In Part I the cases of the linear ($f(z) = az + b$) and bilinear ($f(z) = (az + b)/(cz + d)$) transformations are discussed. For instance, explicit formulae giving the actual transformation which carries any two non-intersecting circles into two concentric circles are given.

In Part II, Algebraic functions and z^α for real values of α are discussed. Included in this part is a section on JOUKOVSKI's transformation $w = az + b/z$ and its generalisations.

This dictionary will be completed by the issue of three further parts:

109 Part III, *Exponential Functions and some related Functions,*
110 Part IV, *Schwarz-Christoffel Transformations,*
111 Part V, *Higher Transcendental Functions.*

112. ALAN BAXTER (1910–1947), *The Fourier Transformer.* 1947.

This is a machine of mixed electrical and mechanical design which evaluates the Fourier transforms $C(n) = \int_0^\infty f(n) \cos nx\, dx$, and $S(n) = \int_0^\infty f(n) \sin nx\, dx$ of a given function $f(x)$. The integrations are carried out electrically but the selection of the wave-number n is mechanical. The input function is followed manually and the motion translated into a proportional A.C. voltage (50 c/s) by an inductance potentiometer. A power supply of equal voltage is derived from a servo-operated Variac transformer. This feeds simultaneously 22 magslip resolvers, each of which multiplies the voltage by the sine and cosine of the particular angle at which its rotor lies. These voltages derived from the magslip resolvers, proportional to $f(x) \cos nx$ and $f(x) \sin nx$, are integrated by modified sub-standard K.W.H. meters.

Each magslip rotates continuously during the transit of the input function, to produce the appropriate angle nx, the angles being selected by a system of gear boxes. The range of wave numbers n is from $\frac{1}{16}$ to 128, a figure which can be further increased if the function is subdivided. One traverse of the function occupies 10 minutes, and produces simultaneously 22 cosine integrals and 21 sine integrals whose wave numbers cover a range of 4:1. The full range from $\frac{1}{16}$ to 128 is covered, if required, by repeated following of the input function.

Additional dispositions of the gear boxes enable the density of wave-numbers in any particular range to be increased at first fourfold, and then a further tenfold, allowing for close investigation of any particularly interesting regions of the transform. It is hoped that the errors in the transform will be less than 1 per cent of its peak value.

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[1] Other aspects of the work of Admiralty Computing Service are described in an article by the present authors, in *Nature*, v. 157, May 4, 1946, p. 571–573; see *MTAC*, v. 2, p. 188. See also A. ERDÉLYI and JOHN TODD, *Nature*, v. 158, 1946, p. 690; AÇS **115**.

RECENT MATHEMATICAL TABLES

Seven reviews of RMT are to be found in our introductory article "Admiralty Computing Service," **7, 9, 18, 20, 80, 89, 96.**

400[C].—JOSEF KŘOVÁK, *Achtstellige Logarithmische Tafel der Zahlen. Osmimístné Logaritmické Tabulky Čísel.* Prague, Geographic Institute of the Minister of the Interior, 1940. iv, 26 p. 14.6 × 21.1 cm.

This little pamphlet is divided into two parts. In the first part (p. 2–14) the arguments are the logarithmic mantissae from 0000 to 6389 corresponding to the numbers 10 000 000 to 43 541 160. In the second part (p. 15–26) are given the mantissae of log N, for $N = 4340(1)10009$. In columns headed d, of each part, are the greatest and least differences which arise in successive lines. Examples in German and Czechish illustrate the interpolation process for getting the 8-figure logarithm of any number.

401[D].—JOSEF KŘOVÁK, *Sechsstellige Tafeln der natürlichen Werte der Funktionen Sinus und Cosinus für Winkel in Zentesimalteilung. Šestimístné Tabulky přirozených hodnot Funkcí Sinů a Cosinů úhlů v setinném dělení.* Prague, Geographic Institute of the Minister of the Interior, 1943?, xii, 52 p. + two slips (in German) listing errors in the first two editions as well as in the current volume. 14.9 × 20.8 cm.

From the errata slips of this undated volume it would seem as if the edition before us was at least the third. And from the preface of the first edition (1941) of Křovák's *Natürliche Zahlen der Funktion Cotangens*, of which the second edition (1943) was reviewed in RMT 362, it is clear that the first edition of the six-place table appeared in 1940.

The table gives the natural values of sine and cosine for each centesimal minute, with differences, each page being devoted to 1^g.

On p. 51, there are "Tafeln für die Berechnung zehnstelliger Logarithmen der Zahlen N und umgekehrt," explanations following on p. 52.

R. C. A.

402[D].—JOSEF KŘOVÁK, **a.** *Zwölfstellige Tafeln der trigonometrischen Funktionen. Dvanáctimístné Tabulky trigonometrických Funkcí.* Prague, Landesvermessungsamt Böhmen und Mähren, second ed., 1944. viii, 271 p. + 2 loose sheets of tables. 21.8 × 29.6 cm.

b. *Koeffizienten zur Berechnung der zweiten Interpolationsglieder. Koeficienty pro výpočet druhých interpolačních členů.* Second ed., Prague, 1944. vi, 26 p. 21.8 × 29.6 cm.

a. This table, which appeared originally in 1928, is a twelve-place sexagesimal table of sines and cosines for each 10″ of the quadrant, and of tangents, at a similar interval, 0 to 45°. It is the first twelve-place table of the kind. But among recent tables Andoyer's fifteen-place tables of these three functions (1915–1916), throughout the quadrant, at interval 10″, will be recalled.

Each page is devoted to 10′ and columns with first and second differences (d_1, d_2) are given.

The companion five-place table **b**, which appeared also with the original edition, facilitates interpolations. There are also two loose-sheet tables for calculating second and third differences: $-z(1-z)d_2/2!$, $z(1-z)(2-z)d_3/3!$.

The author tells us that his values of the sine and cosine functions, insofar as they were correct, were rounded off from the fifteen-place Rheticus-Pitiscus table of 1613, *Thesaurus Mathematicus sive canon Sinuum ad Radium 1.00000.00000.00000 et ad dena quaeque Scrupula secunda Quadrantis. . . .* The values of the tangents (cotangents) were derived, with the aid of calculating machines, from the quotients of the values for sines and cosines. All values for each function were tested by differences, and by comparison with the tables of Andoyer.

R. C. A.

403[D].—J. T. PETERS, *Sechsstellige Werte der trigonometrischen Funktionen von Tausendstel zu Tausendstel des Neugrades.* Ninth ed., Berlin, Wichmann, 1944. iv, 512 p. 18.9 × 25.3 cm.

The preface of this work is dated "Berlin, im August 1938" and this is the year when the first edition was published (iv, 512 p.). A third edition with corrections appeared in 1940 and a seventh edition in 1943. The later "editions" are presumably nothing but "reprints." They reflect however the great popularity of the centesimal division of the quadrant in German war applications.

Pages 2–501 contain the numerical values of the trigonometric functions sin, tan, cos, to 6D, cot to 6S, at interval $0^g.001$ or 10 centesimal seconds. On each page are the values of the functions for 10 centesimal minutes. Differences and proportional parts are given at the foot of each page. On pages 2–21 we also find here a 6S auxiliary table w cot w for each $0^g.01$ to $2^g.00$.

Apart from some worked out examples and constants, the final pages (503–512) contain a number of tables for changes between the sexagesimal and centesimal units, between centesimal units and time, and mils, and for the improvement of the Gauss-Krüger projection.

Typographically the pages are very unattractive.

The Czechoslovakian volumes of the six-place centesimal tables of cotangents, and of sines and cosines, have been already referred to in RMT **362** and **401**.

R. C. A.

404[D].—J. T. PETERS, *Siebenstellige Werte der trigonometrischen Funktionen von Tausendstel zu Tausendstel des Neugrades*. Berlin, Verlag des Reichsamts für Landesaufnahme, 1941. viii, 544 p. 17 × 26 cm.

This was probably the last printed volume of tables coming from Peters' hands. The preface is dated March 1941; he died in the following August. The fine appearance of this 7-place table happily contrasts with that of the 6-place table of the previous review. As in that table the functions are the sin, tan, cot, cos and in the first 2^g $w \cot w$ is also tabulated. Throughout the quadrant the interval is $0^g.001$. Proportional parts are given at the bottom of each page. From 2^g on, the first two decimals of each value are printed at the head of each column. The main table ends on p. 521.

Pages 525–544 are devoted to trigonometric formulae, series, solutions of equations, least squares, mathematical and geodetic constants, tables indicating the relations between sexagesimal and centesimal degrees, centesimal degrees and radians.

Thus endeth the twentieth major mathematical table prepared under the direction of Peters. See *MTAC*, v. 1, p. 168f. The 7-place table of the trigonometric functions for each 0°.001 in the quadrant were sent forth by Peters in 1918, and reprinted in 1930 and 1938. For the American edition, 1942, see *MTAC*, v. 1, p. 12f.

R. C. A.

405[D, E, V].—ÎA. M. SEREBRIĬSKIĬ, "Obtekanie Krylovykh Profileĭ Proizvol'noĭ Formy" (Flow past an aerofoil of arbitrary form), Akad. N., SSSR., Moscow-Leningrad, *Inzhenernyĭ Sbornik* (*Engineering Review*), v. 3, no. 1, 1946, p. 105–136. 16.4 × 25.7 cm. See RMT **194**, v. 1, p. 390.

There are the following tables:

T. 1, p. 111, $\cosh \psi$, defined by equations: $x = \cosh \psi \cos \theta$, $y = \sinh \psi \sin \theta$, for $x = 0(.2).6(.1).8(.05).9, .93(.02).97(.01).99(.005)1(.01)1.05$; $y = 0(.01).02(.02).22$. Tables 1–9 are to 3D. $\cosh \psi = [w \pm (w^2 - x^2)^{\frac{1}{2}}]^{\frac{1}{2}}$, where $w = \frac{1}{2}(1 + x^2 + y^2)$.

T. 2, p. 114, $\psi_n(\theta) = [\frac{1}{2}(1 + \cos \theta)]^n$, for $\theta = 0(5°)20°(10°)180°$; $n = 1(1)10(5)20(10)40(20)80$. Same ranges of θ, n in T. 4–6.

T. 3, p. 117, $\psi_n(\theta) \sin \theta$, for $\theta = 0(5°)20°(10°)180°$; $n = 1(1)10(5)20$. Same ranges of θ, n in T. 7–9.

For **T. 4** and a few others we need the definition of a *conjugate*; as follows: If a function F is expressed in the form $F = \sum_{n=0}^{\infty} A_n \cos n\theta/\xi^n$, then its *conjugate* is $E[F] = \sum_{n=0}^{\infty} A_n \sin n\theta/\xi^n$. [E.g., taking the above $\psi_n(\theta) = [\frac{1}{2}(1 + \cos \theta)]^n$, we have $\psi_2(\theta) = [\frac{1}{2}(1 + \cos \theta)]^2 = \frac{1}{4}(1 + 2\cos\theta + \cos^2\theta) = \frac{1}{4}(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta) = \frac{1}{4}(\frac{3}{2}\cos 0° + 2\cos\theta + \frac{1}{2}\cos 2\theta)$; $E\psi_2(\theta) = \frac{1}{4}(\frac{3}{2}\sin 0° + 2\sin\theta + \frac{1}{2}\sin 2\theta) = \frac{1}{2}\sin\theta + \frac{1}{8}\sin 2\theta$.]

T. 4, p. 122, conjugate of $\psi_n(\theta)$.

T. 5, p. 123, derivative of $\psi_n(\theta)$.

T. 6, p. 124, derivative of conjugate of $\psi_n(\theta)$.

T. 7, p. 126, conjugate of $\psi_n(\theta) \sin \theta$.

T. 8, p. 127, derivative of $\psi_n(\theta) \sin \theta$.

T. 9, p. 128, derivative of conjugate of $\psi_n(\theta) \sin \theta$.

There are graphs of the various functions tabulated T. 1–9.

T. 10, p. 129, values of coefficients $A_{k,n}$ in the development $\psi_n(\theta) = \sum_{k=0}^{n} A_{k,n} \cos k\theta$, with the recurrence formula $A_{k,n} = \{[n - (k-1)]/(n+k)\}A_{k-1,n}$ for $k = 0(1)24$; $n = [1(1)10(5)20(10)40(20)80; 4D]$.

On p. 130, similar values of coefficients $B_{k,n}$ in the development $\psi_n(\theta) \sin\theta = \sum_{k=1}^{n+1} B_{k,n} \sin k\theta$, where $B_{k,n} = .5(A_{k-1,n} - A_{k+1,n})$, $B_{0,n} = 0$, $B_{1,n} = A_{0,n} - .5A_{2,n}$, for $k = 1(1)10(5)20$, $n = [1, 2; 4D]$.

S. A. J.

406[E].—HARVARD COMPUTATION LABORATORY, *Fifteen-place table of* e^{-x}. Bureau of Ships Computation Project. *Publication* no. 20, July 1945. iv, 6 leaves. 21.5 × 27.8. Out of print.

This is a table of e^{-x}, for $x = [0(.05)30; 15D]$. C. E. VAN ORSTRAND, Nat. Acad. Sci., *Memoirs*, v. 14, no. 5, 1921, gave in T. VI, the value of e^{-x}, $x = [0(.1)50; 33\text{–}48D]$.

407[E].—H. S. UHLER, "Special values of $e^{k\pi}$, cosh $(k\pi)$ and sinh $(k\pi)$ to 136 figures," Nat. Acad. Sci., *Proc.*, v. 33, Feb. 1947, p. 34–41. 17.3 × 25.7 cm.

T. I. $e^{\pi/m}$, $\pm m = 1(1)4(2)8(4)16(8)32(16)64$, 96, 192. T. II–III, Sinh (π/m), Cosh (π/m), for $+m$. Various checks for accuracy are stated.

408[E, O].—K. G. HAGSTROEM, "Un problème du calcul stochastique," *Försäkrings Matematiska Studier tillägnade Filip Lundberg*, Stockholm, 1946, p. 104–127, table p. 127. 18.5 × 24.4 cm.

This volume of studies in actuarial mathematics is dedicated to Dr. Lundberg, former general manager of the Life Insurance Co., De Förenade, and Sickness Insurance Co. EIK, on his 70th birthday Dec. 31, 1946. There is a "table of $u(z)$," arranged however as a 3D table of z for $u(z) = 0(.01)2.4$; $z = u^{-1} \ln \cosh u = \frac{1}{2}u - \frac{1}{12}u^3 + \frac{1}{45}u^5 - \frac{17}{2520}u^7 + \frac{31}{14175}u^9 - \cdots$, $u(z) = 2z + 1.3333z^3 + 1.2444z^5 + 1.2529z^7 + 1.2935z^9 + \cdots$.

409[F].—ALBERT GLODEN, "Factorizaci6n de numeros de la forma $x^4 + 1$," *Euclides*, Madrid, v. 5, 1945, p. 620–621. 16.6 × 24.1 cm.

In this note the author republishes some of his factorizations of $x^4 + 1$ (*MTAC*, v. 2, p. 211). Unlike the previous table, this table lists only those factorizations which have been completely decomposed into primes. The variable x ranges irregularly over 132 values between 1008 and 1500.

D. H. L.

410[F].—ALBERT GLODEN, *Table des solutions de la congruence* $x^4 + 1 \equiv 0$ (mod$p$) pour $350.000 < p < 500.000$. Luxembourg, author, 11 rue Jean Jaurès, and Paris, Centre de Documentation Universitaire, 1946, 42 p. 21.8 × 27.4 cm. Offset print.

This table is an extension of four previous tables of CUNNINGHAM $(1 < p < 100000)$, HOPPENOT $(100000 < p < 200000)$, GLODEN $(200000 < p < 300000)$ and DELFELD $(300000 < p < 350000)$, see *MTAC*, v. 2, p. 71–72, 210–211. As in the previous tables, this table gives only the two solutions x_1, x_2 of

$$x^4 \equiv -1 \pmod{p}, \qquad p = 8k + 1,$$

which are less than $\frac{1}{2}p$, the other two being the negatives of these. The method of construction of the table is as follows. First, one compiles two lists of numbers of the forms $a^2 + b^2$ and $c^2 + 2d^2$. In each list each prime p of the form $8k + 1$ will appear exactly once. In fact those numbers which appear more than once are composite. The representations

(1) $$p = a^2 + b^2 = c^2 + 2d^2$$

lead at once to the four solutions

$$x \equiv \pm d(a \pm b)/ac \qquad (\text{mod } p).$$

To reduce these values to integers modulo p involves the solution of a single linear congruence for t

$$act \equiv 1 \ (\text{mod } p).$$

This procedure appears to be somewhat shorter than the one earlier described by the author (*MTAC*, v. 2, p. 71–72).

Under the heading of applications the author gives complete factorizations of nine numbers of the form $x^4 + 1$ (RMT 366). It may be of interest to point out that another application of this table is its use to rediscover the quadratic partitions (1). In fact we may use the tabulated values of x_1, x_2 to construct l and m by:

$$m \equiv \tfrac{1}{2}(x_2 - x_1) \qquad (\text{mod } p),$$
$$l \equiv m(x_2 + x_1) \qquad (\text{mod } p).$$

If we now form the sequence $r_1, r_2, \cdots$ by Euclid's algorithm

$$p = q_0 l + r_1, l = q_1 r_1 + r_2, \cdots,$$

the first two r's less than $\sqrt{p}$ are values of a, b such that $p = a^2 + b^2$. The numbers c, d may be found in like manner from p and m. It is worth noting that the table gives a list of primes of the form $8k + 1$ between 350000 and 500000.

D. H. L.

411[F].—Irving Kaplansky & John Riordan, "The problème des ménages," *Scripta Mathematica*, v. 12, 1946, publ. Jan. 1947, p. 113–124. 16.7 × 24.8 cm.

This interesting paper on a problem due to Lucas,[1] concludes with a small table of "ménages numbers" $u_{n,x}$ for $x = 0(1)n$ and for $n = 2(1)10$. These numbers may be defined as follows. Let

(1) $$a_1, a_2, a_3, \cdots, a_n$$

be a permutation of $1, 2, \cdots, n$. The integer a_k is said to be in a forbidden place if it is in either the k-th or $k + 1$st place in (1). If (1) has precisely x elements in forbidden places it may be said to be of type x. The ménages number $u_{n,x}$ is the number of permutations of $1, 2, \cdots, n$ of type x. When $x = 0$ this number is the same as the number of ways that n married couples may be seated at a round table, ladies alternating with gentlemen, in such a way that no one sits next to one's spouse. This fact accounts for the name given the function $u_{n,x}$ in general. The famous function $u_{n,0}$ has been tabulated by Lucas[1] for $n = 4(1)20$.

D. H. L.

[1] Edouard Lucas, *Théorie des Nombres*, Paris, 1891, p. 495.

412[F].—Gino Loria, "Sulla scomposizione di un intero nella somma di numeri poligonali," Accad. Naz. Lincei, *Atti, Cl. Sci. Fis. Mat. Nat.*, *Rendiconti*, s. 8, v. 1, 1946, p. 7–15. 18.1 × 26.8 cm.

This note gives three tables showing all partitions of each integer $n \leqq 100$ into not more than (i) four squares, (ii) three triangular numbers and (iii) five pentagonal numbers.[1] The number ν of such partitions is also given. Tables of this kind appear never to have been published before this. The number ν is perhaps of more interest than the actual partitions themselves. It is surprising to find that the tables are quite unreliable. A complete recalculation shows the following errata.

Table I

N	
28	*for* $3+3+1$, *read* $3+3+3+1$; *insert* $5+1+1+1$; *for* $\nu = 2$, *read* $\nu = 3$
34	*insert* $4+4+1+1$; *for* $\nu = 3$, *read* $\nu = 4$
52	*insert* $4+4+4+2$; *for* $\nu = 4$, *read* $\nu = 5$
58	*insert* $7+2+2+1$; *for* $\nu = 4$, *read* $\nu = 5$
68	*insert* $5+5+3+3$; *for* $\nu = 3$, *read* $\nu = 4$
71	*delete* $5+4+4+2$; *for* $\nu = 3$, *read* $\nu = 2$
76	*for* $6+2+2$, *read* $6+6+2$
83	*delete* $8+4+1+1$; *for* $\nu = 5$, *read* $\nu = 4$
89	*insert* $6+6+4+1$; *for* $\nu = 4$, *read* $\nu = 5$
93	*delete* $7+$ at end of first line
96	*for* $8+4+2$, *read* $8+4+4$

Table II

16	*insert* $4+2+2$; *for* $\nu = 2$, *read* $\nu = 3$
18	*insert* $3+3+3$; *for* $\nu = 1$, *read* $\nu = 2$
21	*insert* $5+2+2$; *for* $\nu = 3$, *read* $\nu = 4$
22	*insert* $4+3+3$; *for* $5+3+2$, *read* $5+3+1$; *for* $\nu = 2$, *read* $\nu = 3$
23	*for* $4+4+3$, *read* $4+4+2$
27	*insert* $5+3+3$; *for* $\nu = 2$, *read* $\nu = 3$
28	*for* $5+4+3$, *read* $5+4+2$
45	*for* $8+3+3$, *read* $8+3+2$
49	*for* $8+4+3$, *read* $8+4+2$

Table III

24	*for* $2+2+1+1$, *read* $3+2+2+1+1$
28	*for* $3+2+2+1$, *read* $3+2+2+2+1$
53	*for* $3+3+3+2$, *read* $3+3+3+3+2$
62	*for* $5+2+2+2+2$, *read* $5+3+2+2+2$
80	*insert* $4+4+3+3+3$; *for* $\nu = 5$, *read* $\nu = 6$
92	*insert* $6+3+3+3+2$; *for* $\nu = 7$, *read* $\nu = 8$
95	*insert* $6+4+3+2+2$; *for* $\nu = 5$, *read* $\nu = 6$
96	*for* $7+3+3+3+1$, *read* $7+3+3+1+1$
97	*insert* $5+5+4+2$, and $6+4+3+3$; *delete* $5+4+43+2$; *for* $\nu = 6$, *read* $\nu = 7$
100	*for* $4, 4+4+4+3$, *read* $4+4+4+4+3$.

D. H. L.

[1] For positive integers n triangular numbers are given by the formulae $T = \frac{1}{2}n(n+1)$, and pentagonal by $P = \frac{1}{2}n(3n-1)$.

413[I, L, M], [L].—BAASMTC, *Legendre Polynomials*, (*Mathematical Tables, Part-volume A*). Cambridge, Univ. Press, 1946, 42 p. 22 × 28.4 cm. 8s. 6d.
[L, M], J. C. P. Miller, *The Airy Integral, giving Tables of Solutions of the Differential Equation* $y'' = xy$. (*Mathematical Tables, Part-volume B*). Cambridge, Univ. Press, 1946, 56 p. 22 × 28.4 cm. 10 s. With this Part-v, are also the following Auxiliary Tables on Cards.
[I], *Coefficients in the Modified Everett Interpolation Formula, BAASMTC, Auxiliary Tables*, no. I. Cambridge, Univ. Press, 1946, 1 p. 19.5 × 26 cm. 6d. each, or 5s. per dozen.
[I], *Table for Interpolation with Reduced Derivatives*, BAASMTC, *Auxiliary Tables*, no. II. Cambridge, Univ. Press, 1946, 2 p. 19.5 × 26 cm. 6d. each, or 5s. per dozen.

Some years ago the BAASMTC considered the problem of tables whose size did not justify their publication as bound volumes of the Committee's series, but which it did seem desirable to publish, and decided to issue them as 'Part-volumes,' in paper covers, with the aim of ultimately combining them into bound volumes of cognate tables. Considerable delay—mainly arising out of the circumstances of the war—has occurred since the inception of the project, but the first two Part-volumes are now issued. Apart from the (not very) stiff paper covers, they are uniform, in page size, typography, arrangement, etc., with the main series.

Part-volume A contains tables of the Legendre polynomials $P_n(x)$ as follows:

$x = 0(.01)1$, $n = 2(1)6$, exact or 7D, with δ^2 or δ^2_m p. A6, A7.
$x = 1(.01)6$, $n = 2(1)6$, exact, 7 or 8S, with δ^2 or δ^2_m, p. A8–A17.
$x = 0(.01)1$, $n = 7(1)9$, 7D, with δ^2 and δ^4, p. A18, A19.
$x = 1(.01)6$, $n = 7(1)9$, 7 or 8S, with δ^2 or δ^2_m, p. A20–A29.
$x = 0(.01)1$, $n = 10(1)12$, 7D, with δ^2_m and δ^4_m, p. A30, A31.
$x = 1(.01)6$, $n = 10(1)12$, 6 to 8S, with δ^2 or δ^2_m, and δ^4 (for $x = .5$–1.5), p. A32–A41.
$x = 6(.1)11$, $n = 2(1)6$, 7 or 8S, with δ^2 or δ^2_m, p. A42.

The tables were designed by Dr. L. J. COMRIE, and calculated under his supervision when he was secretary of the Committee. Their nucleus, however, dates back much further, to the Committee's report for 1879, which gave exact values of $P_n(x)$ for $x = 0(.01)1$, up to $n = 7$. $P_8(x)$ for the same range was taken from TALLQVIST[1] and HAYASHI[2] (5 errors in Hayashi), and checked. $P_9(x)$ for this range was calculated by use of the recurrence formula, while the values for $n = 10(1)12$ were calculated from the definitions, independently of any previous work, by Dr. A. J. THOMPSON, who has edited the Part-volume.

For values of x greater than 1 the functions were computed under Comrie's supervision. For the smaller values of n they were built up mechanically from the constant nth difference, while for larger values of n values obtained by use of the recurrence formula were checked by differencing. A short introduction by Dr. Comrie gives further details, and there is a page (A5) of formulae.

The integral

$$W(m) = \int_0^\infty \cos \tfrac{1}{2}\pi(t^3 - mt)dt \tag{1}$$

was introduced by AIRY[3] in 1838, and he gave values for $m = -4(.2) + 4$, later extending the table[4] to $|m| = 5.6$. The function $\mathrm{Ai}(x)$ tabulated in Part-volume B is related to the above by

$$\mathrm{Ai}(x) = \tfrac{1}{3}\lambda W(-\lambda x) = \frac{1}{\pi}\int_0^\infty \cos(\tfrac{1}{3}t^3 + xt)dt \tag{2}$$

where $\lambda = (12/\pi^2)^{\frac{1}{3}}$.

It can, however, readily be shown that $y = \mathrm{Ai}(x)$ is a solution of the differential equation

$$y'' = xy, \tag{3}$$

and it is more satisfactory to regard the functions $\mathrm{Ai}(x)$ and $\mathrm{Bi}(x)$ tabulated in the Part-volume as a convenient pair of linearly independent solutions of this differential equation. This equation is an approximation to *any* second order linear differential equation over a limited range not including a singularity. For if the equation be reduced to the canonical form

$$y'' + I(x)y = 0, \tag{4}$$

then near any ordinary point X we have

$$I(x) = I(X) + (x - X)I'(X) + \cdots$$

and inserting this in (4), we recover (3) after a linear change of independent variable. It was this fact that led Dr. HAROLD JEFFREYS to suggest the tabulation. The work has been carried out under the supervision of (and a very large proportion actually done by) Dr.

MILLER. Miller contributes also a scholarly, but very readable, introduction, dealing with the history of the function, description of the tables, their computation and checking, methods of interpolation, and the definitions and basic theory of the functions tabulated.

The general solution of (3) will contain two functions, of which Ai(x) is taken as one, and the other chosen, Bi(x), is defined by a contour integral. Power series and asymptotic expansions for these functions are given. For large real positive values of x, Ai(x) tends to zero and Bi(x) to infinity. For negative real values of x the functions oscillate. It is therefore convenient to write

$$\mathrm{Ai}(x) = F(x)\sin\chi(x), \qquad \mathrm{Bi}(x) = F(x)\cos\chi(x),$$
$$\mathrm{Ai}'(x) = G(x)\sin\psi(x), \qquad \mathrm{Bi}'(x) = G(x)\cos\psi(x).$$

The auxiliary functions F, G, χ, and ψ are slowly varying functions of x, when x is negative and not too small, and are very convenient for tabulation.

The actual tables are:

T. I, Ai(x) and Ai′(x), $x = 0(.01)2$, Ai($-x$) and Ai′($-x$) for $x = 0(.01)20$, 8D, with δ^2 or δ^2_m, p. B18–B39;

T. II, log Ai(x), 8D and Ai′(x)/Ai(x), 7D, for $x = 0(.1)25(1)75$, with δ^2 or δ^2_m, p. B40–B42;

T. III, Zeros and turning-values of Ai(x) and Ai′(x), 8D, the first fifty of each, p. B43;

T. IV, Bi(x) and Reduced Derivatives, $x = 0(.1)2.5$, Bi($-x$) and Reduced Derivatives, for $x = 0(.1)10$, 8 to 10D, p. B44–B46;

T. V, Zeros and turning-values of Bi(x) and Bi′(x), the first 20 of each, 8D, p. B44;

T. VI, log Bi(x), 8D and Bi′(x)/Bi(x), 7D, for $x = 0(.1)10$, p. B47;

T. VII, Auxiliary Functions, $F(x)$, 7D, and $\chi(x)$, in degrees, 6D, $G(x)$, 7D, and $\psi(x)$, in degrees, 6D, for $x = 0(.1)2.5$, with δ^2_m and γ^4, p. B49;
$F(-x)$ and $G(-x)$, 8D, $\chi(-x)$ and $\psi(-x)$, in degrees, 6D, for $x = 0(.1)30(1)80$, with δ^2, p. B50–B56.

Provision is everywhere made for interpolation to the full accuracy of the tables. Usually this is done by giving second differences (δ^2) or modified second differences (δ^2_m). In some ranges, where the latter are inadequate, we are given γ^4, whose leading term is $\delta^4/1000$, and which contains contributions from higher differences, and formulae whereby the necessary correction can be applied with the aid of coefficients supplied on Auxiliary Table I (see below). With Bi(x), the tabular interval is too large for interpolation by differences to be convenient. Here we are given reduced derivatives, and Auxiliary Table II facilitates their use.

The introduction contains a short bibliography, and a graph of the functions Ai(x) and Bi(x), their derivatives, and related functions. Formulae are collected on p. B17 and B48.

Dr. Miller is to be thanked for his labours, and congratulated upon their result and the manner of its presentation.

The modified Everett formula is, when $|\gamma_1^4 - \gamma_0^4| > 1$,

$$f(x+\theta h) = \phi f_0 + \theta f_1 + E_0^2\delta^2_{m0} + E_1^2\delta^2_{m1} + M_0^4\gamma_0^4 + M_1^4\gamma_1^4;$$

when $|\gamma_1^4 - \gamma_0^4| \leqslant 1$,

$$f(x+\theta h) = \phi f_0 + \theta f_1 + E_0^2\delta^2_{m0} + E_1^2\delta^2_{m1} + T^4(\gamma_0^4 + \gamma_1^4)$$

with $\phi = 1 - \theta$, and

$$\delta^2_m = \delta^2 - 0.184\delta^4 + 0.038082\delta^6 - 0.00830\delta^8 + 0.0019\delta^{10} - \cdots$$
$$1000\gamma^4 = \delta^4 - 0.27827\delta^6 + 0.0685\delta^8 - 0.0164\delta^{10} + \cdots$$

Auxiliary Table I gives, on one side of a card, for $\theta = 0(.01)1$, E_0^2 and E_1^2 with second lifferences, 7D, and M_0^4, M_1^4 and T^4, 3D, and ϕ, along with the formulae.

Defining

$$\tau^n = \tau^n f(x) = h^n f^{(n)}(x)/n!,$$

where h is the tabular interval, Taylor's series may be written

$$f(x+\theta h) = f(x) + \theta\tau + \theta^2\tau^2 + \theta^3\tau^3 + \theta^4\tau^4 + \cdots$$

and

$$hf'(x + \theta h) = \tau + 2\theta\tau^2 + 3\theta^2\tau^3 + 4\theta^3\tau^4 + \cdots$$

(on early printings, the *h* of the second formula was unfortunately omitted).

Auxiliary Table II is printed on both sides of the card. One side gives, for $\theta = 0(.01)1$, θ^2, θ^3, θ^4, exact; θ^5, 7D; θ^6, 6D; and θ^7, 5D with one D less for $x \geqslant .5$. The other gives 2θ, $3\theta^2$, $4\theta^3$, exact; $5\theta^4$, 6D; $6\theta^5$, 5D; $7\theta^6$, 4D; and $8\theta^7$, 3D with one D less for $x \geqslant .5$.

The Auxiliary Tables can be purchased separately, and are likely to be found very useful and convenient aids in interpolation. It is to Dr. Miller that the lion's share of credit for their inception and design must go.

W. G. Bickley

Imperial College,
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[1] A. H. H. Tallqvist, Finska Vetenskaps-Societeten, *Acta*, v. 32, no. 6, 1904.
[2] K. Hayashi, *Tafeln der Besselschen, Theta-, Kugel-, und anderer Funktionen*, Berlin, 1930.
[3] G. B. Airy, Camb. Phil. Soc., *Trans.*, v. 6, 1838, p. 379–402.
[4] G. B. Airy, Camb. Phil. Soc., *Trans.*, v. 8, 1849, p. 595–599.

414[K].—Émile F. É. J. Borel & André Chéron, *Théorie Mathématique du Bridge à la Portée de Tous; 134 Tableaux de Probabilités avec leurs Modes d'Emploi. Formules Simples. Applications. Environ 4000 Probabilités. Monographies des Probabilités*, fasc. 5. Paris, Gauthier-Villars, 1940. xx, 392 p. 16 × 25 cm. 175 francs.

The title page description shows the impracticability of attempting to list all the tables in this exhaustive work. These tables fall into 3 main groups: 1, à priori; 2, bidding; 3, play. In the first group the à priori probabilities of all distributions of the 4 suits among the 4 players, of the suits between the 2 partnerships, of the distributions of aces, aces and kings, etc., and the probabilities of voids, singletons, etc., are given. In the second group similar tables are given except now we know the 13 cards in the bidder's hand. The third group of tables mainly covers the probabilities after the dummy has been exposed and we know 26 cards, plus derived tables to cover the cases where part of a suit has been played and we are interested in the distribution of the remainder.

The basic probabilities were calculated exactly as fractions, with the aid of a Pascal triangle complete up to $\binom{52}{26}$ which the authors possess, and are given to varying numbers of decimal places, generally about 6 or 7, but up to 15 places in some cases. Formulae for making the calculations are given in the text, and the use of the Tables in evaluating a hand or selecting the best method of play is illustrated.

The authors point out in the first chapter that ordinary shuffling is quite apt to give results that are far from the random ordering which is assumed in their later calculations.

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415[K].—H. Labrouste & Mme Y. Labrouste, *Analyse des Graphiques résultant de la Superposition de Sinusoïdes.* **a.** *Tables Numériques précedées d'un Exposé de la Méthode d'Analyse par Combinaisons Linéaires d'Ordonnées.* vi, 205 p. 21.9 × 31.6 cm. **b.** *Atlas de Courbes de Sélectivité. Supplément aux Tables Numériques*, 35 plates. 21 × 30.8 cm. Paris, Presses Universitaires de France, 1943. 350 francs.

The problem discussed by the authors is that of harmonic analysis. Suppose that a function $y = y(x)$ is a sum of a finite, though unknown, number of simple oscillations.

How may one find the periods, the phases, and the amplitudes of the terms? The authors' approach to the problems is as follows: Suppose that

$$(1) \qquad y = a \sin(\theta x + \phi) + a' \sin(\theta' x + \phi') + a'' \sin(\theta'' x + \phi'') + \cdots;$$

if such a representation is possible, it is certainly unique. Let us consider $2m + 1$ equidistant points $x_0 - m, x_0 - (m-1), \cdots, x_0, \cdots, x_0 + m$ symmetric with respect to the point x_0, and let $y_{-m}, y_{-(m-1)}, \cdots, y_0, \cdots, y_m$ be the corresponding ordinates of the curve. We write

$$\theta = 2\pi/n, \qquad \theta' = 2\pi/n', \cdots, \qquad \alpha_m = 2\cos(2\pi m/n), \qquad \alpha'_m = 2\cos(2\pi m/n'), \cdots$$

If we set $Y_\mu = y_\mu + y_{-\mu}$, we get

$$Y_\mu = \alpha_\mu a \sin(\theta x_0 + \phi) + \alpha'_\mu a' \sin(\theta' x_0 + \phi') + \cdots,$$

so that any linear combination

$$R_m = K_0 y_0 + K_1 Y_1 + \cdots + K_m Y_m$$

of the quantities $y_0, Y_1, \cdots, Y_m$ can be written in the form

$$R_m = \rho_m a \sin(\theta x_0 + \phi) + \rho'_m a' \sin(\theta' x_0 + \phi') + \cdots$$

where

$$\rho_m = K_0 + K_1\alpha_1 + \cdots + K_m\alpha_m, \qquad \rho'_m = K_0 + K_1\alpha'_1 + \cdots + K_m\alpha'_m, \cdots.$$

It follows that (with x_0 replaced by x) the phases and the periods of the terms composing R_m are the same as the phases and the periods of the terms of y. If the K's are so chosen that all the numbers ρ except one—say except ρ_m—are very small, the graph of R_m/ρ_m gives the first term on the right of (1). To each period $\theta_0 = 2\pi/n_0$ corresponds a "selective" combination R_m. The numbers K_m are the Fourier coefficients of a function large in the neighborhood of the point θ_0 and small elsewhere. The simplest combinations R_m are $s_m = y_{-m} + \cdots + y_0 + \cdots + y_{m-1} + y_m$. The corresponding multipliers of the amplitudes are then

$$\sigma_m = 1 + \alpha_1 + \alpha_2 + \cdots + \alpha_m = \sin(2m+1)\frac{\pi}{n} \Big/ \sin\frac{\pi}{n}.$$

Tables I and II, p. 91–142, of **a** give the values, to 3D, of α_m and σ_m respectively for $m = 0(1)20$, $n = .5(.01)3(.02)5(.05)10(.1)15(.2)25(.5)50(1)100(5)200(10)500$; $m = 21(1)40$, $n = 4(.02)5$[then as above]500; $m = 41(1)50$, $n = 50(1)100(5)200(10)500$. Other tables give values of multipliers corresponding to combinations whose basic element is not $y_\mu + y_{-\mu}$ but $y_\mu - y_{-\mu}$.

In **b** the authors investigate in great detail products (superpositions) of simple combinations and give graphs of the ratio ρ/ρ max as functions of period n. Here ρ is the amplitude multiplier and ρ max is its maximum.

The title-page states that H. Labrouste is a professor in the Faculty of Sciences of the University of Paris and that Mme. Y. Labrouste is an associate physicist at the Institut de Physique du Globe, of the University of Paris.

A. Zygmund

University of Pennsylvania

416[K].—L. W. Pollak, assisted by C. Heilfron, *Harmonic Analysis and Synthesis Schedules for Three to One Hundred Equidistant Values of Empiric Functions.* (Department of Industry and Commerce, Meteorological Service, *Geophysical Publications* v. 1.) Dublin, Stationery Office, 1947. xxxiii, 118 p. 24.5 × 30.2 cm. £2 s2.

This work is designed to facilitate the work of fitting the Fourier series,

$$y = p_0 + p_1 \cos x + p_2 \cos 2x + \cdots + q_1 \sin x + q_2 \sin 2x + \cdots,$$

to a set of n equidistant values.

The formulae devised for this purpose by F. W. Bessel more than a century ago require the values of the functions $\cos(2\pi m/n)$, $\sin(2\pi m/n)$, for all values of $m \leqq n$. In spite of the age of Bessel's formulae, only brief and inadequate tables have been available until recently.

The present work is divided into four parts, an "Introduction," and three tables called respectively "Schedules for Harmonic Analysis and Synthesis," the "Register" and the "Index."

The first table, namely the Schedules, provides 5 decimal values of the two functions given above for values of n from 3 to 100 and for values of $m \leqq n$. With respect to his choice of 5D the author says that the functions "are given to five decimal places in spite of the well-known fact that three decimal places are sufficient for most geophysical investigations. All the values were computed to seven decimal places and I thought it desirable to retain five for those who require greater accuracy, especially as the presence of the additional figures is no handicap to the normal user of the Schedules." The author of this review applauds this decision. He has had occasion to prepare a similar set of tables to $n = 75$ wherein 8 decimal places are retained. The needs of the astronomer, for example, require much greater accuracy than the needs of the geophysicist, the meteorologist, and the economist who are frequent users of these methods.

The Schedules occupy 56 pages of the work, but from the repetitive character of the values tabulated the tables could have been given in half the space. The present generous use of space was adopted, however, because after each value there is given an "identification number" which permits the determination of the corresponding angles by reference to the Index.

The Register provides us, in order of size, the values of the angles $2\pi m/n$, which are given in degrees, minutes and seconds to hundredths of a second, and in radians to 6D. Identification numbers are also provided which by reference to the Index permit the determination of the corresponding values of the sine and cosine.

The Index gives the values of the angles $2\pi m/n$ in terms of the identification numbers. These values are identical with those printed in the Register, but are arranged differently. The corresponding value of the sine is recorded to 5D for each angle together with the value of m and n which determines the angle.

In addition to these principal tables the work gives 12 auxiliary tables used in connection with illustrative examples. A bibliography is appended which contains 17 references. This is quite inadequate and omits reference to such modern works as those of A. HUSSMANN (1938), P. TEREBESI (1930), and K. STUMPFF (1939). See, for example, *MTAC*, v. 1, p. 193, and v. 2, p. 32.

The sines and cosines were computed twice, once by the author and again by C. HEILFRON, the latter using E. GIFFORD, *Tables of the Natural Sines* (see *MTAC*, v. 1, p. 24f). Also about ten per cent. of the values selected at random were checked by F. E. DIXON using Chambers' *Mathematical Tables* (edited by J. PRYDE).

The book is well printed and should be very easy to use in the applications for which it is designed.

H. T. D.

417[K, M].—NIELS ARLEY, *On the Distribution of Relative Errors from a Normal Population of Errors*. Danske Videnskab. Selskab, *Math.-fysiske Meddelelser*, v. 18, no. 3, 1940, 62 p. 14.8 × 24.2 cm.

On p. 61 is a table of r in the formula

$$P(r) = 2\int_r^{(f+1)^{\frac{1}{2}}} \pi^{-\frac{1}{2}}(f+1)^{-\frac{1}{2}}[\tfrac{1}{2}(f-1)]!\left(1-\frac{r^2}{f+1}\right)^{\frac{1}{2}(f-2)} dr/[\tfrac{1}{2}(f-2)]!$$

for $f = 1(1)30(5)50(10)100, 120, \infty$, $P = [.001, .01, .02, .05, .1(.1).9; 3D]$. There is also a table of $(f+1)^{\frac{1}{2}}$ to 4D.

418[L].—HARVARD COMPUTATION LABORATORY, *Six-Place Tables of (a) $J_m(x)$, (b) $Y_m(x)$, (c) $J_m(x) - J_{m-2}(x)$, (d) $Y_m(x) - Y_{m-2}(x)$, (e) $(1/i^m)H_m^{(1)}(x)$, (f) $(1/i^{m-1})dH_m^{(1)}(x)/dx$. Publication no. 21*, July 1945. v, 11 leaves. 21.5 × 27.8 cm. Out of print.

These tables are only for $x = 24, 28, 32, 36, 40$ (a), (b), $m = 0(1)x$; (c), (d), $m = 0(1)x + 1$; and (e), (f), real and imaginary parts, $m = 0(1)x$.

419[L].—NBSMTP, *Tables of Spherical Bessel Functions*, volume 1, New York, Columbia University Press, 1947. xxviii, 375 p. 20 × 26.4 cm. $7.50.

Considering that of the eleven coordinate systems for which the wave equation is separable, the solutions in six involve Bessel functions, and that in four the orders are half odd integers, it is—as Professor PHILIP M. MORSE remarks in the foreword to this volume—surprising that extensive previous tables of these functions do not exist (see FMR, *Index*, §17.21, p. 248, or *MTAC* v. 1, p. 234). The surprise is not lessened by the fact that one of these coordinate systems is the obviously important spherical polar, nor by the fact that the functions are often linked with the name of Stokes.

The combination to which the analysis of the wave equation (and others) gives rise is $x^{-\frac{1}{2}}J_{n+\frac{1}{2}}(x)$ rather than the function $J_{n+\frac{1}{2}}(x)$ itself, and it is a numerical multiple of the former function which is tabulated in the main table in this volume, namely $(\pi/2x)^{\frac{1}{2}}J_{\pm(n+\frac{1}{2})}(x)$.

When we note that

$$(\pi/2x)^{\frac{1}{2}}J_{\frac{1}{2}}(x) = \sin x/x,$$

$$(\pi/2x)^{\frac{1}{2}}J_{-\frac{1}{2}}(x) = \cos x/x,$$

and that the functions of higher order are expressible in the form

$$P_n \cos x + Q_n \sin x,$$

where P_n and Q_n are polynomials in $1/x$ (i.e., that the 'asymptotic' expansions terminate in this case) *some* reason—but still hardly adequate—can be discerned for the delay in tabulating this class of function.

The main table in this volume (p. 2–323) is one of

$$(\pi/2x)^{\frac{1}{2}}J_{\pm(n+\frac{1}{2})}(x)$$

for $n = 0(1)13$, and $x = 0(.01)10(.1)25$, expanded for $n = 13$ in the range $x = 10(.05)10.5$. The standard of accuracy may be loosely described as 8S (at least) for $x < 10$ and 7S for $x > 10$. Actually, for smaller values of x and for the smaller positive orders, there are large ranges where as many as 10S are given. For values of x where the functions have begun to oscillate, it would be more accurate to say that enough *decimals* are given for the above standards to apply to the *maxima*. Functions of equal and opposite order are tabulated together, each pair occupying 23 pages.

Interpolation is provided for by second (or modified second) differences, where these are adequate, and a table (p. 370–375) of the Everett coefficients is given. But for the smaller values of x, and especially for the higher orders, the functions vary so rapidly that (even modified) second differences become inadequate, and, in fact, interpolation by differences becomes not feasible. Hence there are many empty columns headed δ^2 (sometimes when the companion function has δ^2_m provided). The need for interpolation in these ranges is met by the provision of auxiliary tables (p. 326–369) giving $(\pi/2)^{\frac{1}{2}}\cdot x^{-\nu}J_\nu(x)$, with δ^2. (In this part of the table we encounter many columns headed δ^4, entirely blank!)

These tables should prove of very great value. They will enable the solutions of many physical and engineering problems to be explored in considerable numerical detail, where previously the labour would have been quite impracticable for the physicists or engineers concerned. The account of the methods of computation and checking, coupled with the reputation for accuracy which the NBSMTP has earned, enables one to use these tables with complete confidence.

In view of the undoubted value of the tables, and of the amount of useful material they contain, it may seem ungenerous to find fault. But, in the end, nothing but the best is good enough! In some respects the design and arrangement of these tables falls short of the standards which the NBSMTP has itself set, and maintained in its earlier volumes. The number of empty *headed* columns has already been mentioned. Another point is inconsistency in breaking up the entries into groups by spaces. In most of the tables the now usual (and for *many* reasons optimum) groups of five, with breaks every five places from the decimal point, are used, but in places groups of six or more are given. The arrangement of p. 321 is not happy, and there seems no obvious reason why additional horizontal spaces could not have been provided on this (exceptional) page. Attention to such detail would have required an infinitesimal proportional increase in the time and labour which has been expended on this volume.

W. G. BICKLEY

EDITORIAL NOTE: The second volume of NBSMTP, *Spherical Bessel Functions* is to contain the following:

(a) Tables of $(\frac{1}{2}\pi/x)^{\frac{1}{2}}J_\nu(x)$, for $\pm 2\nu = 29(2)43$, $x = 0(.01)10(.1)25$, and $\pm 2\nu = 45(2)61$, for $x = 10(.1)25$.
(b) Tables of $\Lambda_\nu(x) = 2^\nu\Gamma(\nu + 1)J_\nu(x)/x^\nu$, for $x = 0(.1)10$, $2\nu = [1(1)41(2)61; 8\text{–}9S]$, and $x = 10(.1)25$, $2\nu = [1(2)61; 7S \text{ mostly}]$. Also for negative values of ν, in regions where $(\frac{1}{2}\pi/x)^{\frac{1}{2}}J_\nu(x)$ does not difference well.
(c) Table of roots of $J_\nu(x)$ and $J_\nu'(x)$ over the region covered by the main tables.
(d) Other auxiliary tables for purposes of interpolation.

The publication of this volume is expected by September 1947.

420[L].—P. M. WOODWARD & Mrs. A. M. WOODWARD, with the assistance of Miss R. HENSMAN, H. DAVIES, & Miss N. GAMBLE, "Four-figure tables of the Airy function in the complex plane," *Phil. Mag.*, s. 7, v. 37, Apr. 1946, publ. Jan. 1947, p. 236–261. 17 × 25.3 cm. See RMT **260**, *MTAC*, v. 2, p. 35.

"Prefatory Remarks: The immediate need for tables of the Airy function in the complex plane has arisen in connection with theoretical work on the propagation of electromagnetic waves through the earth's atmosphere, and it was for this particular purpose that the present tables were computed and produced as a report at the Telecommunications Research Establishment in February, 1945. It is not claimed that the tables are comprehensive, but they do provide material which, so far as is known at present, is not available elsewhere. The introductory matter, moreover, should be of interest independently of the tables, as it provides general suggestions with regard to interpolation in functions of a complex variable. Much of the labour associated with ordinary second and fourth difference bivariate interpolation may be avoided if suitable use be made of the Cauchy-Riemann equations and if the differences tabulated be modified accordingly."

The functions Ai(z) and Bi(z), $z = x + iy$, are independent solutions of $d^2w/dz^2 = zw$, such that $\mathrm{Ai}(z)\mathrm{Bi}'(z) - \mathrm{Ai}'(z)\mathrm{Bi}(z) = 1/\pi$. The power series

$$w_1 = 1 + \frac{1}{3!}z^3 + \frac{1\cdot 4}{6!}z^6 + \frac{1\cdot 4\cdot 7}{9!}z^9 + \cdots,$$

$$w_2 = z + \frac{2}{4!}z^4 + \frac{2\cdot 5}{7!}z^7 + \frac{2\cdot 5\cdot 8}{10!}z^{10} + \cdots$$

were used for calculation of the functions by means of the relations $\mathrm{Ai}(z) = pw_1 - qw_2$, $\mathrm{Bi}(z) = 3^{\frac{1}{2}}(pw_1 + qw_2)$, where $1/p = (-1/3)!\,3^{\frac{2}{3}}$, $1/q = (-2/3)!\,3^{\frac{1}{3}}$.

There are 8 tables containing the real and imaginary parts of I–IV: Ai(z), Bi(z); V–VIII: Ai′(z), Bi′(z). These cover the region $x = -2.4(.2) + 2.4$, $-y = [0(.2)2.4; 4D]$ in the lower half of the complex plane. The upper half is covered by taking the complex conjugates from the tables. Beneath each tabular entry are given $\frac{1}{2}(\Delta_x^2 - \Delta_y^2)$ and $\Delta_x^2\Delta_y^2$. It is hoped that, apart from any typographical errors which may have passed undetected, the errors in Tables I–IV never exceed 0.6 or 0.7 and those of Tables V–VIII never exceed 0.8 in units of the fourth decimal.

Postscript: Since the preparation of these tables, there have been published ["Annals of the Computation Laboratory of Harvard University," v. 2, ⋯ 1945] eight-figure tables of the modified Hankel functions of order one-third[1]—functions which are closely related to the Airy functions. The tables cover the region $|x + iy| \leqslant 6$, at interval 0.1 in x and y.

Extracts from the text

[1] See *MTAC*, v. 2, p. 176f.—EDITORIAL NOTE.

421[L, M].—J. P. KINZER & I. G. WILSON, "End plate and side wall currents in circular cylinder cavity resonator," *Bell System Technical Jn.*, v. 26, Jan. 1947, "Appendix, integration of $\int_0^x J_n(x)dx/J_n'(x)$," p. 70–79; tables calculated by Miss F. C. LARKEY. 15 × 22.8 cm.

On p. 73–75 are 4D tables of $F_n(x) = \int_0^x J_n(t)dt/J_n'(t)$, and of $G_n(x) = e^{-F_n}$, for $x = 0(.1)9.9$, $n = 1(1)3$. Then on p. 76–79 are 4D tables of $J_n(x)$ for $x = 0(.1)9.9$, $n = 0(1)7$, and of $J_n'(x)$, for $x = 0(.1)9.9$, $n = 1(1)6$.

422[L, P].—V. I. KOVALENKOV, "Obshchee reshenie uravneniiâ Gel'mgol'tsa s uchetom vliiâniiâ zheleza" [A general solution of the Helmholtz equation taking into account the effect of iron], Akad. N., Moscow, *Otdiel Tekhnicheskikh Nauk, Abtomatika i Telemekhanika*, Organ komiteta Telemekhaniki i Abtomatiki, no. 2, 1939. 16.5 × 25.1 cm.

On p. 9 there is a table of values of

$$\frac{y}{1!} + \frac{y^2}{2(2!)} + \frac{y^3}{3(3!)} + \cdots, \tag{1}$$

for $y = 0(.01)5$, 4D through 3.25, 3D or 5S thereafter.

On p. 22 is a table of values of

$$\frac{y}{1!} - \frac{y^2}{2(2!)} + \frac{y^3}{3(3!)} - \frac{y^4}{4(4!)} + \cdots \tag{2}$$

for the same range, 4D through 1.34, 3D or 4S thereafter.

423[L, S].—GUY LANSRAUX, "Calcul des figures de diffraction des pupilles de révolution," *Revue d'Optique*, v. 26, Jan.–Feb. 1947, p. 24–45. 15.6 × 23.9 cm.

On p. 43–45 is a table of $\Lambda_n(x) = L_n(x) = n!\,(\frac{1}{2}x)^{-n}J_n(x)$, for $n = 1(1)30$, and $x = [0(.5)5(1)15; 6D]$.

If $G(x) = e^{-i\theta}[L_1(x) + \frac{1}{2}i\theta L_2(x) + \cdots (1/n!)(i\theta)^{n-1}L_n(x) + \cdots]$,
$G_1(x) = \cos\theta[L_1(x) - (\theta^2/6)L_3(x) + \cdots] + \sin\theta[\frac{1}{2}\theta L_2(x) - (\theta^3/24)L_4(x) + \cdots]$,
$G_2(x) = \cos\theta[\frac{1}{2}\theta L_2(x) - (\theta^3/24)L_4(x) + \cdots] - \sin\theta[L_1(x) - (\theta^2/6)L_3(x) + \cdots]$,
and $I(x) = G_1^2(x) + G_2^2(x)$. On p. 32 are tables $G_1(x)$, $G_2(x)$ to 5D, and of $I(x)$ to 4D, for $\theta = \pm\pi, \pm 2\pi$, $x = 0(.5)5(1)15$; there are also graphs of the six functions on p. 32–33.

There are also tables on p. 37 of:

$G(x) = e^{-k}[L_1(x) + \frac{1}{2}kL_2(x) + \cdots + (k^{n-1}/n!)L_n(x) + \cdots]$, for $k = 0$, to 5D, and for $k = 1, 2, 4$, to 4D; on p. 36 are graphs of the four functions.

On p. 40 are tables of:

$G_1(x) = \frac{1}{2}(\frac{1}{2}\pi)L_2(x) - \frac{1}{7\cdot 3!}(\frac{1}{2}\pi)^3L_7(x) + \cdots$,

$G_2(x) = -L_1(x) + \frac{1}{5\cdot 2!}(\frac{1}{2}\pi)^2L_5(x) + \cdots$, and $I(x) = G_1^2(x) + G_2^2(x)$, for $x = 0(.5)5(1)15$, the first two to 5D and the third to 4D. On p. 41 are graphs of the functions.

In JAHNKE & EMDE, *Tables of Functions*, 1945, p. 180–188, there are tables of $L_n(x)$, for $n = 0(1)8$, $x = [0(.02)9.98; 4–5D]$. But the NBSMTP tables of $L_n(x)$, see *MTAC*, v. 1, p. 363f, are for $n = 0(1)20$, $x = [0(.1)25; 10D]$; $n = 0(1)12$, $x = [0(.01)10; 8D]$. Thus the Lansraux tables of $L_n(x)$ are new only for $n = 21(1)30$, $x = [0(.5)5(1)15; 6D]$.

R. C. A.

424[M].—NIELS ARLEY, *On the Theory of Stochastic Processes and their Application to the Theory of Cosmic Radiation.* Diss. Copenhagen G.E.C. Gads Forlag, 1943. 240 p. 17.3 × 25 cm.

Chapter 8, p. 222–227 is entitled, "On the numerical computation of $\psi(x) = \int_0^x e^t dx$," $t = x^2$, that is, Dawson's or Poisson's integral, tables of which we have listed, *MTAC*, v. 1, p. 322–323, 422–423, v. 2, p. 55, 185. Arley's only references are to JAHNKE & EMDE, and DAWSON. T. 24, p. 224–225, gives $\psi(x)$, for $x = [2(.01)10;\ 4S]$. T. 23, p. 223, is of $f(x) = 2xe^{-t}\psi(x) = 1 + \frac{1}{2x^2} + \frac{1\cdot 3}{(2x^2)^2} + \frac{1\cdot 3\cdot 5}{(2x^2)^3} + \cdots$, $x \gg 1$, for $x = [2(.01)4(.1)10;\ 4\text{–}5S]$, $t = x^2$. Of this table it is stated that "We estimate that the figures are correct within 1 or 2 units in the last figure." T. 24 was calculated from T. 23 by means of the formula $\psi(x) = (2x)^{-1}e^t f(x)$, $t = x^2$.

R. C. A.

MATHEMATICAL TABLES—ERRATA

References have been made to Errata in RMT 412 (Loria); N **74** (FMR).

105.—G. F. BECKER & C. E. VAN ORSTRAND, *Hyperbolic Functions* (*Smithsonian Mathematical Tables*), Washington, 1909—fourth reprint 1931.

In this and previous editions the formulae 83, 84 on p. XV are incorrect. They should read

$$83.\quad \tanh^{-1}\tan u = \tfrac{1}{2}gd^{-1}2u$$
$$84.\quad \tan^{-1}\tanh u = \tfrac{1}{2}gd2u$$

They are given correctly in the fifth reprint 1942. The same error occurs in A. E. KENNELLY, *Tables of Complex Hyperbolic and Circular Functions*, second ed., Cambridge, Mass., 1921, p. (230), and in various textbooks.

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106. W. W. DUFFIELD, *Logarithms, their Nature, Computation and Uses, with Logarithmic Tables of Numbers and Circular Functions to Ten Places of Decimals*. Washington, 1897. See *MTAC*, v. 2, p. 161–165.

A. The fact that Duffield prepared this table by copying from VEGA *Thesaurus*, 1794, is well known; see the references in *MTAC* (above), and also L. J. COMRIE, Br. Astron. Assoc., *Jn.*, v. 36, 1926, p. 341.

DUFFIELD's copying from Vega included nearly 300 end-figure errors, but his work is held in such low esteem that no one appears to have checked anything but the end figures. Recently an opportunity of examining the first six presented itself when reading the proofs of a new 6-figure table now in press for Messrs. W. and R. Chambers. The result of a single comparison was as follows.

Page	Number	
495	32067	the figures 058 3318 should be overlined
500	33411	*for* 8994744, *read* 8894744
526	41764	*for* 8920871, *read* 8020871
530	42680	*for* 2224108, *read* 2244108
580	57482	*for* 3318704, *read* 5318704
686	89330	the overline on 9973340 should be deleted
710	9680.	in number, *for* 7180, *read* 9680
710	96826	the overline on 9919910 should be deleted
716	98771	*for* 5294508, *read* 6294508.