MATHEMATICAL TABLES—ERRATA

References have been made to Errata in the article "On a Scarce Factor Table" (Beeger, Chernac, Legendre); and in RMT **428** (Boll, De Morgan), **429** (Vuagnat), **432** (Lambert), **434** (Kraïtchik, Cunningham & Woodall), **438** (Yarden), **439** (Kraïtchik, Yarden), **445** (Bateman), **446** (Reissner), **447** (Dwight); UMT **60** (Thomson); N **75** (Pal).

110. A. N. Dinnik. *Tablitsi Besselevikh Funktsiĭ drobovogo poriadku* [Tables of Bessel Functions of fractional order], Vseukrain'ska Akad. N., *Prirodnicho-tekhnichniĭ viddil*, Kiev. 1933. Compare *MTAC*, v. 1, p. 287.

On p. 23 Dinnik gives the first 5 zeros of $J_{\pm 5/6}(x)$, $J_{\pm 1/6}(x)$ to 3D. The following errors of more than a unit in the last place were found by recomputation, either by interpolation or from the series for $J_\nu(x)$: first zero, $J_{-5/6}(x)$, *for* 0.844, *read* 0.849; second zero, $J_{-5/6}(x)$, *for* 4.736, *read* 4.136; second zero, $J_{-1/6}(x)$, *for* 5.344, *read* 5.257; fifth zero $J_{-1/6}(x)$ *for* 14.618, *read* 14.668; fifth zero, $J_{1/6}(x)$, *for* 15.184, *read* 15.192; fifth zero, $J_{5/6}(x)$, *for* 16.202, *read* 16.218.

NBSMTP

Editorial Note: In Novocherkask, Donskoĭ Politekh. Institut, *Izvestiia*, v. 2, 1913, p. 356, Dinnik gives the correct value, 5.26, for the second zero of $J_{-1/6}(x)$.

111. FMR, *An Index of Mathematical Tables*. 1946. See *MTAC*, v. 2, p. 13–18, 136, 178–181, 219–220, 277–278.

On p. 235, section 16.11 $P_n(x)$, line 3, "$n = 20$" should replace "$n = 18$" in the sentence "The first forms are given to $n = 18$ in Prévost 1933." In fact, Prévost gives the exact algebraic expressions for $P_0(u)$, $\cdots$, $P_{20}(u)$ on p. 156 and also the Fourier series for $P_0(\cos\theta)$, $\cdots$, $P_{18}(\cos\theta)$, on p. 157. This latter fact should have been noted on p. 236, section 16.21 $P_n(\cos\theta)$.

This information is given correctly in NBSMTP, *Tables of Associated Legendre Functions*, 1945, p. xli, item 28. I checked Egersdörfer's expressions for $P_n(u)$, p. xxxvii, item 10, by comparison with Prévost's up to $n = 20$ and found that they agree.

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112. P. R. E. Jahnke & F. Emde, *Tables of Functions with Formulae and Curves*, 1933 (p. 319), all later editions p. 269. See *MTAC*, v. 1, p. 391f; v. 2, p. 47.

The error found in line − 5, on the page thus referred to, occurs in the numerical coefficient of the term containing $1/n^{z+5}$ in the expansion of $\zeta(z)$; *for* 1/3024, *read* 1/30240. This particular coefficient is equal to $B_3/6!$, the general value being $B_r/(2r)!$, where B_r is the rth Bernoulli number (see p. 322 or 272).

This error was noted by Dr. R. H. Healey and myself in *Amalgamated Wireless Australasia (A.W.A.) Technical Rev.*, v. 6, 1943, p. 134.

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113. Marchant Calculating Machine Co., *Cube Root Divisors*. Publication no. 68, 1944.

The review, *MTAC*, v. 1, p. 356, states that one application of the process will give the required root correct to at least 5S. This is a proper statement on the usual assumption that the 5th-figure error will not exceed 1. However, this table, computed by the writer, states that the root differs from the true one by less than 5 in the sixth figure. Because 6-figure divisors are used which introduce a possible rounding error of 5 in 7th place, the resulting relative error at various points of the table may cause the calculated root to be in error by as much as 7 in 6th place if the divisors are taken from col. 3 for N of 192.5 or more. This is because the bound of error due to spacing of the N's in col. A is about 4 in 6th place for values close to the mid-points of col. A. A check of roots at such mid-points starting upwards from $N = 192.5$ shows that at $N = 242$ using as "nearest divisor" that for 244, the error of the root is 7 in 6th place; the divisor for 244 has a 7th-place error of 5. There may be other cases.

The table readily may be brought within the error-limit of 5 in 6th place by using 7-place divisors in col. 3 for arguments above 191. A computation of such 7-place divisors has been made by multiplying the constant $3 \times 100^{2/3}$ by the values of $x^{2/3}$ taken from NBSMTP, *Tables of Fractional Powers* (see *MTAC*, v. 2, p. 205), for x equal to the arguments of col. A $\times 10^{-2}$. This recalculation shows no errors of the original amounts. The desired 7th-figure terminals will be supplied upon application to the writer at 2254 Bancroft Way, Berkeley 4, Calif.

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114. J. de MENDIZÁBAL TAMBORREL, *Tables des Logarithmes à Huit Décimales des Nombres de 1 à 125000· · ·*. Paris, 1891.

The 10,000 logarithms from 100,000 to 110,000 in the above tables were compared with the proof of a table of 8-figure logarithms appearing in a new table now in press for Messrs. Chambers. The only error found was the following:

log 101597 *for* 8083, *read* 8088.

L. J. C.

EDITORIAL NOTE: Since Mendizábal Tamborrel tells us that he copied his logarithms of the numbers 100000 to 108000 from the tables of H. L. F. Schrön, 1799–1875 (see *MTAC*, v. 1, p. 40) who gives the logarithm of this number correctly, the error is a case of miscopying. It may be well to put on record here references to errata in Schrön's tables (of which there were many editions after the first in 1860) as listed in *Archiv d. Math. Phys.* These are as follows: v. 34, 1860, p. 368; v. 35, 1860, p. 120; v. 36, 1861, p. 384; v. 41, 1864, p. 120, 240, 496; v. 43, 1865, p. 120, 244; v. 45, 1866, p. 236; v. 46, 1866, p. 360; v. 47, 1867, p. 120, 362; v. 51, 1870, p. 128.

115. L. M. MILNE-THOMSON & L. J. COMRIE, *Standard Four-Figure Mathematical Tables*. London, 1931. See *MTAC*, v. 1, p. 16, 84, 95, 192, 335, 432.

The following end-figure errors of exactly a unit each were found in the course of preparing a new 6-figure edition of Chambers' *Mathematical Tables*. On page 196:

	$\sec^{-1}x$		$\text{cosec}^{-1}x$	
x	*For*	*Read*	*For*	*Read*
1.72	.9505.	.9504.	.6203·	.6204·
1.73	.9546.	.9545.	.6162·	.6163·
1.74	.9586·	.9585·	.6122.	.6123.

These errors occurred in the course of building up 7-figure values of these functions from their first differences; two compensating errors of 1000 in the seventh decimal (1 in the fourth decimal) were introduced. As $\text{cosec}^{-1}x = \frac{1}{2}\pi - \sec^{-1}x$, both functions were affected. Such errors could not occur with the use (introduced since these tables were published) of the National machine for differencing and subtabulation. No other errors are known in these tables.

L. J. C.

116. NBSMTP, *Table of Natural Logarithms*, v. 4, 1941.

P. 462, argument 9.6061, *for* 2.26239 48763 487638, *read* 2.26239 83133 487638.

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EDITORIAL NOTES: Dr. Lowan reported that every surmise as to the cause of this serious error has had no real basis in the evidence at hand.

With regard to the error noted in MTE 109, p. 168, where 5.33452 58209 12879 must replace 5.33452 58202 12879, Dr. Lowan wrote as follows: "The curious thing about this error is that both the original manuscript page 168 and the corresponding negative are correct. Moreover in two of the fifteen copies of the table of exponential functions available at the NBSMTP the correct though faint digit 9 appears in the group 58209. It thus appears that an extremely careless worker tampered with the plate, converting the 9 to a 2. It is fortunate from our point of view that in making this change, the new figure 2 is conspicuously different from the typed 2; it is precisely this fact which made it possible for us to discover this error."

Concerning the error on p. 304, MTE 109, Dr. Lowan wrote: "The original manuscript is correct, although the fourth digit, 9, is rather faint. In the process of retouching the negative the faint digit was changed to an 8 by a careless worker who did not deem it necessary to consult the manuscript page!"

117. *A Webb & Airey—Adams—Bateman—Olsson Error.*

If α and γ are arbitrary constants, which may be complex, in the equation

$$(1) \qquad xy'' + (\gamma - x)y' - \alpha y = 0$$

the complete solution is

$$y = AM(\alpha, \gamma, x) + Bx^{1-\gamma}M(\alpha - \gamma + 1, 2 - \gamma. x)$$

where

$$M(\alpha, \gamma, x) = 1 + \frac{\alpha}{1(\gamma)}x + \frac{\alpha(\alpha+1)}{1(2)(\gamma)(\gamma+1)}x^2 + \cdots$$

except when γ is zero or a negative integer; this case can be excluded with no loss of generality. When γ is a positive integer the coefficient of B is either infinite or identical with the coefficient of A. In this case one of the solutions takes a form analogous to the second solution of Bessel's equation of integral order, and the complete solution of (1) is

$$\begin{aligned}(2) \quad y = {} & C\{\ln x + \psi(1-\alpha) - \psi(\gamma) - \psi(1)\}M(\alpha, \gamma, x) \\ & + C\sum_{n=0}^{\gamma-2} (-1)^{\gamma+n}\Gamma(\gamma)B(n+\alpha-\gamma+1, \gamma-n-1)x^{n+1-\gamma}/n! \\ & + C\left[\frac{\alpha x}{\gamma}\left(\frac{1}{\alpha} - \frac{1}{\gamma} - 1\right) + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\frac{x^2}{2!}\left(\frac{1}{\alpha} + \frac{1}{\alpha+1} - \frac{1}{\gamma} - \frac{1}{\gamma+1} - 1 - \tfrac{1}{2}\right)\right. \\ & + \frac{\alpha(\alpha+1)(\alpha+2)}{\gamma(\gamma+1)(\gamma+2)}\frac{x^3}{3!}\left(\frac{1}{\alpha} + \frac{1}{\alpha+1} + \frac{1}{\alpha+2} - \frac{1}{\gamma} - \frac{1}{\gamma+1} - \frac{1}{\gamma+2}\right. \\ & \left.\left. - 1 - \tfrac{1}{2} - \tfrac{1}{3}\right) + \cdots \text{ to infinity}\right]\end{aligned}$$

where $\psi(\gamma) = \Gamma'(\gamma)/\Gamma(\gamma)$ is the psi function and $B(m, n)$ is the Beta function.

This result was obtained by W. J. ARCHIBALD, "The complete solution of the differential equation for the confluent hypergeometric function," *Phil. Mag.*, s. 7, v. 26, 1938, p. 415–419. An incorrect result was given by H. A. WEBB & JOHN R. AIREY, "The practical importance of the confluent hypergeometric function," *Phil. Mag.*, s. 6, v. 36, 1918, p. 132, since the terms in x^{-1}, x^{-2}, $\cdots x^{-\gamma+1}$ were omitted. This erroneous result was copied by

H. BATEMAN, *Partial Differential Equations of Mathematical Physics*. Cambridge University Press, 1932, p. 457, and by EDWIN P. ADAMS, *Smithsonian Mathematical Formulae and Tables of Elliptic Functions*. Washington, 1922 and 1939, p. 186. In the American edition of Bateman's work, New York, 1944, p. 457, he gave a corrected result, except that his arbitrary constant for the terminating series should not equal the constant for the logarithmic term.

R. GRAN OLSSON, "Tabellen der konfluenten hypergeometrischen Funktion erster und zweiter Art," *Ingenieur-Archiv*, vol. 8, 1937, p. 99–103, also p. 376–377, used the incorrect results of WEBB & AIREY to compute tables of the second solution for $\gamma = 2, 3$.

The corrected series for the Whittaker function was first given by R. STONELEY, "The transmission of Rayleigh waves in a heterogeneous medium," R.A.S., *Mo. Not.*, *Geophys. Suppl.*, v. 3, 1934, p. 227, which he obtained by comparing the terms of the series for $W_{k,m}(z)$, when $2m$ is an integer, which was given by S. GOLDSTEIN, "Operational representation of Whittaker's confluent hypergeometric function and Weber's parabolic cylinder function," London Math. Soc., *Proc.*, s. 2, v. 34, 1932, p. 103–125, with the corrected Webb & Airey formula.

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UNPUBLISHED MATHEMATICAL TABLES

60[F].—JOHN THOMSON, *Table of Twelve-Figure Logarithms*, mss. in the possession of the Royal Astronomical Society. Compare UMT **30**, v. 1, p. 368.

This MS table is described at length by J. W. L. GLAISHER in R.A.S., *Mo. Not.*, v. 34, 1874, p. 447–475. Glaisher lists many errors found in other tables with the aid of this manuscript.

The first 8 figures of the 10,000 12-figure logarithms from 100,000 to 110,000 (all in Volume A_4) were recently compared with the proofs of a table appearing in a new set of tables now in press for Messrs Chambers. The following errors were recorded. Each logarithm in Thomson is written in full, in four triads, and with the characteristic or index.

Number	Triad	*For*	*Read*
100934	1	104	004
101051	2	440	540
101068	2	513	613
101171	Index	4	5
102553	2	958	948
102707	2	599	600
104317	2	455	355
104819	3	112	012
105148	1	121	021
105471	1	123	023
106525	3	342	542
106947	2	178	168
107021	2	468	469
109477	3	287	887

It may not be without interest to record that when the present writer tried in 1926 to see these tables at the library of the Royal Astronomical Society, he was informed that they were out on loan. After persistent efforts by the Secretary, they were received two years later from Dr. Glaisher, who had had them for 47 years! But for this intervention, they might have gone to a bookseller when his library was disposed of after his death in 1928. Now after three quarters of a century, they have once more been useful as an independent check on a published table.

L. J. C.