Last paragraph: "The foregoing Underwood-Elliott Fisher Accounting machine [Model D], a standard business machine, accomplishes more than Babbage expected of his Difference Engine and represents a marked improvement over other machines used to difference tables. The ease with which the operations of the machine can be changed, its comparative simplicity, easy manipulation, and low cost, greatly enhances [*sic*] its practicality and demonstrates [*sic*] how a machine of this type lends itself to specialized computational work. Unquestionably, many more machines of this general nature exist which appear to be unknown to scientific computers. There is no doubt that the great effort currently expended on computational work could be appreciably reduced by stimulating a wider knowledge of the capacities of existing business machines and by promoting a broader extension of their applications."

8. O. AMBLE, "On a principle of connexion of Bush integrators," *Jn. Sci. Instruments*, v. 23, 1946, p. 284–287. See *Math. Rev.*, v. 8, 1947, p. 288.

9. G. HÄGG & T. LAURENT, "A machine for the summation of Fourier series," *Jn. Sci. Instruments*, v. 23, 1946, p. 155–158. See *Math. Rev.*, v. 8, 1947, p. 56.

NOTES

75. BHOLANATH PAL'S TABLES OF ROOTS OF THE EQUATIONS $P_n^m(x) = 0$ AND $dP_n^m(x)/dx = 0$ REGARDED AS EQUATIONS IN n.—These tables are given in Calcutta Math. Soc., *Bull.*, v. 9, no. 2, 1919, p. 95, and v. 10, 1919, p. 188–194. The tables contain 4S values of n, for each of 87 zeros of the equations, $\theta = 15°(15°)45°$, $x = \cos\theta$, except for 9 3S values. The tables give also numerous numerical details of the calculations. In Amer. Math. Soc. *Bull.*, v. 53, Feb. 1947, p. 154–155 it was stated by C. W. HORTON that Pal erred in listing for $P_n^2(x) = 0$ the roots $n = 4.77, 2.26, 1.52$, corresponding respectively to the values $\theta = 15°, 30°, 45°$. Horton also supplied 9 other early zeros which Pal had overlooked. These are for $P_n^0(x) = 0$ and $dP_n^1(x)/dx = 0$, $\theta = 15°(15°)45°$, and for $dP_n^2(x)/dx = 0$, $\theta = 15°, 30°$. These 9 values are given by Horton in tables including exact reprints of Pal's 87 values of the zeros.

Pal points out that the roots of these equations are of very great importance in a number of physical problems involving a conical boundary. In illustration Pal gives references to papers by CARSLAW[1] dealing with scattering of sound waves, and to the discussion by LAMB[2] of the problem of the determination of the oscillations of a sea bounded by parallels of latitude. In his computations Pal used an asymptotic expansion due to G. N. WATSON (Camb. Phil. Soc., *Trans.*, v. 22, 1918, p. 277–308).

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[1] H. S. CARSLAW, *Math. Annalen*, v. 75, 1914, p. 133f., 592; *Phil. Mag.*, s. 6, v. 20, 1910, p. 690–691.

[2] H. LAMB, *Hydrodynamics*, third ed., Cambridge, 1906, §200, p. 292; German transl., Leipzig and Berlin, 1907, p. 359f; and sixth ed., 1932, §201, p. 306.

76. MARTIN WIBERG, HIS TABLES AND DIFFERENCE ENGINE.—Brown University has recently acquired a copy of Wiberg's large volume, *Tables de Logarithmes Calculées et Imprimées au moyen de la Machine à Calculer*, Stockholm, Compagnie anonyme de Forsete, 1876. xii, 561 p. 17 × 27 cm. The text and table headings are entirely in French, and the "Avertissement," p. iii–iv, by the author is dated "Stockholm, en novembre 1875." "1875" was the date of a Swedish edition entitled *Logarithmtabeller uträknade*

och tryckte med Räknemaskin (of which there is a copy in the Boston Public Library), and there were also English and German (1876, *Fortschritte*) editions.

There are the following six 7-place tables in the volume: T. I, p. 1–185, log N, $N = 1(1)100000$, with P.P. and values of S and T for $0(50'')1000''$-$(10'')9990''$, i.e. to $2°46'30''$. T. II, p. 186–189, ln N, $N = 1(1)1000$; T. III–IV, p. 190, Multiples, 0(1)99, of ln M, and of $1/M$; T. V, p. 191–291, log sin x and log tan x for $x = 0(1'')5°$; T. VI, p. 292–561, log sin x, log tan x, log cot x, log cos x, for $x = 0(10'')90°$, with Δ and P.P.

The title of Wiberg's volume suggests that the tables were calculated anew, but concerning such calculation there is not one word in the text. The last sentence of the "Avertissement," is "La disposition des tables est principalement en conformité de celles de Bremiker et de Dupuis." The tables thus referred to are, presumably, JEAN DUPUIS, *Tables de Logarithmes à sept Décimales d'après Callet, Véga, Bremiker, etc.*, Paris, 1862, and the 7-place table, VEGA, *Logarithmisch-Trigonometrisches Handbuch*, edited by BREMIKER, Berlin 1856. [Bremiker's own table, first published in 1852, was a 6-place table.] The "disposition" of each of these tables is in broad outline the same as that of Wiberg.

The original of the machine by means of which the tables were "calculated and printed" was examined by a commission, appointed by the French Academy of Sciences, and consisting of MATHIEU, CHASLES & DELAUNAY. Their report to the Academy is published in Acad. d. Sci., Paris, *C.R.*, v. 56, 1863, p. 330–339; this suggests how the tables may have been calculated. It is noted that already in 1863 interest tables had been calculated and published by the aid of the machine. The report concludes: "Nous proposons à l'Académie d'accorder son approbation à cette belle et ingénieuse machine." There is a brief statement about the machine in M. D'OCAGNE, *Le Calcul Simplifié*, third ed., Paris, 1928, p. 77–78. This second Swedish Difference Engine performed exactly the same things as that of the SCHEUTZS, but by the introduction of new mechanical devices it occupied very much less space. A picture of the machine is given on the outside paper cover of this volume of tables. A copy of an edition of the Wiberg interest tables referred to above is in the British Museum, and has the following title: *Med Maskin uträknade och stereotyperade Ränte-tabeller, jemte en Dagräknings-Tabell* . . . Second enlarged ed., Stockholm, 1860.

The printing of tables by the Scheutz machines (in the publications we have previously listed, *MTAC*, v. 2, p. 242–243) was exceedingly unattractive. Wiberg tells us that the delay of more than a decade in publishing a volume of tables, was largely due to long research for achieving typographic excellence. The result, involving an attractive face of type, with top and bottom tails, and of different sizes, is certainly a great advance over the product of the Scheutz machines, some 20 years earlier.

The Wiberg Tables (1876) do not seem to be very generally known; they are not mentioned in J. HENDERSON, *Bibliotheca Tabularum Mathematicarum*, 1926, or in the printed catalogues of Library of Congress, British Museum, Astor Library, Univ. of Edinburgh, Royal Observatory of Edinburgh, Hamburg Math. Soc., Frankfurt City Library, Amer. Math. Soc., and they were not seen by FMR. These Tables are not in the University Libraries of Columbia, Cornell, Harvard, Illinois, Michigan, or in the New York Public

Library or Library of Mass. Institute of Techn. Those two copies of Wiberg's Tables, to which we have referred, seem to be the only ones in this country at present. Does any reader know that the accuracy of the Tables has been investigated?

Wiberg was an exhibitor at the Philadelphia Exhibition of 1876. In U. S. CENTENNIAL COMM. *International Exhibition 1876. Reports and Awards*, ed. by F. A. WALKER, v. 7, Washington, 1880, p. 165, is the following under Wiberg's name and address: "Exhibits logarithmic tables calculated and printed by machine of his invention, by which means the results are free from errors; also a deep-sea drag, of excellent design, for exploration of the sea bottom."

Wiberg was born in 1826, got his doctorate at the University of Lund in 1850, and died at Stockholm in 1905. According to "Poggendorff," v. 3_2, 1898, among his inventions was an apparatus for railway train heating, and a machine for measuring the speed of a train. There is a portrait and sketch of Wiberg in HERMAN HOFBERG, *Svensk Biografiskt Handlexicon*, new ed., Stockholm, 1906, v. 2.

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77. π.—The famous American astronomer and mathematician, SIMON NEWCOMB (1835–1909) once remarked concerning the calculation of π: "Ten decimal places are sufficient to give the circumference of the earth to the fraction of an inch, and thirty decimals would give the circumference of the whole visible universe to a quantity imperceptible with the most powerful telescope." E. KASNER & J. NEWMAN, *Mathematics and the Imagination*, New York, 1940, p. 78.

78. SEISMOLOGICAL TABLES INVOLVING SIMPLE MATHEMATICAL FUNCTIONS.—The tables about to be analyzed are in *Prince* BORIS BORISOVICH GALITZIN, *Seismometrische Tabellen . . . Nachtrag zu der Abhandlung "Ueber ein neues aperiodisches Horizontalpendul mit galvanometrischer Fernregistrierung," Comptes Rendus des séances de la Commission Seismique Permanente*, v. 4, part 1, St. Petersburg, 1911. ii, 266 p. The volume contains 17 tables all of which may be described as if they were simple mathematical functions. We indicate the contents of Tables **1–9**.

T. 1, p. 19–21, $(1 - x^2)^{\frac{1}{2}}$, and $e^{\pi t}$, where $t = (1 + x^2)^{\frac{1}{2}}/x$, for $x^2 = [.01(.01)1; 2\text{–}3S]$.
T. 2, p. 23–64, T_p/T, for $T_p = 1(1)40$, $T = [10.1(.1)30; 3D]$, Δ.
T. 3, p. 65–71, $\log (1 + x^2)$, for $x = [.01(.01)4; 4D]$, Δ.
T. 4, p. 73–82, $\log [2x/(1 + x^2)]^2$, for $x = [.01(.01)4; 4D]$, Δ.
T. 5, p. 83–207, $\log U$, $U = (1 + x^2)[1 - 2xm^2/(1 + x^2)]^{\frac{1}{2}}$ for $x = [.01(.01)2; 4D]$, Δ, and $m^2 = -.1(.01) + .2, .6(.01).9$.
T. 6, p. 209–213, $(2\pi)^{-1} \tan^{-1} [2hx/(x^2 - 1)]$, $h = (1 - m^2)^{\frac{1}{2}}$, for $x = [.1(.1)4; 3D]$, Δ, and $m^2 = -.2(.1) + .9$.
T. 7, p. 215–217, $(2\pi)^{-1} \tan^{-1} [2x/(x^2 - 1)] + \frac{1}{4}$, for $x = [.1(.1)4; 3D]$, Δ.
T. 8, p. 219–222, $\Delta m = \frac{1}{3}m^3/D^2 - \frac{1}{5}m^5/D^4 + \frac{1}{7}m^7/D^6$, $D = 10^3$, $m = [51(1)400; 1D]$.
T. 9, p. 223–232, $\log (1 + .5372x^2)^{\frac{1}{2}}$, for $x = [.001(.001).8; 5D]$, Δ.

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79. Tables of 2^n.—A table for $n = 1(1)120$ is given by Peters & Stein, *Zehnstellige Logarithmentafel*, v. 1, *Anhang*, Berlin, 1922, p. 13–32; for $n = 1(1)140$ by H. W. Weigel, *$x^n + y^n = z^n$? Die elementare Lösung des Fermat-Problems.* Leipzig, 1933; for $n = 1(1)50$, 52(4)100(5)150(6)180 by BAASMTC, *Table of Powers.* Cambridge, 1940; for $n = 1(1)256$ by Aleksander Katz, *Riveon Lematematika*, v. 1, April, 1947, p. 83–85; for $n = 1(12)721$ by William Shanks, *Contributions to Mathematics Comprising chiefly the Rectification of the Circle.* London, 1853, p. 90–95. In *MTAC*, v. 2, p. 246, references were made to manuscript tables, for $n = 1(2)1207$ by Dr. J. W. Wrench, Jr., and for non-consecutive values of n up to 671 by Professor H. S. Uhler. In *Intermédiaire des Recherches Mathématiques*, v. 2, July 1946, p. 73, no. 0550, what purports to be the value of 2^{600} is given by D. Fau who asks if higher powers of 2 have been calculated. The seventy-sixth digit in the 181-digit value given should be 2, not 8, and the hundred and fifty-fifth and fifty-sixth digits 86, should be 68. Professor Uhler's later calculation has been to $n = 1000$, and checked as agreeing with the value obtained by Dr. Wrench in each of its 302 digits.

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QUERIES

23. Hendrik Anjema.—What is known besides facts indicated below concerning this author of *Table of Divisors of all the Natural Numbers from 1 to 10000*, Leyden, 1767 (of which there were also Dutch, French, German and Latin editions of the same date)? In this volume's "Avertisement of the Booksellors" are the following notes regarding Anjema, "After having taught with success & applause, for several years, the Mathematicks in the University of Franequer, he resolved to devote the leisure hours, which an Employment given by the states of Friesland had left him, to the advantage of his old Disciples. He formed the design, of giving a Table of Divisors of all the natural numbers to the amount of 100 000, & he had already brought it so far as 10000, when unhappily he died." What was the year of death of the author of this posthumously published work? A. v. Braunmühl, in M. Cantor, *Vorlesungen über Geschichte der Mathematik*, v. 4, 1908, misspells the name as "Ajema" (p. 434, 1099).

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QUERIES—REPLIES

30. Log Log Tables (Q4, v. 1, p. 131; QR9, p. 336, 12, p. 373).—In Marcel Boll, *Tables Numériques Universelles*, Paris, 1947, p. 446–449, there is a 5D table of log log x, for $x = 1.001(.001)1.18(.002)1.3(.005)1.5(.01)2(.02)3(.05)5(.1)10(.5)20(1)50(2)100(5)200(10)300(20)500(50)1000(100)2000(200)5000(500)10000(1000)20000(5000)50000(10000)100000$.

In *Electrical World*, v. 122, Oct. 14, 1944, p. 118–119, is an article by Jerry Ailinger, "24-scale slide rule solves vector problems." This "Rota-Vec-Trig" rotating slide rule invented and designed by the author is designated as a log log vector and trigonometric slide rule. For greater flexibility there are log log scales in the hexagonal sliding section. There are also new log log scales of decimal quantities of full unit length for greater accuracy,