

partial list of those tables with the preparation of which Dr. Lowan was more or less directly connected, appears in R. C. ARCHIBALD, *Mathematical Table Makers*, New York, 1948.

Dr. Lowan was born in Jassy, Roumania, in 1898. He graduated as chemical engineer at the Polytechnic Institute of Bucharest in 1924, the year that he arrived in America; in 1929 he became a naturalized American citizen. During 1928–1931 he was a research physicist for the Combustion Utilities Corp., Linden, N. J., and received the M.Sc. degree from New York University in 1929. His Ph.D. degree was granted by Columbia University in 1934.

As a recognition of Dr. Lowan's notable scientific services we take pleasure in presenting his portrait as our frontispiece for this issue.

EDITORS

104. ROOTS OF CERTAIN TRANSCENDENTAL EQUATIONS.—The FMR, *Index* does not indicate any existing tables of the roots of the equations $\tan x + x = 0$ and $\tan x + 2x = 0$.

The functions arise in a study of the extreme values of

$$f(x) = x \sin x.$$

It is evident that a solution may be obtained with a slight modification of the method developed by EULER¹ and independently by Lord RAYLEIGH.² It is assumed that

$$x = (n + \frac{1}{2})\pi + y = \phi + y \quad (n = 0, 1, 2, \dots),$$

where y is a positive quantity which is small when x is large. Then

$$\tan y = (\phi + y)^{-1}$$

and

$$y = \phi^{-1} - \phi^{-2}y + \phi^{-3}y^2 - \dots - \frac{1}{3}y^3 - \frac{2}{15}y^5 - \frac{17}{315}y^7 - \dots$$

Solving this equation by successive approximation we obtain

$$(1) \quad x = \phi + \phi^{-1} - \frac{4}{3}\phi^{-3} + \frac{53}{15}\phi^{-5} - \frac{1226}{105}\phi^{-7} + \frac{13597}{315}\phi^{-9} - \dots,$$

where $\phi = (n + \frac{1}{2})\pi$.

When $n < 2$, this equation is not suitable for computation, because of slow convergence. The first two roots can best be determined by the use of trigonometric tables. For the higher roots the variation of the tabular values of the tangent become so rapid that use of the series expansion is preferable and the convergence of the series increases rapidly with increasing n .

For the second equation, $\tan x + 2x = 0$, we have

$$\tan y = \frac{1}{2}(\phi + y)^{-1}$$

and

$$(2) \quad x = \phi + \frac{1}{2}\phi^{-1} - \frac{7}{24}\phi^{-3} + \frac{163}{480}\phi^{-5} - \frac{6637}{13440}\phi^{-7} + \dots$$

The convergence of this series for values of $n > 1$ is very rapid. Comparison of the second root calculated from trigonometric tables and by the series

using five terms agree to 5 places of decimals, the value of the fifth term in the series being 10^{-5} .

The values of the roots are given below

	$\tan x = -x$	$\tan x = -2x$
x_1	2.02876	1.83660
x_2	4.91318	4.81584
x_3	7.97867	7.91705
x_4	11.08554	11.04083
x_5	14.20744	14.17243
x_6	17.33638	17.30764
x_7	20.46917	20.44480
x_8	23.60428	23.58314
x_9	26.74092	26.72225
x_{10}	29.87859	29.86187
x_{11}	33.01700	

The calculations were carried to 6D and rounded off to 5D. It is not believed that in any case the last figure is in error by more than one unit.

It has been brought to the author's attention by R. P. EDDY, of the Naval Ordnance Laboratory, that in LOTHAR COLLATZ, *Eigenwertprobleme und ihre numerische Behandlung*, Leipzig, 1945, p. 145, are given 4D values of the first 3 roots of $\tan x = -x$, the first 2 roots of $\tan x = \pm 2x$, and the first 4 roots of $\tan x = x$.

It might also be noted that the first 7 roots, 6–10D, of the equations (i) $\cot x + x = 0$, or $J_{-\frac{3}{2}}(x) = 0$, (ii) $\tan x - x = 0$, or $J_{\frac{3}{2}}(x) = 0$, (iii) $\tan x - 3x/(3 - x^2) = 0$, or $J_{\frac{3}{2}}(x) = 0$, (iv) $\tan x + (3 - x^2)/x$, or $J_{-\frac{3}{2}}(x) = 0$, are to be found in NBSMTP, *Tables of Spherical Bessel Functions*, v. 2, 1947, p. 318–319.

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¹ *MTAC*, v. 1, p. 203; see also p. 336, 459 and v. 2, p. 95.—EDITOR.

² RAYLEIGH, *Theory of Sound*, second ed., rev. and enl. by R. B. LINDSAY. New York, 1945, v. 1, p. 334.

105. NOTE ON THE FACTORS OF $2^n + 1$.—I have established the primality of

$$N = (2^{92} + 1)/17 \\ = 29\ 12800\ 09243\ 61888\ 82115\ 58641.$$

This is the fifth largest prime known, the four largest ones being

$$\begin{array}{ll} 2^{127} - 1 & (\text{LUCAS (?) 1876, FAUQUEMBERGUE 1914}) \\ 2^{107} - 1 & (\text{POWERS, Fauquembergue 1914}) \\ (10^{31} + 1)/11 & (\text{D. H. LEHMER 1927}) \\ 2^{89} - 1 & (\text{Powers 1911, Fauquembergue 1912}) \end{array}$$

My work is in four steps and is based on the converse of FERMAT's theorem as modified by Lehmer, and may be described briefly as follows.

In step I, the sequence $3, 3^2, 3^4, 3^8, \dots$ was computed (mod N) by successive squaring. It was found that

$$3^{2^{90}} \equiv -81 \equiv -3^4 \pmod{N}.$$

Hence

$$3 \cdot 3^{2^{92}} = 3^{17N} \equiv 3^{17} \pmod{N}.$$

That is, N "behaves like a prime."