NOTES

99. DANIEL FRIEDRICH ERNST MEISSEL.—There have been several Germans who, while teaching in secondary schools, contributed something notable to mathematics. KARL WEIERSTRASS (1815–1897) taught in secondary schools until he was 41 years old, when, after the publication of important papers in Crelle's *Jn.*, he was appointed a professor of mathematics at the University of Berlin. HERMANN G. GRASSMANN (1809–1877), who spent his life as teacher in a Stettin Gymnasium, was a man of extraordinary versatility, and author of *Ausdehnungslehre* (1844, 1862), a contribution towards the vector analysis of a later generation. Dr. JOHANNES TROPFKE (1866–1939), long director of the Kirschner Oberrealschule in Berlin, wrote the best reference work in existence for the history of elementary mathematics (1902–1940; there was a second ed. of the 7 v. and a third ed. of v. 1–4). We shall now assemble some of the facts concerning the life and publications of another man whose main life work was in secondary schools.

MEISSEL was born in Neustadt-Eberswalde 31 July 1826, and died at Kiel 11 March 1895. His education was begun in the Friedrich Wilhelm-Gymnasium in Berlin and continued as he studied mathematics under JACOBI at the University of Berlin (1847–1850) and got his doctorate at the Univ. of Halle in 1850 (Diss.: *De serie quadam Jacobiana*). During 1852–56 he was in Berlin a docent at the Bergakademie and Bauakademie. From 1856 to 1871 he was director of the Provinzial-Gewerbeschule at Iserlohn, and from 1871 to the time of his death director of the Ober-Realschule in Kiel.

He was the author of two text-books, one on the calculus (1854), and the other on arithmetic and algebra (1861), and of various articles and tables published in Crelle's *Jn.*, Grunert's *Archiv*, Poggendorff's *Annalen*, *Astr. Nachrichten*, *Math. Annalen*, Progr. Iserlohn (three, 1862–70), and Progr. Kiel (twelve, 1874–1894). These Programmen are listed in E. WÖLFFING, *Mathematischer Bücherschatz* (1903), and other publications in the Royal Soc. *Catalogue of Scientific Papers*, and in "POGGENDORFF," v. 2–4. We now present a chronological list of Meissel's mathematical tables.

1_1. *Sammlung mathematischer Tafeln berechnet und herausgegeben. Erste Lieferung* [:*Tafel der Elliptischen Functionen enthaltend die Werte von Log. Vulg. q auf acht Decimalen für das von Minute zu Minute Fortschreitende Argument*]. Iserlohn, Selbstverlag, 1860, ii, 20 p. No more published. $\log q$, θ = [0(1′)90°; 8D]. "The largest single-entry table of elliptic functions in existence"; see *MTAC*, v. 3, p. 275–276; for this, and errors.

1_2. J. L. F. BERTRAND, *Traité de Calcul Différentiel et de Calcul Intégral*, Deuxième partie, *Calcul Intégral*. Paris, 1870, p. 711–717. FLETCHER shows (*MTAC*, v. 3, p. 261) that (because of its errors) the 5D table of $\log q$, $\theta = 0(5')90°$ was an unacknowledged abridgment of Meissel's table. He also points out that this table of Bertrand was then copied by LÉVY (1898), and POTIN (1925).

2_1. *Tafel der Bessel'schen Functionen* I_k^0 *und* I_k^1 *von* $k = 0$ *bis* $k = 15.5$ *berechnet.* Akad. d. Wissen., Berlin, *Abh.* for 1888, Berlin, 1889, p. 4–23. A table of $J_0(k)$ and $J_1(k)$, k = [0(.01)15.5; 12D]. [For errors in $J_0(.62)$, $J_0(1.71)$, $J_0(1.89)$, $J_1(7.87)$ see *MTAC*, v. 1, p. 298.] The first 10 zeros of $J_0(x)$, to 10D, are given on p. 3.

2_2. A. GRAY & G. B. MATHEWS, *A Treatise on Bessel Functions and their Applications to Physics.* London, 1895 [GRAY & MATHEWS, 1895], p. 247–266, 244; second ed. prepared by A. GRAY & T. M. MACROBERT, London, 1922; reprinted 1931 and 1936 [GRAY, MATHEWS & MACROBERT, 1922], p. 267–286, 300. For errors in both eds. in $J_0(.62)$, $J_0(3.07)$, $J_1(7.87)$, and in the first edition $J_0(1.89)$, $J_0(5.90)$, see *MTAC*, v. 1, p. 290, 298.

2_3. E. JAHNKE & F. EMDE, *Funktionentafeln*, 1909; 4D abridgments, p. 111–123; reprints 1923, 1928. Second ed., 1933, p. 228–235, 237. Third ed., 1938, and 1945, p. 156–163, 166.

2_4. The zeros of $J_0(x)$ are included in 3_3.

3_1. *Über die Bessel'schen Functionen* I_k^0 *und* I_k^1. Ober-Realschule in Kiel, *Jahres-Bericht 1889–90*, Kiel, 1890, p. 4. Tables of the first 50 zeros of $J_1(x) = 0$, with corresponding values of $J_0(x_n)$, each to 16D.

3_2. GRAY & MATHEWS, 1895, p. 280; GRAY, MATHEWS & MACROBERT, 1922, p. 301. Meissel's correct value for $x_1^{(9)}$ was copied incorrectly in both editions.

3_3. H. T. DAVIS & W. J. KIRKHAM, "A new table of the zeros of the Bessel functions $J_0(x)$ and $J_1(x)$ with corresponding values of $J_1(x)$ and $J_0(x)$," Amer. Math. Soc., *Bull.*, v. 33, 1927, p. 769–770. A 10D rounding off of Meissel, and other material. There is a 9-unit error in the tenth decimal of $x_1^{(9)}$.

3_4. The Meissel values of 3_3, corrected, were reprinted in BAASMTC, *Bessel Functions*, Part 1, 1937, p. 171.

3_5. E. JAHNKE & F. EMDE, *Funktionentafeln*, 1909; 4D abridgments of $x_0^{(s)}$ and $x_1^{(s)}$, and 4S of $J_0(x_1^{(s)})$, p. 122–123; reprints, 1923, 1928. Second ed., 1933, p. 237. Third ed., 1938, and 1945, p. 166.

4. "Abgekürzte Tafel der Bessel'schen Functionen $I_k^{(h)}$ (Auszug aus einer grösseren Tafel mit 18 Decimalen)," *Astr. Nach.*, v. 128, 1891, cols. 153–156. Mainly a 6D abridgment of no. **7**, of $J_k(h)$, $h = 1(1)10, 16, 20$; $7 \leq k \leq 35$. Also 8D values of $J_k(1000)$ for $k = 967, 968, 981(1)1000$. For errors when $k = 967$ and 968, see *MTAC*, v. 2, p. 47–48.

5. "Neue Entwickelungen über die Bessel'schen Functionen," *Astr. Nachrichten*, v. 129, 1892, cols. 283–284. Table of $10^8 \cdot J_{2n}(n)$, n = [10(1)14; 8D], [15(1)19; 10D], [20, 21; 12D].

6. *Entwurf einer Tafel aus welcher die sechs Elemente einer beliebigen Menge sphärischer Dreiecke sofort entnommen werden können.* Ober-Realschule in Kiel, *Jahres-Bericht* 1893–94, Kiel, 1894, p. 1–7. See *Astron. Nachrichten*, v. 95, 1879, col. 69–74.

7_1. Tables of $J_n(x)$, x = [1(1)24; 18D], $n = 0(1)N - 1$, $17 \leq N \leq 61$, GRAY & MATHEWS, 1895, p. 266–279; GRAY, MATHEWS & MACROBERT, 1922, p. 286–299. These tables were first published here in 1895. There are 5 errors; the following 4 are noted in *MTAC*, v. 1, p. 290: $J_4(5)$, $J_{23}(6)$, $J_{30}(14)$, $J_{31}(16)$. In FMR, *Index*, p. 246 Dr. MILLER notes that in $J_{26}(6)$ the last three digits, 415, should read 507. In recently published Harvard tables $J_n(x)$, for $n = 0(1)39$ and x = [1(1)99; 10D] are given. But J. C. P. MILLER & C. E. GWYTHER are extending Meissel's table to cover the range x = [0(1)100; 18D].

7_2. JAHNKE & EMDE, 1909, 1923, 1928, 4S abridgment, p. 149–157; $17 \leq N \leq 61$. Second ed., 1933, p. 242–249. Third ed. 1938 and 1945, p. 171–177. Error in $J_9(21)$ except in 1945 ed. In this edition however there are 3 other errors: $J_{23}(6)$ and $J_{31}(16)$ should have their last digit values changed by unity, and in $J_{26}(6)$ *for* $.0^{14}4415$, *read* $.0^{14}4507$.

7_3. J. W. STRUTT, BARON RAYLEIGH, "The problem of the whispering gallery," *Phil. Mag.*, s. 6, v. 20, 1910, p. 1002; also in his *Scientific Papers*, v. 5, 1912, p. 618; 4D abridgment of $J_{18}(x)$, $J_{21}(x)$, $x = 11(1)24$.

For a copy of the portrait of Dr. Meissel (1890) which we have reproduced we are greatly indebted to Dr. W. D. C. DANIELSON, director of the Humboldt Schule in Kiel. The portrait is a copy of the one hanging in the Aula of the Schule. This portrait was secured through the friendly cooperation of Professor FRITZ EMDE.

Apart from sources, mentioned above, concerning MEISSEL and his work, we may note the following:

1. C. N. A. KRUEGER (1832–1896), "Todes-Anzeige," *Astron. Nachrichten*, v. 137, 1895, col. 239–240. Krueger here told readers of the *A.N.* that he had a large number of the Meissel's 1860 tables (no. **1**) which he could place at their disposal!

2. *Leopoldina*, Halle, v. 31, 1895, p. 102. Brief note.
3. Ober-Realschule in Kiel, *Jahres-Bericht*, 1894–95, Kiel, 1895.
4. J. H. ECKARDT, *Aus der Schuljungenzeit. Erinnerungen an den Buchwaldschen Hof*, Kiel, 1911. Reported by Dr. DANIELSON.

R. C. A.

100. A NEW FACTORIZATION OF $2^n + 1$.—In a letter dated 20 Dec. 1948, AIMÉ FERRIER (b. 6 May 1896), Principal of Collège de Cusset, Allier, France, sent us the following communication:

"J'ai établi successivement:

(i) que $N = \frac{1}{3}(2^{67} + 1)$ est composé [9.X.48], en appliquant la reciproque de la contraire du theorème de FERMAT. LEHMER ayant établi qu'aucun nombre $2^n + 1$ pour $n < 150$, n'a de diviseur inférieur à 4 600 000, il en résultait que N n'a que 2 facteurs premiers.
(ii) que l'un des diviseurs est $536n + 1$, l'autre $536n + 403$.
(iii) enfin [21.XI.48] que

$$2^{67} + 1 = 3 \cdot 7\ 327\ 657 \cdot 6\ 713\ 103\ 182\ 899."$$

This completes the factorization of $2^n + 1$ up to $n = 70$. Mr. Ferrier is the author of the work on prime numbers which we reviewed *MTAC*, v. 3, p. 95; see also v. 2, p. 341.

101. NEWMAN'S *Mathematical Tracts*.—When we wrote our Note about FRANCIS WILLIAM NEWMAN (1805–1897), and mathematical tables which he had computed and published (*MTAC*, v. 1, p. 454–459), we knew of only the first edition of his *Mathematical Tracts*, Part I, 1888, ii, 1–80 p. and Part II, 1889, iv, 81–139 p. Through information furnished to us by Dr. ALAN FLETCHER we learned that Bowes & Bowes had in 1912 published a reprint of these two parts of the *Tracts*, in a single volume, now out of print. Brown University has recently acquired the last copy in stock.

R. C. A.

102.—TABLES OF $x \tan x$.—Mr. JOHN TODD of King's College, London, has reminded us that we omitted to refer to ENGLUND'S table (*MTAC*, v. 2, p. 20) in our EDITORIAL NOTE, *MTAC*, v. 3, p. 296.

QUERIES

30.—GIRARD AND SNELL TABLES.—D. BIERENS DE HAAN, *Bibliographie Néerlandaise Historique-Scientifique des Ouvrages Importants . . . sur les Sciences Mathématiques et Physiques*, Rome, 1883, lists two mathematical tables by these authors. The first published work of Albert Girard (1595–1632), editor of the works of SIMON STEVIN, was *Tables des Sinvs, Tangentes & Secantes, selon le raid de 100000 parties. Avec un traicté succinct de Trigonometrie. . . .* The Hague, Elzevir, 1626, 120 p., of which there is a copy in Library of Congress. Second editions corrected and enlarged (132 p.) in French and Latin were also published by Elzevir in 1629. There is a copy of this French edition in the New York Public Library. The last published book of Willebrord Snell (1580 or 1581–1626), before his death, was *Canon Triangulorum, hoc est sinuum, tangentium et secantium Tabulae, ad taxationem*