu - 1}; \mu \neq 1,$$

and

$$C(2, 1; x) = -x^{-1} \ln (1 - x)$$

with

$$C(m, \mu; 1) = (m - 1)/(\mu - 1).$$

JOHN TODD & D. H. SADLER

MECHANICAL AIDS TO COMPUTATION

In the introductory article, "Admiralty Computing Service," there are reviews of publications **37**, *Electronic Differential Analyser;* **40**, *Rangefinder Performance Computer;* **112**, *The Fourier Tranformer.*

There is an interesting biographical sketch, and a portrait of HOWARD HATHAWAY AIKEN (1900–), in *Current Biography*, v. 8, no. 3, March 1947.

In *Wisconsin Engineer*, v. 51, Dec. 1946, p. 10–12 is an article by WALTER GRAHAM, "Do you know your slide rule?", in which the author explains the slide rule solution of equations of the type $x^x = k$, $a^x = x^b$, $\tan x = kx$, and $\sin x = kx$. See *MTAC*, v. 1, p. 203, **Q 8** and v. 2, p. 194, **25**.

Dr. LOTHAR SCHRUTKA, professor of mathematics in the Technische Hochschule, Vienna, is the author of a third much revised edition of his *Theorie und Praxis des logarithmischen Rechenschiebers*, Vienna, Deuticke, 1943. xii, 101 p. The first edition appeared in 1911, and the second in 1929. There are 32 titles in the bibliography, p. 95–96, and there is a full index, p. 97–101.

H. H. AIKEN & GRACE M. HOPPER, "The Automatic Sequence Controlled Calculator," *Electrical Engineering*, v. 65, Aug.–Nov. 1946, p. 384–391, 449–454, 522–528. See *MTAC*, v. 2, p. 185f.