The work is in 6 chapters entitled I. Numerical Calculation and its Aids, II. Interpolation, III. Approximate Integration and Differentiation, IV. Practical Equation Theory, V. Analysis of Empirical Functions, VI. Approximate Integration of Ordinary Differential Equations.

In simply turning over the pages one is struck with the large number of figures. The book contains as many as 132 figures, an average of 1 to every three pages. This is partly due to the fact that there is more space than usual devoted to graphical methods and computational instruments. In the author's foreword we find: "I still believe it necessary to describe the graphical methods, since I am of the opinion that they are of practical importance." Whether or not the reader feels that this opinion has been shaken by the last two decades he will find the treatment of graphical and instrumental subjects very complete and well done. The account of the planimeter is especially good.

There is much space devoted to definite examples. Not only is the numerical work shown in great detail (indicating that in many cases the computer is using merely paper and pencil) but also the numerical problem is often set up *ab initio* from a physical situation.

Each chapter has about 6 sections. Of the 35 sections of the book over such a wide range of topics it is impossible to discuss each one here. They are for the most part unconnected so that a reader may use the book as a sort of an encyclopaedia. Many of the sections are necessarily too short to give a complete account of the particular subject. The author has wisely preferred to illustrate the fundamental ideas and to leave the reader a short bibliography collected at the end of each section with which further to pursue the topic. Here the reader who is unfamiliar with German will be disappointed, since nearly all the references are to works in German. It would have been very useful had the translator supplied additional if not alternative data in English. Also many of the references are out of date, all being at least 20 years old.

Section 3 on the slide rule was rewritten by the translator to deal with American type rules. No mention is made however of the existence of the circular type rules.

Section 6 is written by T. W. SIMPSON and gives a fine account of the three standard American desk calculators and the two dozen different operational techniques not described in the manufacturers' booklets. As might be expected, there are difficulties in nomenclature not encountered elsewhere in the book. These are largely overcome by careful writing. This section contains the only real tables in the book. These are two tables facilitating square and cube rooting. For description of these tables see *MTAC*, v. 1, p. 356–357, v. 2, p. 350–351.

The printing is excellent with the exception of several of the line drawings which have been poorly reproduced.

This book should find its way into many a computing room. Its use as a text in an upper division college course in numerical methods is also clearly indicated. Many of its sections like §16 (Mean value Methods) are excellent lecture material. Others like §22 (GRAEFFE's Method) could be easily amplified to whatever extent the instructor desired.

D. H. L.

NOTES

103. ARNOLD NOAH LOWAN.—In *Scripta Mathematica*, v. 15, p. 33–63, March, 1949, Dr. Lowan has recently presented a detailed account of the work of the Computation Laboratory, which since its foundation in January 1938 has been under the sponsorship of the National Bureau of Standards. During all of this time Dr. Lowan has been the director of the technical planning of this group. Since, after recent reorganization by the NBS, only a very few computers or planners are left in the CL, Dr. Lowan's extraordinarily successful period as director has this month been brought to a close. The wisdom of the NBS procedure in this regard may be doubted.

The great publication output of members of the NBSCL and preceding organizations, 1939–1949, is listed in the article referred to above, and a partial list of those tables with the preparation of which Dr. Lowan was more or less directly connected, appears in R. C. ARCHIBALD, *Mathematical Table Makers*, New York, 1948.

Dr. Lowan was born in Jassy, Roumania, in 1898. He graduated as chemical engineer at the Polytechnic Institute of Bucharest in 1924, the year that he arrived in America; in 1929 he became a naturalized American citizen. During 1928–1931 he was a research physicist for the Combustion Utilities Corp., Linden, N. J., and received the M.Sc. degree from New York University in 1929. His Ph.D. degree was granted by Columbia University in 1934.

As a recognition of Dr. Lowan's notable scientific services we take pleasure in presenting his portrait as our frontispiece for this issue.

EDITORS

104. ROOTS OF CERTAIN TRANSCENDENTAL EQUATIONS.—The FMR, *Index* does not indicate any existing tables of the roots of the equations $\tan x + x = 0$ and $\tan x + 2x = 0$.

The functions arise in a study of the extreme values of

$$f(x) = x \sin x.$$

It is evident that a solution may be obtained with a slight modification of the method developed by EULER [1] and independently by Lord RAYLEIGH.[2] It is assumed that

$$x = (n + \tfrac{1}{2})\pi + y = \phi + y \quad (n = 0, 1, 2, \cdots),$$

where y is a positive quantity which is small when x is large. Then

$$\tan y = (\phi + y)^{-1}$$

and

$$y = \phi^{-1} - \phi^{-2}y + \phi^{-3}y^2 - \cdots - \frac{1}{3}y^3 - \frac{2}{15}y^5 - \frac{17}{315}y^7 - \cdots.$$

Solving this equation by successive approximation we obtain

$$\text{(1)} \quad x = \phi + \phi^{-1} - \frac{4}{3}\phi^{-3} + \frac{53}{15}\phi^{-5} - \frac{1226}{105}\phi^{-7} + \frac{13597}{315}\phi^{-9} - \cdots,$$

where $\phi = (n + \frac{1}{2})\pi$.

When $n < 2$, this equation is not suitable for computation, because of slow convergence. The first two roots can best be determined by the use of trigonometric tables. For the higher roots the variation of the tabular values of the tangent become so rapid that use of the series expansion is preferable and the convergence of the series increases rapidly with increasing n.

For the second equation, $\tan x + 2x = 0$, we have

$$\tan y = \tfrac{1}{2}(\phi + y)^{-1}$$

and

$$\text{(2)} \qquad x = \phi + \frac{1}{2}\phi^{-1} - \frac{7}{24}\phi^{-3} + \frac{163}{480}\phi^{-5} - \frac{6637}{13440}\phi^{-7} + \cdots.$$

The convergence of this series for values of $n > 1$ is very rapid. Comparison of the second root calculated from trigonometric tables and by the series using five terms agree to 5 places of decimals, the value of the fifth term in the series being 10^{-5}.

The values of the roots are given below

	$\tan x = -x$	$\tan x = -2x$
x_1	2.02876	1.83660
x_2	4.91318	4.81584
x_3	7.97867	7.91705
x_4	11.08554	11.04083
x_5	14.20744	14.17243
x_6	17.33638	17.30764
x_7	20.46917	20.44480
x_8	23.60428	23.58314
x_9	26.74092	26.72225
x_{10}	29.87859	29.86187
x_{11}	33.01700	

The calculations were carried to 6D and rounded off to 5D. It is not believed that in any case the last figure is in error by more than one unit.

It has been brought to the author's attention by R. P. EDDY, of the Naval Ordnance Laboratory, that in LOTHAR COLLATZ, *Eigenwertprobleme und ihre numerische Behandlung*, Leipzig, 1945, p. 145, are given 4D values of the first 3 roots of $\tan x = -x$, the first 2 roots of $\tan x = \pm 2x$, and the first 4 roots of $\tan x = x$.

It might also be noted that the first 7 roots, 6–10D, of the equations (i) $\cot x + x = 0$, or $J_{-\frac{3}{2}}(x) = 0$, (ii) $\tan x - x = 0$, or $J_{\frac{3}{2}}(x) = 0$, (iii) $\tan x - 3x/(3 - x^2) = 0$, or $J_{\frac{5}{2}}(x) = 0$, (iv) $\tan x + (3 - x^2)/x$, or $J_{-\frac{5}{2}}(x) = 0$, are to be found in NBSMTP, *Tables of Spherical Bessel Functions*, v. 2, 1947, p. 318–319.

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[1] *MTAC*, v. 1, p. 203; see also p. 336, 459 and v. 2, p. 95.—EDITOR.

[2] RAYLEIGH, *Theory of Sound*, second ed., rev. and enl. by R. B. LINDSAY. New York, 1945, v. 1, p. 334.

105. NOTE ON THE FACTORS OF $2^n + 1$.—I have established the primality of

$$N = (2^{92} + 1)/17$$
$$= 29\ 12800\ 09243\ 61888\ 82115\ 58641.$$

This is the fifth largest prime known, the four largest ones being

$2^{127} - 1$	(LUCAS (?) 1876, FAUQUEMBERGUE 1914)
$2^{107} - 1$	(POWERS, Fauquembergue 1914)
$(10^{31} + 1)/11$	(D. H. LEHMER 1927)
$2^{89} - 1$	(Powers 1911, Fauquembergue 1912)

My work is in four steps and is based on the converse of FERMAT's theorem as modified by Lehmer, and may be described briefly as follows.

In step I, the sequence 3, 3^2, 3^4, 3^8, $\cdots$ was computed (mod N) by successive squaring. It was found that

$$3^{2^{90}} \equiv -81 \equiv -3^4 \pmod{N}.$$

Hence

$$3 \cdot 3^{2^{92}} = 3^{17N} \equiv 3^{17} \pmod{N}.$$

That is, N "behaves like a prime."

In step II, by combining previously computed values of $3^k \pmod{N}$ in the appropriate way it was found that $3^{N-1} \equiv 1 \pmod{N}$.

In step III the following theorem of Lehmer was used. Let p be a prime factor of $N - 1$ and let $N - 1 = mp = hp^a$. Then if N divides $b^{N-1} - 1$ but is prime to $b^m - 1$, all the divisors of N are of the form $p^a x + 1$. Since, in our case

$$N - 1 = 2^4(2^{88} - 1)/17 = 2^4 \cdot 3 \cdot 5 \cdot 23 \cdot 89 \cdot 353 \cdot 397 \cdot 683 \cdot 2113 \cdot 2931542417,$$

the best value of p is 2931542417. It was found that

$$3^m - 1 \equiv 16\ 79443\ 67320\ 76409\ 93695\ 68642 \pmod{N},$$

a number prime to N. Hence the theorem applies with $b = 3$ and $a = 1$. The factors of N (if any) are of the form $2931542417x + 1$. It is well known that these factors are also of the form $184x + 1$, and hence of the combined form

$$539403804728x + 1.$$

In step IV the 30 numbers of this form not exceeding the square root of N were examined as possible factors of N. All but 8 of these have prime factors under 100 and hence cannot be primes. None of the 8 others divides N; hence N is a prime.

I have also investigated $N_k = (2^k + 1)/3$ for $k = 67$ and 71. Both are composite since

$$(1) \qquad \begin{aligned} 3^{2^{67}} &= 24486\ 86690\ 62763\ 73758 && \pmod{N_{67}} \\ 3^{2^{71}} &= 6\ 00827\ 62146\ 43042\ 03171 && \pmod{N_{71}} \end{aligned}$$

I discovered the factors of

$$N_{67} = 7327657 \cdot 671\ 31031\ 82899$$

later (see *MTAC*, v. 3, p. 451). The factors of N_{71} are unknown.

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EDITORIAL NOTES: The four steps of Mr. FERRIER give a valid and positive proof of the primality of $(2^{92} + 1)/17$. However steps II and IV may be shown to be unnecessary. Step IV may be obviated by the following simple reasoning: If N were composite we would have

$$N = (mr + 1)(nr + 1) = A^2 - B^2,$$

where $r = 539403804728$, $m \geqslant n > 0$ and $2A = (m + n)r + 2$. Since the least prime factor of N exceeds r,

$$2A < r + N/r < 10^{14} < r^2.$$

Now

$$N + 1 = 1 + (mr + 1)(nr + 1) \equiv 2 + (m + n)r \equiv 2A \pmod{r^2}.$$

Hence the remainder on division of $N + 1$ by r^2 must be less than 10^{14}. A very rough calculation shows it to be about $3 \cdot 26 \cdot 10^{22}$ however.

The elimination of the more lengthy step II requires a little more reasoning. By the same method of proof the theorem used in step III may be modified to the following: Let p be a prime factor of $N - 1 = mp = hp^a$, and let k be prime to p ($k = 17$ in the above example). Then if N divides $b^{kN} - b^k$ but is prime to $b(b^m - 1)$, all the factors of N are of the form $p^a x + 1$. Thus step II is unnecessary.

Mr. Ferrier's result (1) was obtained also by D. H. L. in October 1946. This comforting agreement, though no longer of much importance in the presence of the factors of N_{67}, adds extra strength to Mr. Ferrier's assertion that N_{71} is composite.