Admiralty Computing Service[1]

In 1942, in order to use more efficiently the scientific staff available in the Admiralty, the Director of Scientific Research set up, within the branch directed by Dr. J. A. CARROLL, an Admiralty Computing Service to centralise, where possible, the computational and mathematical work arising in Admiralty Experimental Establishments.

Mr. JOHN TODD undertook the organisation and supervision of the Service. By agreement with the Astronomer Royal additional staff were attached to H. M. Nautical Almanac Office to carry out the computational work under the direction of the Superintendent, Mr. D. H. SADLER. In addition, arrangements were made to permit the employment of experts from the Universities and elsewhere as consultants.

The work undertaken by Admiralty Computing Service was in general of one of two classes: heavy computation, or difficult mathematics. Altogether more than one hundred separate investigations were carried out ranging from projects involving several thousand hours' computing to small problems for which a solution could be obtained in a few hours. In addition, a considerable amount of advisory work has been undertaken, usually informally. For instance the *Five-figure Logarithm Tables*, reviewed in RMT **188**, were designed by Admiralty Computing Service for the Ministry of Supply, as one item in a comprehensive program for providing the optical industry with the tables they required. For various reasons, mainly owing to the increased use being made of machines and to the availability of the U. S. reprint of PETERS' seven-figure table of natural trigonometric functions, other tables were never published; but see under UMT **57**.

Shortly after the formation of Admiralty Computing Service it became apparent that research work in Admiralty (and other) Establishments would be greatly facilitated if their members were informed in certain mathematical and computational techniques not usually covered in undergraduate courses, and of which no adequate account was available in easily accessible literature. Accordingly the preparation of a series of monographs of an expository nature was begun [see ACS **53**, **68**, **71**, **101**, **102**, **106** (revised edition of ACS **26**), **107**].

It was soon realised that while centralisation within a Department was an improvement, nothing less than centralisation on a national scale could be really efficient. At the end of 1943, an approach was therefore made to Sir EDWARD V. APPLETON, Secretary of the Department of Scientific and Industrial Research, asking for consideration of the formation of a National Mathematical Laboratory. Discussions, in which the experience gained by Admiralty Computing Service played an important part, have now resulted in the formation of a Mathematics Division of the National Physical Laboratory. Staff have been released from Admiralty Computing Service to form a nucleus for the computational sub-division of the new organisation. It is anticipated that the computational needs of the Admiralty will be met by outside organisations and the mathematical needs by an even larger use of the service of consultants, working under the general direction of the Director of Physical Research, Admiralty.

Among the consultants employed during the war were Dr. N. ARONSZAJN, Professor W. G. BICKLEY, Dr. L. J. COMRIE (Scientific Computing Service, Ltd.), Professor E. T. COPSON, Dr. J. ĆOSSAR, Dr. A. ERDÉLYI, Professor P. P. EWALD, Dr. H. KOBER, Dr. J. MARSHALL, Dr. J. C. P. MILLER, Professor E. H. NEVILLE.

It is perhaps as well to explain that Admiralty Computing Service started on a scale severely limited by difficulties of staff recruitment; its main object in its early stages was to obtain the numerical results required in problems of war research and to make those results available to the particular establishment concerned as early as possible. Publication of reports was then a secondary matter, and, in fact, it has generally been so regarded as far as the purely computational work is concerned. For this reason, it was not until the beginning of 1944 that the numbering of Admiralty Computing Service reports was systematised in the SRE/ACS series. All reports issued prior to that date, either with an NAO (=Nautical Almanac Office) serial number or with an SRE/MA (=Headquarters) reference number, were renumbered in the new series and will be referred to here by those numbers.

All the reports mentioned below were issued by the Admiralty Computing Service of the Department of Scientific Research and Experiment (Admiralty), Great Britain; generally the computational reports were prepared and reproduced by H. M. Nautical Almanac Office, while the mathematical reports were edited and reproduced at Headquarters.

The following 21 SRE/ACS Reports have already been reviewed in *MTAC* under the heading of Recent Mathematical Tables (one under UMT):

ACS	RMT	No.	p.	ACS	RMT	No.	p.
19	**262**	13	36	**65**	**226**	12	**446**
21	**260**	13	35	**68**	**206**	11	**424**
22	**267**	13	39	**71**	**206**	11	**424**
31	**260**	13	35	**82**	**252**	13	**31**
39	**260**	13	35	**90**	**293**	14	**80**
46	**260**	13	35	**91**	[UMT **41**]	13	**52**
47	**334**	16	175	**93**	**277**	14	**70**
52	**268**	13	40	**97**	**333**	16	**174**
53	**206**	11	424	**102**	**288**	14	**76**
55	**260**	13	35	**108**	**352**	17	**215**
62	**266**	13	38				

There follows a summary of 19 other items of work (ACS **7**, **8**, **9**, **18**, **20**, **26**, **37**, **40**, [**47**], **51**, **80**, **89**, **96**, [**97**], **101**, **106**, **107**, **109**, **110**, **111**, **112**) which appear to have a certain permanent value to the computer and mathematician. It is hoped to arrange for the publication in full of some of the reports. Suggestions as to those which appear suitable for this treatment will be appreciated. Details of other work, not conveniently described, apart from its background, will appear elsewhere.

Copies of the reports are only available for distribution to Government Departments and similar agencies but arrangements have been made for copies of some of the reports to be deposited with the Editors where they may be consulted. A very limited number of photostat copies of the unpublished tables is available for distribution or loan to institutions or individuals with a special computational requirement.

7. *Summation of Certain Slowly Convergent Series*. Stencilled typescript, on one side of 4 p.; undated but issued January 1943. 20 × 32.5 cm.

This note draws attention to a device which was apparently first applied in computational work by P. P. EWALD in his work on crystal-structure, *Ann. d. Phys.*, v. 64, 1921, p. 253–287, Gesell. d. Wissen., Göttingen, *math.-phys. Kl.*, n. s. v. 3II, no. 4, 1938, p. 55–64. The analytical basis of the device is the JACOBI Imaginary Transformation of Theta-function Theory (see E. T. WHITTAKER & G. N. WATSON, *A Course in Modern Analysis*, fourth ed., 1927, p. 475); there is a physical basis, too, which consists in replacing (taking the electrostatic analogy) point charges by Gaussian space-distributions. Applied to the series $S = \sum_{n=0}^{\infty} (n + \frac{1}{2})^{-1} e^{-\frac{1}{10}(n+\frac{1}{2})}$ accuracy comparable with that obtained by summation of 50 terms of the original series may be obtained by taking a single term of one of the two infinite series into which S is transformed, and three terms of the other.

8. *Mechanical Quadratures*. Stencilled typescript, 11 p. + 1 diagram; undated but issued December 1942. 20 × 32.5 cm.

Part I is an exposition of the method of (Approximate or) Mechanical Quadratures in which an estimate for $\int_a^b f(x)w(x)dx$ is given as $\Sigma\lambda_n f(x_n)$ where the x_n are the zeros of the orthogonal polynomials associated with the distribution $w(x)$ in the interval (a, b) and the λ_n are certain constants, called the Christoffel numbers.

Brief discussions are given of the cases when $w(x) = 1$, see RMT92,132; $w(x) = e^{-t}$, $t = x^2$, when the polynomials are the Hermite polynomials, see RMT131,250; $w(x) = e^{-x}$ when the polynomials are the Laguerre polynomials, see RMT252; and $w(x) = x^{\alpha}e^{-x}$ when the polynomials are the Sonine polynomials.

Part II applies these methods to a particular case of estimating a probability integral of the form $\int_0^{\infty} f(x)e^{-t}dx$, $t = x^2$.

A third part is in preparation; this will deal with the two-dimensional case and in it an account will be given of some recent Russian work.

The most promising of the methods discussed appears to be the Laguerre case and consequently the definitive table of the λ_n, x_n, referred to above, see RMT252, was prepared.

9. *Table of* $f(x, y) = (2\pi)^{-1} \int_0^{2\pi} e^{-x\cos\theta - y\cos^2\theta} d\theta$. Stencilled typescript, 2 p.; undated but issued April 1943. 20.5 × 33.2 cm.

The function $f(x, y)$ is tabulated for $x, y = [0(.25)5; 3\text{–}4D]$, without differences, with a reservation that "the tabular values are unlikely to be in error by more than one unit in the last figure retained."

The table was computed from the relation

$$e^y f(x, y) = e^{yj}(x)$$

where j is regarded as an operator such that

$$j^r(x) = \frac{(2r)!}{2^r r!\, x^r} I_r(x)$$

where $I_r(x)$ is the Bessel function of purely imaginary argument. The BAASMTC values of $i_r(x) = x^{-r}I_r(x)$ were used in the computation.

18. *Cable Tables*. Stencilled typescript, 10 p.; undated, but issued in August 1943. 33.5 × 40.5 cm. With separate introductory text, mimeographed, 2 p. 20.5 × 33.5 cm.

These tables are a re-issue of earlier tables prepared by H. M. Nautical Almanac Office prior to the formation of Admiralty Computing Service.

The problem is connected with the form of a heavy cable in a uniform stream, but it is thought that the tables are of more general interest and may have other applications.

Defining

$$\ln f(\theta) = \int_0^\infty \frac{\sin U\,dU}{\cos U + \mu \sin^2 U},$$

the quantities tabulated are:

$$y/\bar{y} = \mu\{f(\theta) - 1\}, \qquad s/\bar{y} = \mu \int_0^\infty \frac{f(U)dU}{\cos U + \mu \sin^2 U},$$

and the difference $(s - y)/\bar{y}$, for the ranges: $\theta = 0(1°)90°$; $\mu = .05, .1(.1).5, .4(.2)2(.5)5(1)12$, with 3D in $y/\bar{y}$ and $s/\bar{y}$, but 4D in $(s - y)/\bar{y}$ for $41° \leq \theta \leq 90°$.

No differences have been provided and interpolation is not always linear, though $(s - y)/\bar{y}$ behaves smoothly even for small μ and θ near 90°, where both $y/\bar{y}$ and $s/\bar{y}$ are not easily interpolable. It is not expected that the last figure will be in error by more than one unit, though no great effort was made to ensure end-figure accuracy.

20. *Trajectories of a Body Moving with Resistance Proportional to the Square of the Velocity.* Stencilled typescript; 6 p. + 1 p. diagrams. 20.2 × 33 cm.

Tabulations are connected with projectile trajectories when the motion is under gravity, with a resistance proportional to the square of the velocity.

The functions tabulated are

$$X = \int_{\theta_0}^{\theta} \frac{\sec^2 \psi d\psi}{F(\psi) - F(\alpha)}, \qquad Y = \int_{\theta_0}^{\theta} \frac{\tan \psi \sec^2 \psi d\psi}{F(\psi) - F(\alpha)}, \qquad S = \int_{\theta_0}^{\theta} \frac{\sec^3 \psi d\psi}{F(\psi) - F(\alpha)},$$

$$T = \int_{\theta_0}^{\theta} \frac{\sec^2 \psi d\psi}{\sqrt{F(\psi) - F(\alpha)}}, \qquad \text{and} \qquad \dot{S} = \frac{dS}{dT} = \frac{\sec \theta}{\sqrt{[F(\theta) - F(\alpha)]}},$$

where $F(\theta) = \sec \theta \tan \theta + \ln (\sec \theta + \tan \theta) = \int_0^\theta \sec^3 \psi d\psi$.

In tables I(a) to II(b), $\theta_0 = 89°$, and since this makes most of the values of X, Y, S, T negative, the quantities actually tabulated are $-X$, $-Y$, $-S$, $-T$, $\dot{S}$. The tabulations are generally to 3D. No attempt at great accuracy has been made in the calculations. In the main tables it can be stated that

(i) the maximum error possible is ten units in the last figure retained: this error, if it occurs at all, will be systematic and will therefore not enter with its full weight.
(ii) errors of more than three units in the last place are unlikely.

Tables have been prepared for $\alpha = -85°(5°)85°$ with θ as independent variable, and for $\alpha = -90°(5°)-80°$ with T as independent variable.

Table I(a) : $\alpha = -10°(5°) + 85°$, $\theta - \alpha$ from 0 to 10°.
Table I(b) : $\alpha = 70°(5°)85°$, $\theta - \alpha$ small, less than 0°.1.
Table II(a): $\alpha = -85°(5°) + 80°$, θ from $-5°$ (or some larger value depending on α) to 89° for $\alpha \leq -50°$ and to 85° for $\alpha \geq -45°$.
Table II(b): α from $-85°$ to 85°, θ from 85° to 90°.
Table III : $\alpha = -90°(5°) - 80°$, argument $T = 0(.1)1(.2)2.8$.
Table IV : an auxiliary table giving $F(\theta)$ for $\theta = [0(0°.1)70°; 4D]$, and $\cos^2 \theta F(\theta)$ for $\theta = [60°(0°.1)90°; 4D]$.

The report contains instructions for the use of the tables and indicates the method of computation.

26, 106. E. T. Copson, *The Asymptotic Expansion of a Function defined by a Definite Integral or Contour Integral.* Mimeographed, 1943, 45 p. **106.** Second revised ed., 1946, 63 p. 20.2 × 33 cm.

This monograph gives an account of the methods used in the asymptotic evaluation of integrals. It includes the method of integration by parts, Laplace's method, Kelvin's principle of stationary phase, the use of Watson's lemma, method of steepest descent and saddle point method. Applications are made to the gamma function, incomplete gamma functions, Bessel functions, scattering of sound waves, Airy integrals, Legendre and Hermite polynomials, and other functions.

In the second edition the section on Airy's integral has been revised in order to bring the notation in line with that used by the British Association Tables, and an application to a problem in probability has been added. The Bibliography (with brief notes on the scope of each item) will be of assistance to those who require further information.

37. J. M. JACKSON, *An Electronic Differential Analyser*. Mimeographed, 1944. 19 foolscap leaves + 4 plates. Reprinted by the Navy Department, Washington, D. C., Office of Research and Inventions, July, **1946**.

The processes required in a differential analyser, e.g. addition (subtraction), multiplication, integration, and differentiation, can all be performed by simple electronic circuits. The chief drawback is that the lower frequencies being differentiated suffer attenuation and phase shift, and that the process of integration can occur only for a limited time. Both these troubles can be appreciably relieved by liberal use of negative feed back.

A differential analyser was built on these lines and, within the limits of its accuracy, about 5%, gave good service. The main advantages, apart from the low initial cost, are the rapidity of operation, and the simplicity of setting up, each unit, whether adder, multiplier, or integrator, being simply plugged into the correct position in the chain. The output of the machine operated a pen and ink recorder. It is felt that a more detailed investigation of the possibilities of the method should be made in cooperation with experienced electronic engineers.

40. *Rangefinder Performance Computer*. Stencilled typescript, July 1944, 3 p. 20.4 × 33 cm.

This device calculates the mean error and root mean square error made by an operator being trained in the use of a mechanism such as a range-finder. It thus reduces greatly the labor both of selecting trainees according to natural ability and of assessing the value of their training.

The true reading, and that obtained from the operator's use of his mechanism, are fed simultaneously into a differential gear which rotates a uniselector. The instantaneous error appears in the uniselector expressed as an integer up to ±24. At instants determined either by a timing device or by the operator himself, this error is transferred to the computing mechanism, which, by uniselectors and relays, adds it to the sum of all previous errors, and by means of a built-in table of squares, adds its square to the sum of the squares of the previous errors. It also counts the number of errors inserted. These operations require up to 4 seconds; the results, including the sign, are shown on an illuminated display panel. The machine can accommodate up to 100 errors, or can be set to stop automatically at any previous number. It can be reset to zero almost instantaneously.

The calculation is completed either by a hand machine or by inserting the numbers from the display panel into a suitable network of inductance potentiometers.

[47]. *Tabulation of the Function* $f(x,y) = \int_0^\infty e^{-k}[J_0(kx)\cosh(ky) - 1]dk/\sinh k$. Photo-offset print of handwriting and machine-printed tables, February 1945. 9 p. + 1 folding diagram. 21 × 34.5 cm. See RMT **334**.

The function $f(x, y)$ is the solution to a two-dimensional potential problem, being that solution in the strip $0 \leq y \leq 1$ of the differential equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial y^2} = 0,$$

which satisfies the conditions $\frac{\partial f}{\partial y} = 0$ on $y = 0$ for $x \neq 0$, $\frac{\partial f}{\partial y} = (1 + x^2)^{-3/2}$ on $y = 1$, and $\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sim O\left(\frac{1}{x^2 + y^2}\right)$, both near the origin and for large values of x.

The function is tabulated for $x = 0(.1)5$; $y = [0(.1)1; 4D]$, with second differences in both the x- and y-directions. There is also a diagram showing the contours in the x, y-plane for $f(x, y) = -1(.1) + .3$, and the values of $f(x, y)$ for $x = 0(.2)5$; $y = 0(.2)1$ obtained by the use of relaxation technique.

Expansion as a series gives:

$$f(x, y) = \sum_{n=1}^{\infty} [\{x^2 + (2n - y)^2\}^{-\frac{1}{2}} + \{x^2 + (2n + y)^2\}^{-\frac{1}{2}} - 1/n]$$

and this was used, in various forms, to obtain values to 6D on the lines $x = 0$, $x = 5$, $y = 0$ and $y = 1$ and for one or two interior points as a check. Computation for the remaining interior points at interval .2 in both x and y was performed by applying the technique of relaxation to solve the simultaneous equations arising from the finite-difference equivalent of the partial differential equation defining the function. As far as it is known this is the first application of the relaxation method specifically including in the equations to be solved the corrections for the effect of higher order differences. Their inclusion allows the use of a larger interval than would otherwise be permissible. The excellence of the agreement of the independent calculations and the method of computation suggests that no value is in error by much more than one unit in the last figure retained.

51. *Table of Angular Quarter Squares.* Photostat, July 1944. 11 p. 20 × 25 cm.

This table gives, for the range 0(3′)100°, the angular equivalent in degrees and minutes of the radian measure of the quarter square of the radian measure of the argument. It was prepared for the Admiralty Compass Observatory for use in analysing compass deviations.

The preparation of the table is trivial and it was actually built up from a constant second difference on a sexagesimal National accounting machine, the machine being arranged to print the final copy directly.

80. *Probability Charts for Destructive Tests.* Sheets mimeographed on one side only, November 1945. 5 p. of introductory text, 4 folding charts. 20.2 × 33 cm.

Tables are given to 2D of $\log\left[1000\, C^{N-M}_{n-c}\, C^{M}_{c} \big/ C^{N}_{n}\right]$ as a function of n for the range of values $N = 50, 100, 200, 400$; $M = 0(1)10, 12, 15$; $c = 0(1)2$; $n < \frac{1}{4}N$.

Values for the limiting case $N = \infty$ are also tabulated as a function of n/N.

89. *Solution of Integral Equations occurring in an Aerodynamical Problem.* Photo-offset print of handwriting and machine printed tables, July 1945. 17 p. 20.5 × 33 cm.

The actual tables consist of 2 p. only, on which are tabulated:

in Table I, $K'(x)$, $G(x)$, $G'(x)$, $S(x)$, $U(x)$ and $W(x)$ for $x = [0(.2)10; 5D]$;

in Table II, $\omega'(x)$, $\lambda'(x)$ and $f(x)$ for $x = [0(.2)10; 5D]$. These functions are defined as follows:

$$K'(x) = \frac{2\sqrt{2}}{\pi}\frac{1}{2x\sqrt{x+2}}\{(1 + x)E_1(k) - F_1(k)\}$$

$$G(x) = \frac{2\sqrt{2}}{\pi}\frac{1}{\sqrt{x+2}}F_1(k), \qquad G'(x) = \frac{2\sqrt{2}}{\pi}\frac{1}{2x\sqrt{x+2}}\{E_1(k) - F_1(k)\},$$

where $E_1(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{\frac{1}{2}} d\phi$, and $F_1(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{-\frac{1}{2}} d\phi$ are the elliptic integrals of the second and first kind respectively and $k^2 = x/(2+x)$.

From these S, U and W are derived by means of the equations

$$S(x) = K'(x) - \int_0^x K'(x-y)S(y)dy, \qquad U(x) = G(x) - \int_0^x G(x-y)S(y)dy,$$

$$W(x) = -U'(x) = \{G(0)S(x) - G'(x)\} + \int_0^x G'(x-y)S(y)dy.$$

For the functions in Table II,

$$\omega'(x) = \frac{2\sqrt{2}}{\pi} \frac{1}{\sqrt{x+2}} \{(x+2)E_1(k) - F_1(k)\},$$

$$\lambda'(x) = \frac{\sqrt{2}}{\pi} \frac{1}{x\sqrt{x+2}} \{(2x^2 + 4x + 1)E_1(k) - (2x+1)F_1(k)\},$$

and $f(x)$ is defined by the integral equation: $f(x) = \omega'(x) - \int_0^x \lambda'(x-y)f(y)dy$.

The problems from which the two tables arise are essentially the following:

(i) To solve the integral equation $g(z) = \int_0^z \frac{(z+1-t)}{\{(z-t)(z+2-t)\}^{\frac{1}{2}}} h(t)dt$ for $h(z)$ in terms of the general function $g(z)$. The solution is

$$h(z) = w(z) - \int_0^z S(z-t)w(t)dt$$

where $w(s) = \frac{\sqrt{2}}{\pi} v'(s)$, and $v(s) = \int_0^s g(z)(s-z)^{-\frac{1}{2}}dz$ and so can be obtained by quadrature for any given function $g(z)$.

(ii) To determine the function $f(z)$ from the equations $f(z) = \int_0^z k_2(z-t)h(t)dt$, where $1 = \int_0^z k_1(z-t)h(t)dt$, and $k_1(x) = \frac{(x+1)^2}{\{x(x+2)\}^{\frac{1}{2}}}$, $k_2(x) = \frac{(x+1)}{\{x(x+2)\}^{\frac{1}{2}}}$.

Simple elimination leads to the equation

$$\int_0^z k_1(z-t)f(t)dt = \int_0^z k_2(t)dt = \{z(z+2)\}^{\frac{1}{2}},$$

a form identical with the first problem, the general solution of which could be used to compute $f(z)$.

The main interest in the report lies in the method of numerical solution for the integral equations of the second kind defining $S(z)$ and $f(z)$. A simple direct method is developed by replacing the integral by an accurate quadrature. It is also shown that the following explicit formulae for $S(z)$, $U(z)$ and $f(z)$ may be obtained by the application of the Laplace transformation:

$$S(z) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{e^{u(1-z)}}{K_1^2(u) + \pi^2 I_1^2(u)} \frac{du}{\sqrt{u}}, \qquad U(z) = \int_0^\infty \frac{e^{-uz}}{K_1^2(u) + \pi^2 I_1^2(u)} \frac{du}{u^2}$$

and $f(z) = e^{-.6435z}(1.2120 \cos .5012z + .1898 \sin .5012z) - \int_0^\infty \frac{e^{-uz}}{K_1'^2(u) + \pi^2 I_1'^2(u)} \frac{du}{u^2}$. Here K_1, I_1 are the Bessel functions of pure imaginary argument, primes denote derivation with respect to the argument, u, and the first term in $f(z)$ arises from the fact that $K_1'(z)$ has 2 (and only 2) zeros in the complex plane: $z = -.6435 \pm .5012i$. These expressions were used to provide an independent check on the accuracy of the numerical solution, and it is difficult to conceive a check which is more satisfactory. They do not, however, appear to be particularly suitable for systematic numerical computation.

This problem has had a certain amount of publicity owing to the disparity between the original estimate of 300 man-years, made by the Establishment concerned, and the actual time of 50 hours taken by Admiralty Computing Service.

96. *Computation of an Integral occurring in the Theory of Water Waves.* Photo-offset of handwriting and machine printed tables, September 1945. 6 p. 20.5 × 32 cm.

The theory demands numerical values of

$$C(u) = \int_0^\infty \cos \sigma u \operatorname{sech} kh \, d\sigma$$

where $\sigma^2 = gk \tanh kh$, for various values of the parameter h, g being a constant. It is readily seen that a single table of

$$f(v) = \sqrt{\frac{h}{g}}\, C(v) = \int_0^\infty \cos vy \operatorname{sech} x \, dy$$

will enable $C(u)$ to be calculated quickly and easily for any combination of h, g and u.

$f(v)$ has accordingly been tabulated for v = [0(.01)5(.1)10; 4D]; no differences are given, but first differences are always small.

A short auxiliary table is included of sech x with argument y where $y^2 = x \tanh x$; sech x to 4D, with δ^2 for y = 0(.1)3.5.

[97]. *Tables of the Incomplete Airy Integral.* Photo-offset typescript and machine printed figures, April 1946. 5 p. of tables (photo reduced machine figures) and 10 p. of introductory text. 20.8 × 32.6 cm. See RMT **333**.

This report contains tables of the integral

$$F(x, y) = \frac{1}{\pi}\int_0^\pi \cos (xt - yt^3)dt$$

for $x = -2.5(.1) + 4.5$; y = [0(.02)1; 4D]. No Δ given, but the table can be interpolated in the x-direction using second differences; accurate interpolation in the y-direction requires the use of fourth differences, but the use of second differences only will give rise to a maximum error of 2 units only. The last figure tabulated should not be in error by more than one unit, and all the computations have been done in units of the seventh decimal, though two of these were often lost in the course of the integrations.

The title of the report is obtained by considering the function in the form:

$$F(x, y) = \frac{1}{\pi Y}\int_0^{\pi Y} \cos (XT - \tfrac{1}{3}T^3)dT$$

where $Y = (3y)^{1/3}$ and $X = x/Y$. It is thus seen that, for the purpose of extending the tabulation of the complete integral, the present choice of variables is not ideal.

The report contains an account of the methods of computation used and gives the necessary ascending and asymptotic series. The principal method of solution was the direct numerical integration in the x-direction of the differential equation satisfied by the function

$$\frac{\partial^2 F}{\partial x^2} + \frac{x}{3y} F = \frac{1}{3\pi y} \sin (x\pi - y\pi^3).$$

A detailed account is given of the application of the National accounting machine to the integration of second-order differential equations of this type; the method is one which will have great value for use with automatic digital computing machines.

Special methods had to be developed for use for small values of y, and generally the function is one of surprising computational difficulty.

101, 107, [109, 110, 111]. H. Kober, *Dictionary of Conformal Representations, Parts I–II.* Mimeographed on one side of leaves, 1945, 1946. 36 and 48 leaves. 20.2 × 33 cm.

Each page of this dictionary is divided into two columns; that on the left gives diagrams or formulae relating to z-plane while that on the right gives the corresponding diagrams or formulae for the $w = f(z)$ plane.

In Part I the cases of the linear ($f(z) = az + b$) and bilinear ($f(z) = (az + b)/(cz + d)$) transformations are discussed. For instance, explicit formulae giving the actual transformation which carries any two non-intersecting circles into two concentric circles are given.

In Part II, Algebraic functions and z^α for real values of α are discussed. Included in this part is a section on JOUKOVSKI's transformation $w = az + b/z$ and its generalisations.

This dictionary will be completed by the issue of three further parts:

109 Part III, *Exponential Functions and some related Functions,*
110 Part IV, *Schwarz-Christoffel Transformations,*
111 Part V, *Higher Transcendental Functions.*

112. ALAN BAXTER (1910–1947), *The Fourier Transformer.* 1947.

This is a machine of mixed electrical and mechanical design which evaluates the Fourier transforms $C(n) = \int_0^\infty f(n) \cos nx\, dx$, and $S(n) = \int_0^\infty f(n) \sin nx\, dx$ of a given function $f(x)$. The integrations are carried out electrically but the selection of the wave-number n is mechanical. The input function is followed manually and the motion translated into a proportional A.C. voltage (50 c/s) by an inductance potentiometer. A power supply of equal voltage is derived from a servo-operated Variac transformer. This feeds simultaneously 22 magslip resolvers, each of which multiplies the voltage by the sine and cosine of the particular angle at which its rotor lies. These voltages derived from the magslip resolvers, proportional to $f(x) \cos nx$ and $f(x) \sin nx$, are integrated by modified sub-standard K.W.H. meters.

Each magslip rotates continuously during the transit of the input function, to produce the appropriate angle nx, the angles being selected by a system of gear boxes. The range of wave numbers n is from $\frac{1}{16}$ to 128, a figure which can be further increased if the function is subdivided. One traverse of the function occupies 10 minutes, and produces simultaneously 22 cosine integrals and 21 sine integrals whose wave numbers cover a range of 4:1. The full range from $\frac{1}{16}$ to 128 is covered, if required, by repeated following of the input function.

Additional dispositions of the gear boxes enable the density of wave-numbers in any particular range to be increased at first fourfold, and then a further tenfold, allowing for close investigation of any particularly interesting regions of the transform. It is hoped that the errors in the transform will be less than 1 per cent of its peak value.

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[1] Other aspects of the work of Admiralty Computing Service are described in an article by the present authors, in *Nature*, v. 157, May 4, 1946, p. 571–573; see *MTAC*, v. 2, p. 188. See also A. ERDÉLYI and JOHN TODD, *Nature*, v. 158, 1946, p. 690; AÇS **115**.

RECENT MATHEMATICAL TABLES

Seven reviews of RMT are to be found in our introductory article "Admiralty Computing Service," **7, 9, 18, 20, 80, 89, 96.**

400[C].—JOSEF KŘOVÁK, *Achtstellige Logarithmische Tafel der Zahlen. Osmimístné Logaritmické Tabulky Čísel.* Prague, Geographic Institute of the Minister of the Interior, 1940. iv, 26 p. 14.6 × 21.1 cm.

This little pamphlet is divided into two parts. In the first part (p. 2–14) the arguments are the logarithmic mantissae from 0000 to 6389 corresponding to the numbers 10 000 000 to 43 541 160. In the second part (p. 15–26) are given the mantissae of log N, for $N = 4340(1)10009$. In columns headed d, of each part, are the greatest and least differences which arise in successive lines. Examples in German and Czechish illustrate the interpolation process for getting the 8-figure logarithm of any number.