

EDITORIAL NOTE: All of these corrections agree with the 8D values given in SRE/ACS 82, RMT 252, *MTAC*, v. 2, p. 31. The corresponding 7D values which Koshliakov gives for  $\log x_i$  and  $\log A_i$  must also be amended; for example: for  $\log x_1 = 9.4208764$ , read 9.4208800; for  $\log A_1 = 9.7174704$ , read 9.7174672. Similarly for the 5D table of  $A.f(x_i)$ , where  $f(x) = x/(1 - e^{-2x})$ .

122. WILLIAM SPENCE, *An Essay on the Theory of the Various Orders of Logarithmic Transcendents*, London and Edinburgh, 1809. Also in Spence, *Mathematical Essays*, 1819 and 1820. See *MTAC*, v. 1, p. 457-459; v. 2, p. 180.

On p. 63 of the 1809 edition and p. 64 of the 1819 and 1820 editions is a table of the values of the function  $C_1(x)$  which is apparently the first table of  $\tan^{-1} x$ ;  $x = [1(1)100; 9D]$ . Comparison with the NBSMTP *Table of Arc Tan x* reveals the following errors in Spence:  $x = 38$ , for 1.54448 7135, read 1.54448 6609;  $x = 43$ , for 1.54754 4702, read 1.54754 4703;  $x = 61$ , for 1.54440 4352, read 1.55440 4352. Since Spence did not round off his tables, the corrected values are not rounded off.

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123. J. W. WRENCH, JR., *Values of Stieltjes' sums*  $S_k = \sum_1^{\infty} n^{-k}$ .

In the table of the final 7-digit terminal figures in 37D values of  $S_k$ ,  $k = 2(1)33$ , *MTAC*, v. 2, p. 138, Dr. Wrench has erred in the value 6043727 given in connection with  $S_{21}$ . I find beyond the 30th digit 6043730 459. With this correction, the checking relation (using 37D values of  $S_k$ ) now yields a discrepancy of only  $0.5 \times 10^{-37}$  instead of the previously noted (p. 138)  $3.5 \times 10^{-37}$ .

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## UNPUBLISHED MATHEMATICAL TABLES

63[A, B, D, E].—J. W. WRENCH, JR., *A New Table of  $\pi^n/n!$*  Manuscript in the possession of the author, 4711 Davenport St., N. W., Washington 16, D. C.

The present table consists of values to 205D of  $\pi^n/n!$ , for  $n = 1(1)160$ . The first 110 entries were calculated from my table<sup>1</sup> of  $\pi^{\pm n}$ ,  $n = 1(1)110$ , 205S, at least, using appropriate data from UHLER's tables<sup>2</sup> of  $n!$  and  $1/n!$ . Beyond  $n = 110$  each entry was calculated from its predecessor, and every fifth number was calculated independently as a check.

The table as a whole was checked by computing therefrom 205D approximations to  $\sin \pi$  and  $\cos \pi$ . The respective values found were  $-2 \times 10^{-205}$  and  $-1 - 2 \times 10^{-205}$ . Other data immediately obtainable were 205D values for  $e^{\pm \pi}$ ,  $\sinh \pi$ , and  $\cosh \pi$ . The product of the approximations to  $e^{\pi}$  and  $e^{-\pi}$  was found to equal  $1 - 7.036 \times 10^{-205}$ , nearly; indicating an additive correction to the calculated value of  $e^{-\pi}$  of about  $3.04 \times 10^{-206}$ , which was subsequently confirmed by a second calculation of  $e^{\pm \pi}$  using data carried to about 210D. Comparison of my values of these constants was made with the corresponding data published to 138S by UHLER,<sup>3</sup> and complete agreement to that degree of accuracy was found.

Then the tabular entries were collated with HAYASHI's table<sup>4</sup> of  $\pi^n/n!$ ,  $n = 1(1)16$ , 14-40D, and one error was detected therein, namely, in the 20th decimal place of  $\pi$ . It should be mentioned that Hayashi's decimals are not rounded. Six terminal digit errors ranging in magnitude from one to three units, and corresponding to  $n = 6, 10, 11, 12, 13$ , and 14, were discovered in the companion table of  $\pi^n$ .

On the basis of the present table there were computed tables of  $(\pi/2)^n/n!$  and  $(\pi/4)^n/n!$  to 40D, which were used to test the accuracy of smaller tables of ANDOYER,<sup>5</sup> PETERS & STEIN,<sup>6</sup> and BRANDENBURG.<sup>7</sup> No errors were detected in the first two mentioned; the errors in the last have already been noted by the writer in *MTAC*, v. 2, p. 46-47, MTE 71.

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<sup>1</sup> *MTAC*, v. 1, p. 452, UMT 38.

<sup>2</sup> H. S. UHLER, (a) *Exact Values of the first 200 Factorials*. New Haven, 1944. (b) "A new table of reciprocals of factorials and some derived numbers," *Conn. Acad. Arts and Sci., Trans.*, v. 32, 1937, p. 381-434.

<sup>3</sup> H. S. UHLER, "Special values of  $e^{k\pi}$ ,  $\cosh(k\pi)$ , and  $\sinh(k\pi)$  to 136 figures," *Nat. Acad. Sci., Proc.*, v. 33, 1947, p. 34-41.

<sup>4</sup> K. HAYASHI, *Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen*. Berlin, 1930.

<sup>5</sup> M. H. ANDOYER, *Nouvelles Tables Trigonométriques Fondamentales (Valeurs naturelles)*. Paris, v. 1, 1915.

<sup>6</sup> J. T. PETERS & J. STEIN, *Zehnstellige Logarithmentafel*, v. 1, *Anhang*, p. 95. Berlin, 1922.

<sup>7</sup> H. BRANDENBURG, (a) *Siebenstellige trigonometrische Tafel*. Second ed., Leipzig, 1931; (b) *Sechstellige trigonometrische Tafel*. Ann Arbor, Mich., 1944.

64[F].—R. J. PORTER, *Table giving the complete classification of primitive binary quadratic forms for negative determinants from  $-D = 1001$  to  $-D = 2000$* . Ms. computed from April, 1946 to August, 1947, the property of the author, residing at 266 Pickering Road, Hull, England. Compare *MTAC*, v. 1, p. 451.

The Ms. differs slightly in form from that described in *MTAC*, v. 2, p. 183-184, for  $-D = 2$  to  $-D = 1000$ . In tabulating the periods, only the first half of each is given; the remaining half, comprising forms opposite to those in the first half, can be supplied by inspection.

The following results were obtained, including those already given in the article quoted above. The total number of forms is 44141. Of these, 3391 are given by 161 determinants of only one genus each, in which the longest period (of 69 forms) is afforded by the determinants  $-1831$  and  $-1979$ ; 15550 forms are given by 729 determinants of two genera each (of which, for the determinant  $-1889$ , each contains 36 forms); 19272 forms are afforded by 859 determinants each of four genera, the most frequent type being that where each genus contains six forms—this happens in 140 cases; 5768 forms are given by 243 determinants with eight genera each, none containing more than seven forms; and, finally, there are 160 forms given by 7 determinants ( $-840$ ,  $-1320$ ,  $-1365$ ,  $-1560$ ,  $-1680$ ,  $-1785$ , and  $-1848$ ) which yield sixteen genera (none of more than two forms each).

There are 53 irregular determinants (of which 18 in the first thousand have been previously given). The other 35 are  $-1075$ ,  $-1088$ ,  $-1107$ ,  $-1187$ ,  $-1220$ ,  $-1228$ ,  $-1259$ ,  $-1267$ ,  $-1312$ ,  $-1315$ ,  $-1323$ ,  $-1332$ ,  $-1356$ ,  $-1440$ ,  $-1508$ ,  $-1513$ ,  $-1539$ ,  $-1568$ ,  $-1582$ ,  $-1590$ ,  $-1598$ ,  $-1600$ ,  $-1675$ ,  $-1701$ ,  $-1725$ ,  $-1755$ ,  $-1763$ ,  $-1764$ ,  $-1780$ ,  $-1836$ ,  $-1872$ ,  $-1886$ ,  $-1918$ ,  $-1931$ ,  $-1971$ . The exponents of irregularity are either 2 or 3, 25 of the former and 28 of the latter.

R. J. PORTER

65[U].—NBSCL, *Table of Conversion Angles*. Manuscript prepared in 1947 to be published by the U. S. Hydrographic Office, Washington, D. C. 19 sheets,  $28 \times 37.5$  cm.

This is a table of differences to the nearest tenth of a degree between the Rhumb Line Course and the Great Circle Course (assuming the earth to be a perfect sphere) for latitude of departure =  $0(5^\circ)85^\circ$ , latitude of destination =  $0(5^\circ)90^\circ$ —either in the same or in the opposite hemisphere of the point of departure. Difference between longitudes =  $0(5^\circ)120^\circ$ .

Table 1, p. 15-16, *American Practical Navigator* (originally by N. BOWDITCH), U. S. Hydrographic Office, no. 9, revised edition of 1938, is for Radio Bearing Conversion. For difference of longitude =  $1^{\circ}(0^{\circ}.5)16^{\circ}.5$  and middle latitude  $4^{\circ}(1^{\circ})60^{\circ}$  the table gives the correction to be applied to radio bearing to convert to Mercator bearing. This table is to be replaced by the much more extensive new table.

With the new table

(1) Great circle directions can easily be converted to rhumb line directions for plotting radio bearing on a Mercator chart. Since radio waves travel along great circles, such corrections are necessary.

(2) Rhumb lines may be converted to great circle directions. Usually tangents or chords of the great circle, for about  $5^{\circ}$  of longitude, are sailed. The Table will be of value in planning courses, and will quickly show when the difference between rhumb line courses and great circle courses is significant.

If values are desired within the  $5^{\circ}$  intervals of the table at higher latitudes they may readily be found by double interpolation. The table may be of service in both marine and air navigation.

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## AUTOMATIC COMPUTING MACHINERY

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### TECHNICAL DEVELOPMENTS

The leading article of this issue of *MTAC*, "A Bell Telephone Laboratories' Computing Machine—I," by Dr. FRANZ L. ALT, is our current contribution under this heading.

### DISCUSSIONS

#### *Decimal Point Location in Computing Machines*

**General.** For various reasons, the majority of large scale automatic-sequence digital computers in existence or in the design stage use fixed decimal (or binary) point numbers. In all the machines of which the writer has knowledge, the decimal (or binary) point is located at the extreme left so as to make all numbers fall in the range  $-1$  to  $+1$ . It is the writer's opinion that this choice is not the best, and that location of the decimal (or binary) point several digits away from the extreme left is better in almost every respect. Some of the considerations leading to this opinion are presented in this paper. For the sake of simplicity and definiteness, unless otherwise stated, the discussion refers to a machine in which data are stored in some form of memory which restricts numbers to a specified fixed number of decimal digits. This machine is assumed capable of automatically following a prescribed sequence of instructions which include addition, subtraction, multiplication, division, and transfer operations. The results of the operations are rounded off on the right to the same number of digits as the original data, after positioning the digits in accordance with a single fixed decimal point location. It is further assumed that the machine has no provision for handling excess digits on the left (exceeding capacity of the machine) created by any operation, so that such an occurrence implies an error in programming.