424[M].—NIELS ARLEY, *On the Theory of Stochastic Processes and their Application to the Theory of Cosmic Radiation.* Diss. Copenhagen G.E.C. Gads Forlag, 1943. 240 p. 17.3 × 25 cm.

Chapter 8, p. 222–227 is entitled, "On the numerical computation of $\psi(x) = \int_0^x e^t dx$," $t = x^2$, that is, Dawson's or Poisson's integral, tables of which we have listed, *MTAC*, v. 1, p. 322–323, 422–423, v. 2, p. 55, 185. Arley's only references are to JAHNKE & EMDE, and DAWSON. T. 24, p. 224–225, gives $\psi(x)$, for $x = [2(.01)10;\ 4S]$. T. 23, p. 223, is of $f(x) = 2xe^{-t}\psi(x) = 1 + \frac{1}{2x^2} + \frac{1\cdot 3}{(2x^2)^2} + \frac{1\cdot 3\cdot 5}{(2x^2)^3} + \cdots$, $x \gg 1$, for $x = [2(.01)4(.1)10;\ 4\text{–}5S]$, $t = x^2$. Of this table it is stated that "We estimate that the figures are correct within 1 or 2 units in the last figure." T. 24 was calculated from T. 23 by means of the formula $\psi(x) = (2x)^{-1}e^t f(x)$, $t = x^2$.

R. C. A.

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT 412 (Loria); N **74** (FMR).

105.—G. F. BECKER & C. E. VAN ORSTRAND, *Hyperbolic Functions* (*Smithsonian Mathematical Tables*), Washington, 1909—fourth reprint 1931.

In this and previous editions the formulae 83, 84 on p. XV are incorrect. They should read

$$83.\ \tanh^{-1}\tan u = \tfrac{1}{2}gd^{-1}2u$$
$$84.\ \tan^{-1}\tanh u = \tfrac{1}{2}gd2u$$

They are given correctly in the fifth reprint 1942. The same error occurs in A. E. KENNELLY, *Tables of Complex Hyperbolic and Circular Functions*, second ed., Cambridge, Mass., 1921, p. (230), and in various textbooks.

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106. W. W. DUFFIELD, *Logarithms, their Nature, Computation and Uses, with Logarithmic Tables of Numbers and Circular Functions to Ten Places of Decimals.* Washington, 1897. See *MTAC*, v. 2, p. 161–165.

A. The fact that Duffield prepared this table by copying from VEGA *Thesaurus*, 1794, is well known; see the references in *MTAC* (above), and also L. J. COMRIE, Br. Astron. Assoc., *Jn.*, v. 36, 1926, p. 341.

DUFFIELD's copying from Vega included nearly 300 end-figure errors, but his work is held in such low esteem that no one appears to have checked anything but the end figures. Recently an opportunity of examining the first six presented itself when reading the proofs of a new 6-figure table now in press for Messrs. W. and R. Chambers. The result of a single comparison was as follows.

Page	Number	
495	32067	the figures 058 3318 should be overlined
500	33411	*for* 8994744, *read* 8894744
526	41764	*for* 8920871, *read* 8020871
530	42680	*for* 2224108, *read* 2244108
580	57482	*for* 3318704, *read* 5318704
686	89330	the overline on 9973340 should be deleted
710	9680.	in number, *for* 7180, *read* 9680
710	96826	the overline on 9919910 should be deleted
716	98771	*for* 5294508, *read* 6294508.

In *Science*, n.s., v. 7, 1898, p. 109–111, there is a devastating editorial on this table entitled "Logarithms on the 'Spoils System.'" It is unsigned, but from the fearless vigour of its style, and the fact that SIMON NEWCOMB was the mathematics member of the editorial committee,[1] there can be little doubt of the authorship. A few quotations, even after a lapse of nearly half a century, may not be without interest. "Anybody who knows anything about the subject knows that useful tables of logarithms include from four to seven places. The number of problems in which a table of more than seven places would be used is extremely small, and all extension of figures over what are actually used are a nuisance and a real hindrance. That the United States government should suddenly print for free distribution several thousands of copies of this compilation must create, among those who understand, a strong suspicion of a dearth of other printable material." Then, with biting sarcasm: "Their arrangement might have been worse, but only by printing the numbers in one annual report and their logarithms in the next." Of Duffield's claim that he did not know of Vega when the computations were begun: "This great work of Vega, which every tyro in computing knows, was published in 1794. This is more than a hundred years ago, and it is not easy to understand how one could seriously think of repeating such a performance without finding that it had already been done." Then, with his tongue in his cheek: "The author thinks he has discovered some serious mistakes in Vega, but he delicately refrains from telling what they are."

L. J. C.

[1] Simon Newcomb was mathematical editor of *Science* 1895–1903. EDITOR.

B. Since L. J. C. had many years ago sent to me a copy of his 1926 review of Henderson's *Bibliotheca Tabularum Mathematicarum*, 1926, to which he refers above, it was a decided oversight on my part to omit a reference to it in RMT **319**. The paragraph of the review which is here pertinent is the following: "However, he did not take Duffield at his face value, and so made the discovery of his dishonesty in attempting to pass a copy of Vega as his own computation. Peters was also aware [1922] of this fraud, and it was noticed independently by the present writer in 1924. Henderson's hypothesis that Duffield was original up to 26000 is untenable. It is far more probable that in the copy of Vega's *Thesaurus* which Duffield used some previous owner had entered the corrections given by Lefort up to 26000. This supposition is supported by the fact that of the five errors in Duffield before 26000 four were not given by Lefort; three were first pointed out by Glaisher, and in one case Lefort had omitted the asterisk which denoted that the error was in Vega as well as in Vlacq." Before commenting on this I should point out that I did not with sufficient clearness indicate that of the two errata lists of Lefort the first, of 1858, was a list of errors in Vlacq, *Arithmetica Logarithmetica*, 1628, upon which Vega's table of logarithms was based; and the second, of 1875, was simply a list of errors in Vega's *Thesaurus*. In the first list, however, Lefort added a star to indicate where an error in Vlacq persisted in Vega.

L. J. C. kindly reported to me that the "five errors" in Duffield before 26000, to which he referred, were in connection with the numbers 10033 (Lefort with * omitted), 11275 (Glaisher, R.A.S. *Mo. Not.*, v. 32, p. 258), 11699 (Glaisher, *idem*, v. 32, p. 258), 22312 (Lefort), 24580 (Glaisher, *idem*, v. 32, p. 258 and v. 34, p. 471). It is true that Lefort 1858 does not list an error in Vega in connection with 10033 but Lefort 1875 *does* list such an error. Thus in the five Duffield errata two were listed by Lefort. With regard to the error associated with 11275 Glaisher remarks the logarithm of 11275 is 4.05211, 65505, 49998, 14..., and it is a matter of indifference whether the tenth figure of the mantissa be increased or not. But Glaisher lists the end figures of Duffield 65506 as an "error" and the "correction" 65505. That this error was also in Peters was pointed out by L. J. C. in MTE **104**. Thus Peters listed only four Duffield errata before 26000, not five.

R. C. A.

107. MAURICE KRAÏTCHIK, *Recherches sur la Théorie des Nombres*, v. 1. Paris, 1924.

On p. 77–80 is a table of the factors of the two Fibonacci sequences

$$1, 1, 2, 3, 5, 8, 13, \cdots, U_n, \quad \text{and} \quad 1, 3, 4, 7, 11, 18, \cdots, V_n.$$

The following two errata may be noted:

for $U_{37} = 79 \cdot 149 \cdot 2221$, ***read*** $73 \cdot 149 \cdot 2221$,
for $U_{67} = 44945570212853$, ***read*** $269 \cdot 116849 \cdot 1429913$.

This latter error appears to be due to POULET, since the entry is attributed to him.

Table I, p. 131–191 gives, in effect, for each prime less than 300000 the exponent e of 2 modulo p, that is the smallest e for which $2^e - 1$ is divisible by p. Actually, to save space, the table gives $\gamma = (p - 1)/e$. This table is the most extensive of its kind and has been used to a considerable extent in connection with tables of factors of $2^n \pm 1$ and other problems involving the binomial congruence. Immediately after its publication this table was compared with a set of similar tables of CUNNINGHAM & WOODALL[1] extending to $p < 100000$ and the resulting errata of 44 items appear in *Messenger Math.*, v. 54, 1924, p. 184 (given also in *Guide to Tables in the Theory of Numbers*. Washington, 1941, p. 155.)

The purposes for which this table is most frequently used require information about primes whose exponents are comparatively small. It therefore seemed desirable to find independently those primes whose exponents do not exceed 2000 in the range $100000 < p < 300000$. A comparison of these results with Kraitchik's table yields most of the entries in the errata list given below.[2] Since this comparison involves less than 1.5% of the entries in Kraitchik's table the user of this table, who is interested in exponents beyond 2000, is exposed to considerable risk.

p	*For*	*Read*	p	*For*	*Read*
101737	4	8	165233	4	92
102043	2	9	[4]165313	96	672
[2]104161	60	30	194867	7	217
106649	8	4	216217	24	168
107857	7	14	220243	3	213
108497	8	16	246739	3	177
108967	39	78	247381	1	217
[3]109121	8	248	247531	5	185
111487	6	102	250867	-2	1
[2]114601	2	6	254039	2	142
119929	2	114	255071	1	2
[2]121081	4	20	[2]267481	1	2
[2]127681	8	152	272959	2	938
141023	98	14	[5]284689	2	216

TIETZE notes[6] that on p. 191, after 297 967, for 297 671, should be 297 971. For other misprints of primes see my *Guide*, p. [156].

D. H. L.

[1] A. J. C. CUNNINGHAM & H. J. WOODALL "Haupt-exponents of 2," *Quart. Jn. Math.* v. 37, 1905, p. 122–145; v. 42, 1911, p. 241–250; v. 44, p. 41–48, 1912, p. 237–242, 1913; v. 45, 1914, p. 114–125.

[2] Dr. A. E. WESTERN has already noted five of these in *MTAC*, v. 1, p. 429.

[3] This error caused the omission of the entry: 96135601, 881 in the writer's table of composite solutions of $2^n \equiv 2 \pmod{n}$, *Amer. Math. Mo.*, v. 43, 1936, p. 351; see MTE **102.**

[4] The discovery of this error leads to the following factorization into primes
$$2^{123} + 1 = 3^2 \cdot 83 \cdot 739 \cdot 165313 \cdot 8831418697 \cdot 13194317913029593$$

[5] Kraitchik had here $284687 = 13 \cdot 61 \cdot 359$, which is therefore not a prime; the value of γ was also incorrect.

[6] Akad. d. Wissen., Munich, *Abh.*, n.s. Heft 55, 1944, p. 9; see RMT **369.**

108. NBSMTP, *Tables of Sine, Cosine and Exponential Integrals*, v. 1, 1940.

P. 59, argument column, *for* 1.1405, *read* 0.1405.

109. NBSMTP, *Tables of the Exponential Function* e^x, 1939. See *MTAC*, v. 1, p. 438.

P. 168, x = 1.6742, *for* 5.33452 58202 12879, *read* 5.33452 58209 12879.
P. 304, x = .2333, *for* .79181, *read* .79191.

UNPUBLISHED MATHEMATICAL TABLES

56[B].—GREAT BRITAIN, Admiralty Computing Service, *Tables of* $x^{1/4}$, $x^{-1/4}$, $x^{3/4}$, $x^{-3/4}$. Machine printed copy prepared by and in the possession of H. M. Nautical Almanac Office. Compare RMT **339**, *MTAC*, v. 2, p. 205.

Several requirements arose for quarter powers during the course of the computational work undertaken by Admiralty Computing Service at H. M. Nautical Almanac Office during 1943–1945. In the same period the Office was faced with the training of new staff with no previous computing experience. It was accordingly decided to make systematic tables of the four powers $-\frac{3}{4}$, $-\frac{1}{4}$, $+\frac{1}{4}$, $+\frac{3}{4}$ for a comprehensive range of argument; by this means considerable individual calculation for special investigations was avoided and the new staff provided with excellent material for elementary training in computing and tabulation.

Copy has been prepared for two tables in both of which the four functions are arranged side by side in the order $+\frac{1}{4}$, $-\frac{1}{4}$, $+\frac{3}{4}$, $-\frac{3}{4}$ for the range x = 1(.01)10(.1)100(1)1000(10)10000.

Table A. An accurate table with at least 7S with manuscript first differences written in small figures interlinearly. The number of decimals retained is:

Range x	Power $+\frac{1}{4}$	$-\frac{1}{4}$	$+\frac{3}{4}$	$-\frac{3}{4}$
1–10	6	7	6	7
10–100	6	7	6	8
100–1000	6	7	5	9
1000–10000	6	7	4	9

Table B. A "working" table to 11 or 12S intended solely to give the tabulated values to the greatest accuracy to which they are available; therefore no differences are provided and the end-figure may be in error by several units. The number of decimals (D) and the error (E) in the last figure which is unlikely to be exceeded are given in the following table:

Range	Power							
x	$+\frac{1}{4}$		$-\frac{1}{4}$		$+\frac{3}{4}$		$-\frac{3}{4}$	
	D	E	D	E	D	E	D	E
1–10	10	2	10	2	10	2	10	2
10–100	10	3	10	2	9	2	11	4
100–1000	9	2	10	2	8	2	12	5
1000–10000	9	2	10	2	7	2	12	2

The original aim was to provide a table giving 7S accuracy throughout, interpolable with only trivial second difference corrections. Basic values were calculated to 10 or 11S for x = 1(.01)3(.05)6.5(.1)10 for powers $\pm\frac{1}{4}$; x = 1(.01)4(.05)7.5(.1)10 for powers $\pm\frac{3}{4}$. These were multiplied by the appropriate powers of 10 to give powers of $10x$, $100x$ and $1000x$ over the same ranges of x. Values of all 16 functions were then obtained for a uniform interval of .01 in x, over the whole range x = 1 to 10, by standard methods of interpolation to fifths and tenths on the National machines. The copy in each case was prepared by integrating on the National machine from differences produced by end-figure